THE ACCOUNTANT AND Geometrician:

Containing the

DOCTRINE

OF

CIRCULATING DECIMALS, LOGARITHMS, BOOK-KEEPING, and PLANE GEOMETRY.

Deſigned for the Uſe of SCHOOLS, as well as private GENTLEMEN.

By BENJAMIN DONN, H
Teacher of the Mathematicks, *&c.* late of
Bideford, now of the City of *Briſtol.*

Nemo Arithmeticæ ignarus hic ingrediatur.

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N.º 76

BRITISH MUSEUM

TO

The Right Worſhipful the MAYOR,

The Worſhipful the ALDERMEN,

The SHERIFFS,

AND

COMMON-COUNCIL,

OF THE

CITY *of BRISTOL*,

As Encouragers of uſeful LEARNING,

And in grateful Acknowledgment of Favours received,

This VOLUME *is humbly inſcribed,*

By their moſt obedient and obliged Servant,

BENJAMIN DONN.

THE PREFACE.

IN the *General Preface* to the *Mathematical Eſſays*, publiſhed in the Year 1758, the Publick were acquainted with our Deſign of preſenting them with a *New Courſe of Mathematical Learning*; a Thing generally allowed to be much * wanted. For the *great* and *numerous Improvements* which have been made ſince any Courſe has been publiſhed in the *Engliſh* Language, make a *new Courſe* abſolutely neceſſary. For as thoſe Sciences, with their Improvements, are diſperſed in a Multitude of Authors, the young Student knows not how to proceed; and even to a Maſter it is no eaſy Taſk to direct. Whereas, if theſe Sciences are brought into a regular Courſe as they depend on each other, the Student will learn with more *Eaſe*, *Pleaſure*, and *Diſpatch*.

As this, if well executed, is evidently a Work of publick Utility, it is hoped all Mathematical Maſters treat of mixt Mathematicks, or their Application to the ſeveral Neceſſities and Conveniencies of Life; ſuch, for Inſtance, as Navigation, Surveying, Fortification, *&c.*

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* " Mathematical Learning, during the laſt and preſent
" Centuries, has made a moſt ſurpriſing Progreſs, and Truth,
" aſſiſted by the uncontroverted Principles of this Science, has
" baniſhed hypothetical Chicanery from the Regions of Philoſophy.
" It is therefore no Wonder, that a great Variety of
" Authors, deſirous of extending ſo valuable a Branch of Science,
" ſhould have written on every Part of mathematical
" Learning. But ſtill, a Courſe of Mathematicks and Natural
" Philoſophy, tracing the Science from its firſt Principles, and
" exhibiting the Demonſtrations on which each Rule or Problem
" is founded, is ſtill wanting; there being none in our
" own Language that can, with any Shew of Juſtice, be called
" a Courſe of Mathematicks and Natural Philoſophy, according
" to the modern Improvements, and properly adapted to
" Learners. This Defect Mr. Donn has undertaken to ſupply."
Monthly Review for July, 1758.

The Reader will perceive by the Date of theſe ESSAYS, that they were written in the Year 1759: The Delay in the Publication has been occaſioned, by our having been ever ſince that Time engaged in making an actual Survey of the County of *Devon*; on which Account we could not attend to the Publication of them until that Survey was compleated.

London, BENJAMIN DONN.
January 1, 1765.

*** Soon will be opened by the AUTHOR, in the *Library-Houſe*, at *Briſtol*, a *Mathematical School*, in which young Gentlemen and Ladies may be taught, in a rational Manner, any Branch of the Mathematicks and Natural Philoſophy, according to the new Improvements. Such Gentlemen and Ladies as are not inclined to attend at the *School*, may have private Lectures at their own Houſes.

AN
ESSAY
ON THE
DOCTRINE and APPLICATION
OF
Circulating or Infinite Decimals.

CONTAINING

Not only the PRACTICAL RULES, but alſo
the *Reaſons* and *Demonſtrations* of them.

By BENJAMIN DONN.

——— *Si quid noviſti rectius iſtis,*
Candidus imperti; ſi non, his utere mecum.
HOR.

BRITISH MUSEUM

P R E F A C E.

THE Art of *Circulating Decimals* is but of modern Invention, Dr. *Wallis* being undoubtedly the firſt who touched on it, in his *Hiſtory of Algebra*, publiſhed in 1656; in which he made the following curious Remarks, *viz.* " Diviſion in Species (Algebra) is much of the ſame Nature (but more univerſal) with that (in Numbers) of reducing common Fractions to Decimals; which ſometimes end in a determinate Quotient: Which then happens, and then only, when (the Fraction being firſt reduced to the ſmalleſt Terms) the Denominator (or Diviſor) is compounded of no other prime Numbers than 2 and 5, (of which 10 is compounded.) But if the Denominator (ſo reduced) be compounded of any other prime Numbers, (than 2 or 5,) the Quotient will be interminate. —— In which Caſe there is yet always this Concinnity, that, after ſome Time, the Numbers do again return, and circulate in the ſame Order as before: ſometimes in the Repetition of one ſingle Figure; ſometimes of two, or more; but always, if not ſooner, it doth at leaſt begin to return in ſo many Places as are the Number of Units in the Diviſor. The Number of Figures therefore which do thus circulate, is never more than the Number of Units in the Diviſor, wanting one: But many Times it is only an aliquot Part of ſuch Number, or

“ ſome leſſer Number which is not an aliquot
“ Part of it.

“ All which is not ſo to be underſtood, as if
“ this Circulation did always take its Begin-
“ ning from the firſt Place of decimal Frac-
“ tions. For when the Denominator or Divi-
“ ſor is compounded of 2 or 5, or any Powers
“ of theſe, it begins not till ſome Time after;
“ that is, nor till the Influence of theſe Com-
“ ponents ceaſe to operate; that is, not till
“ after ſo many Places as is the Number of ſo
“ many Dimenſions of 2 or 5 aſſumed in that
“ Compoſition.”

Theſe, with two or three more Obſervations of the Doctor's equally curious, excited ſome ingenious Men to purſue the Subject in a more copious Manner. The firſt who began to apply circulating Decimals to arithmetical Computations, was (we believe) the Reverend Mr. *Brown*, in his *Syſtem of decimal Arithmetick*, in the Year 1670, in which he has ſhewn how to multiply ſuch interminate Decimals as have a ſingle Repetend, and which alſo produce a ſingle Repetend in the Product. But this being only a ſingle Caſe of the Practice, Mr. *Cunn* aſſumed the Subject, in his *Treatiſe of Fractions*, publiſhed in the Year 1714; in which Book he has given Examples in both Multiplication and Diviſion of compound Circulates. But having (as Mr. *Malcolm* thinks) choſen to expreſs his Rules in a ſuch Manner, as to ſet them as far out of View as poſſible, Mr. *Malcolm*, in his *Arithmetick*, publiſhed in the Year 1730, attempted to ſet them in a clearer Light, and not only attempted, but ſucceeded. This

Gentleman added ſeveral curious and new Obſervations on this Subject. The only Book of Note remaining unnoticed on this Subject, is the *Decimal Arithmetick* of Mr. *Marſh*, publiſhed in the Year 1742; this Author is very full and clear in explaining the Practice, but has omitted the Demonſtrations. Other Writers on this Subject are, *Jones* 1706, *Wright* 1734, *Martin* 1735, *Pardon* 1738, and *Wingate* by *Dodſon* 1751.

The Doctrine of *Circulating Decimals* is of Uſe in performing decimal Operations, when the greateſt Accuracy is required.

Our principal Aim in this ESSAY has been to give a conciſe, but at the ſame Time an intelligible Diſcourſe, on the Doctrine of *Circulating Decimals*, with the *Inveſtigations* and *Demonſtrations* of the *practical Rules*; which ſome Authors have endeavoured to conceal, and others have been ignorant of. In a Word, we hope the Reader will find as much uſeful Knowledge in this ſhort ESSAY, as in any Six Shilling Book, written expreſſly on the Subject.

Bideford, February 3,
1759.

B. DONN.

ERRATA.

In the Eſſay on *Circulating Decimals.*

P. 8. of the Preface, l. 5 *b*, for *a ſuch* r. *ſuch a.*

In the Eſſay on *Logarithms.*

In the Title, for *Wolfius* r. *Wolfii.*

P. 26. l. 10 *b*, for *Time.* r. *Time,*

In the Eſſay on *Geometry.*

P. 6. l. 7 *b.* for *and* r. *a.*

P. 13. l. 17. r. *to be.*

P. 28. l. 2 *b.* for $x = rr$. read x, rr;

P. 51. the * in the Note refers to Article 111.

P. 81. l. 4. for *Log.* r. *Ruler*;

P. 104. *l.* 17. r. *in the.*

N. B. b ſignifies to count from the Bottom of the Page.

A

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⁂ For the *Contents* of the other ESSAYS in this Volume, ſee at the End of the Preface to each reſpective ESSAY.

The ACCOUNTANT and GEOMETRICIAN.

ESSAY I.

On CIRCULATING DECIMALS.

CHAP. I.

Of NOTATION, *and the Properties of* CIRCULATING DECIMALS.

IT may be remembered by every Perſon who has learnt common Decimals, that, in putting a Vulgar Fraction into a Decimal, (which is done by Diviſion,) though the Operation be continued to any Number of Places, yet, ſometimes, there will be a Remainder; and frequently ſome Figure, or Figures, in the Quotient will be continually repeated *ad infinitum*: Thus, $\frac{1}{3} = .3333$, &c. for ever, where 3 is continually repeated. Alſo, $\frac{2}{11} = .181818$, &c. in which 18 is continually repeated. Such Decimals are ſaid to be *infinite*, or *indeterminate*, or *circulating*.

2. Though in finding the equivalent decimal Expreſſion of any Vulgar Fraction, it is ſufficiently exact to take 5 or 6 decimal Places, even though they circulate, for the Value of the Vulgar Fraction, in moſt Purpoſes of Life; yet, in ſome aſtronomical, and other Calculations, a greater Exactneſs may be required; and how to manage ſuch Decimals more accurately is the Subject of this ESSAY: And, in Order to the more regular treating thereof, it may be neceſſary to give the following Definitions.

3. The circulating Part of a decimal Expreſſion is very properly called the *Repetend*, or *Circulate*.

4. *Repetends* are either ſingle, or compound: A ſingle Repetend is, when one Figure continually circulates, thus, in .333, &c. 3 continually circulates, and is a ſingle Repetend.

5. A *Compound Repetend* conſiſts of two, or more Figures continually circulating, as in .181818, &c. where the Figures 18 are continually repeated. And here note, that for the more ready Writing of Repetends, it may be convenient to diſtinguiſh them, by placing a Dot (.) over the firſt Figure of the Repetend if it be a ſingle one, or over the firſt and laſt if it be a compound Repetend, thus, for .3333, &c. *ad infinitum*, we more compendiouſly write $.\dot{3}$; and for .18181818, &c. *ad infinitum*, we write $.\dot{1}\dot{8}$; and for .234234234, &c. *ad infinitum*, we ſet down $.\dot{2}3\dot{4}$.

Note, Some, inſtead of a Dot, daſh the Circulate, thus, for $\dot{2}3\dot{4}$, they write $.\cancel{2}3\cancel{4}$.

6. Circulating Decimals are either *pure* or *mixt*. A *Pure Repetend*, or *Circulate*, is, when there are no ſignificant Figures but what belong to the Repetend, or ſuch as have an 0, or 0's, betwixt the Repetend and decimal Point; thus, $.\dot{3}$, or $\dot{2}\dot{3}$, or $\dot{1}.8\dot{2}$, or $.00\dot{6}$, or $4\dot{5}.\dot{4}5$, &c.

7. Such *Repetends* as have ſome ſignificant Figure or Figures, prefixed before the Circulation begins, are called *mixt Circulates* or *mixt Repetends*, becauſe ſuch conſiſt of two Parts, a finite, and a circulate, as, $.3\dot{4}$ conſiſts of $.3$ and $.0\dot{4}$ being $= .3 + .0\dot{4}$; alſo, $.5\dot{6}$, or $.23\dot{4}\dot{5}$, or $25.\dot{6}$, or $10.\dot{1}3\dot{2}$, or $2\dot{3}.33$, &c. or $2\dot{5}\dot{4}.54$, &c. are *mixt Circulates.*

8. Repetends are either *ſimilar* or *diſſimilar*. *Similar* Repetends or Circulates are ſuch as conſiſt of the ſame Number of Figures, and begin alſo at the ſame Place, either before or after the decimal Point; thus, $.\dot{2}$ and $.\dot{3}$ are ſimilar Circulates, as are alſo $\dot{2}.3\dot{4}$ and $\dot{4}.3\dot{6}$. As to *diſſimilar* Repetends, they need no Definition, becauſe ſuch as are not *ſimilar* muſt be *diſſimilar*.

9. Decimals are either *certain*, or *incertain*: *Certain Decimals* are ſuch as are either *finite*, or *circulating*, and are equal to ſome Vulgar Fractions. But *incertain Decimals* are ſuch, whoſe Numerators, tho' continued ever ſo far, will not have any conſtant Circulation of Figures: Theſe occur in the Extraction of the Roots of ſurd Numbers, and their Value cannot be expreſſed by any Vulgar Fraction; and therefore, in ſuch Caſes, we muſt be contented to approximate, by continuing the Operation to ſo many decimal Places, that the reſt may be omitted as inconſiderable. And even in certain Decimals, many Times the Repetend conſiſts of ſuch a Number of Places, as to make the Operation very laborious, ſo as to be in ſome Caſes convenient only to continue the Numerator to ſo many Places, as the rejecting the others may cauſe no conſiderable Error; and in ſuch Caſes it may be frequently the beſt Method to work by Vulgar Fractions, inſtead of Decimals.

10. Having laid down the neceſſary Definitions, it may be proper to give the moſt uſeful Properties of circulating Decimals, which we ſhall do in the following Theorems.

Theorem I. Any *finite Decimal* may be conſidered as *infinite*, or *circulating*, by annexing or ſuppoſing o's to be placed continually on the Right Hand of its Numerator; for Example, .45 may be expreſſed thus, $.45\dot{0}00$, &c. *ad infinitum*. For, by annexing o's to the Numerator, by the Nature of Decimals, as many o's muſt be annexed to the Denominator, and ſo the Numerator and Denominator continue in the ſame Ratio to each other, and conſequently, the new, or infinite Decimal, is equal to the given finite one.

11. *Theorem* 2. Any given Repetend, whether ſingle or compound, pure or mixt, may be transformed into another Repetend, conſiſting of an equal, or greater Number of Figures, at Pleaſure, (and ſtill retain its former Value,) by only writing down the given Repetend, as often as is neceſſary for its Transformation, and then dotting off as many Places as are required, for a new Repetend. Thus, $.\dot{4}$ may be transformed to $.\dot{4}\dot{4}$, or $.\dot{4}4\dot{4}$, or $.4\dot{4}$, or $4\dot{4}\dot{4}$, &c. Alſo, $.\dot{5}\dot{7} = .\dot{5}75\dot{7} = .57\dot{5}\dot{7} = .5\dot{7}\dot{5}$, &c. and $.34\dot{3}\dot{8} = .34\dot{3}83\dot{8} = .343\dot{8}\dot{3}$, &c. And this needs no Demonſtration; for it is evident, that in the different Methods of transforming them, as above, there will be the ſame Figures ſuppoſed to be infinitely continued: for Inſtance, in $.\dot{4}$, which ſignifies 4 infinitely continued, 'tis manifeſtly the ſame as if we ſuppoſed two Figures, 44, to be continually annexed on its Right Hand; for $.\dot{4} =$ $.4 +$

$.4 + .04 + .004$, &c. $= .4444$, &c. and $.\dot{4}\dot{4} = .44 + .0044 + .000044$, &c. $= .444444$, &c. as before, and ſo of any other.

12. *Theorem* 3. Diſſimilar Circulates may be made ſimilar, by making all the Repetends begin at the ſame Place where that Repetend begins which is loweſt, either on the right or left of the decimal Point; and then to make them end together, let each Repetend be made to conſiſt (ſee the laſt THEOREM,) of as many Places as the Number of Units in a common Multiple of the ſeveral Numbers of Places in all the given Repetends; but in Order to have no more Places than are neceſſary, let the Multiple be the leaſt poſſible. How to find ſuch a Multiple, ſee Chap. 40. of our MATHEMATICAL ESSAYS.

EXAMPLE I.

$$\left.\begin{array}{r}.\dot{3}\\ .2\dot{7}\\ .04\dot{5}\end{array}\right\}\text{made ſimilar will be}\left\{\begin{array}{l}.33\dot{3}\\ .27\dot{7}\\ .04\dot{5}\end{array}\right.$$

EXAMPLE II.

Diſſimilar.	*Made ſimilar.*
$.\dot{6}$	$.\dot{6}\dot{6}$
$.\dot{2}\dot{3}$	$.\dot{2}\dot{3}$

Here the Number of Places in the given Repetends being 1 and 2, the Multiple is 1 × 2 = 2.

EXAMPLE III.

Diſſimilar.	*Made ſimilar.*
$.521\dot{7}$	$.521\dot{7}7777\dot{7}$
$3.6\dot{4}\dot{3}$	$3.643\dot{4}3434\dot{3}$
$17.\dot{1}2\dot{3}$	$17.123\dot{1}2312\dot{3}$

Here, the Number of Places in the given Repetends being 1, 2, and 3, the Multiple is 1 × 2 × 3 = 6, which is alſo the leaſt poſſible.

The Reaſon of this is ſo evident from THEOREM II. that we need ſay nothing further concerning it.

13. *Theorem* 4. Any pure decimal Circulate is equal to that Vulgar Fraction, whoſe Numerator is the given Circulate, and its Denominator a Number conſiſting of as many 9's as there are Places of Figures in the given Circulate, with as many 0's on the Right of the 9's, as there are 0's betwixt the circulate and decimal Point.*

14.

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* Before we proceed to demonſtrate this Theorem, it will be proper to premiſe: That in any geometrical Progreſſion, decreaſing *ad infinitum*, if we put g = the firſt, or greateſt Term, d = the common Diviſor, then the Sum of the whole Progreſſion will be $\frac{g \times d}{d-1}$. For, by the Note to Art. 441. of the MATHEMATICAL ESSAYS, $\frac{gd-a}{d-1}$ = the Sum of any geometrical Progreſſion, where g = the greateſt Term, a = the leaſt, and d = the common Multiplier in an increaſing Progreſſion, or the common Diviſor in a decreaſing Progreſſion. Now in a Progreſſion decreaſing *ad infinitum*, the laſt, or leaſt Term, muſt be infinitely little, or nothing; ∴ a muſt be repreſented by nothing, and conſequently, $\frac{gd-a}{d-1} = \frac{gd-0}{d-1} = \frac{gd}{d-1}$. Q. E. D.

We are now prepared to proceed to the Demonſtration of the Theorem. Let it then be obſerved, that $.\dot{3}$ may be writ thus, $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$, &c. and $.0\dot{4} = \frac{4}{100} + \frac{4}{1000} + \frac{4}{10000}$, &c. alſo $.00\dot{3}\dot{6} = \frac{36}{100000} + \frac{36}{10000000} + \frac{36}{1000000000}$ &c. Hence it may be obſerved, that in each Series the Numerator of each Term continues the ſame, and therefore the ſeveral

Terms

14. EXAMPLES. $.\dot{4} = \frac{4}{9}$; $.\dot{3} = \frac{3}{9}$; $.0\dot{6} = \frac{6}{90}$; $.00\dot{3} = \frac{3}{900}$; $.\dot{3}\dot{7} = \frac{37}{99}$; $.\dot{3}4\dot{2} = \frac{342}{999}$;

.031

Terms are in * reciprocal Proportion to their Denominators; and therefore the ſeveral finite Decimals, or Terms of which a Circulate is compoſed, form a Series of Terms in geometrical Proportion decreaſing, whoſe common Diviſor which we denote by d, in the firſt of our Series $\frac{3}{10} + \frac{3}{100}$, &c. is $= 10$, (for $10 \times 10 = 100$, and $100 \times 10 = 1000$, &c. multiplying the Denominator of any Fraction being really dividing it) and d in the ſecond Series, $\frac{4}{100} + \frac{4}{1000}$, &c. is $= 10$ alſo; and in the laſt Series, $d = 100$; hence the Sum of any ſuch Series, or circulating pure Decimal is $= \frac{gd}{d-1}$; but d may be repreſented by 1, with as many 0's as there are Places of Figures in the Circulate, the Number of which we will call n; then the Expreſſion $\frac{gd}{d-1}$ becomes $\frac{g \times 1 \text{ with } n, 0\text{'s}}{1 \text{ with } n, 0\text{'s} - 1}$; but 'tis manifeſt, that 1 with n, 0's $- 1 = 9$, &c. taking as many 9's as there are 0's, that is, n, 9's; hence the Expreſſion becomes $\frac{g \times 1 \text{ with } n, 0\text{'s}}{n, 9\text{'s}} = g \times \frac{1 \text{ with } n, 0\text{'s}}{n, 9\text{'s}}$: But g the firſt Term of the Series, or the Circulate, is = that Fraction whoſe Numerator is the given Circulate, and the Denominator 1, with as many 0's as the laſt Figure of the Repetend is diſtant from the decimal Point; that is, putting $r =$ the given Circulate, and $m =$ the Number of Places from the decimal Point to the laſt Figure of the Circulate, $g = \frac{r}{1 \text{ with } m\, 0\text{'s}}$; and $\therefore g \times \frac{1 \text{ with } n, 0\text{'s}}{n, 9\text{'s}} = \frac{r}{1 \text{ with } m, 0\text{'s}} \times \frac{1 \text{ with } n, 0\text{'s}}{n, 9\text{'s}} = {}^{*}\frac{r \times 1 \text{ with } n, 0\text{'s}}{n, 9\text{'s} \times 1 \text{ with } m, 0\text{'s}} =$ by Abbreviation

*Art. 554. of Math. Eſſays.

*Art. 600.

*r,

$.0\dot{3}\dot{1} = \frac{31}{990}$; $.000\dot{1}23\dot{1} = \frac{1231}{9999000}$.

15. *Theorem* 5. Any pure Circulate, in which there is an integral Figure, or integral Figures, is equal to that Vulgar Fraction, whoſe Numerator is the given Circulate, with as many 0's as the Figure in the higheſt Place of the Repetend is diſtant Places of (integral) Figures from the decimal Point; and its Denominator, a Number conſiſting of as many 9's as the Circulate has Places of Figures. †

16.

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*Art.577. of Math. Eſſays. * $\frac{r}{\overline{n,\ 9\text{'s}} \times \overline{1 \text{ with } \overline{m-n},\ 0\text{'s}}}$; which is the ſame as the above Theorem. Q. E. D.

† Demonstration. Things being as in the Demonſtration of the laſt Theorem, the Value of a pure decimal Circulate is $= \frac{r}{\overline{n,\ 9\text{'s}} \times \overline{1 \text{ with } \overline{m-n},\ 0\text{'s}}}$. But when the firſt Figure, or Figure in the higheſt Place of the Circulate, is in the Place next the decimal Point, m is $= n$; and ∵ $m - n = 0$; ∴ the above Expreſſion becomes $\frac{r}{\overline{n,\ 9\text{'s}}}$: Now, from the Nature of Decimals, when the firſt Figure of the Circulate is in the Place of Units, or firſt integral Place, counting from the decimal Point) it muſt be 10 Times the Value it would have been, in the Place of 10ths; and ∵ in ſuch Caſe, the Expreſſion $\frac{r}{\overline{n,\ 9\text{'s}}}$ will be changed into $\frac{r}{\overline{n,\ 9\text{'s}}} \times 10 = {}^{*}\frac{r \times 10}{\overline{n,\ 9\text{'s}}} = \frac{r \text{ with one } 0}{\overline{n,\ 9\text{'s}}}$; and if the firſt Figure of the Circulate is in the Place of Hundreds or ſecond Place from the decimal Point, it muſt be 10 Times as much as if it was in the firſt Place from the decimal Point, and ∵ $= \frac{r \text{ with one } 0}{\overline{n,\ 9\text{'s}}} \times 10 = \frac{r \text{ with two } 0\text{'s}}{\overline{n,\ 9\text{'s}}}$; and if the firſt Figure of the Circulate is in the Place of Thouſands, or third Place from the decimal Point, it muſt be 10 Times the Value it would have been

*Art.600. of Math. Eſſays.

16. Examples. $\dot{3}\cdot\dot{5} = \frac{350}{99}$; $\cdot 23\cdot 23 = \frac{2300}{99}$;

$\dot{3}\dot{5}3\cdot 5 = \frac{35000}{99}$; $\dot{3}2\cdot 1\dot{7} = \frac{321700}{9999}$;

$\dot{1}343\dot{5}9\cdot 13435$, &c. $= \frac{13435000000}{99999}$.

17. *Theorem* 6. Any mixt Circulate, whoſe Repetend conſiſts of decimal Places only, it is manifeſt may have its finite Value found, by finding by *Theorem* 4. the finite Value of its circulating Part, and adding it to the finite Part of the Circulate; or, which is the ſame Thing, it is equal to a Vulgar Fraction whoſe Numerator is the Sum of the Product of the finite Part multiplied by as many 9's as there are Places of Figures in the Repetend, and the Repetend or Circulate itſelf; and the Denominator the ſame as the Denominator of the circulating Part, found by *Theorem* 4. *

been in the ſecond Place, and $\because = \frac{r \text{ with two o's}}{n, 9\text{'s}} \times 10 = \frac{r \text{ with three o's}}{n, 9\text{'s}}$; hence, generally, ſuch Circulate is $= \frac{r \text{ with } p, \text{ o's}}{n, 9\text{'s}}$, ($p$ being put to repreſent the Number of Places from the decimal Point, where the firſt Figures of the Circulate ſtands.) Q. E. D.

[*] Let $f =$ the Numerator of the finite Part, and for $m - n =$ the Number of decimal Places in the finite Part put b; then by the Nature of Decimals, $\frac{f}{1 \text{ with } b, \text{ o's}} =$ the finite Part; and by writing b for $m - n$ in the Expreſſion for the circulating Part, (in Demon. of *Thm.* 1.) $\frac{r}{n, 9\text{'s} \times 1 \text{ with } \overline{m - n}, \text{ o's}}$, it becomes

18. Examples. $.2\dot{3} = \frac{2 \times 9 + 3}{90} = \frac{21}{90}$;

$.3\dot{4}\dot{6} = \frac{3 \times 99 + 46}{990} = \frac{343}{990}$; $23.\dot{4} = \frac{23 \times 9 + 4}{9}$

$= \frac{211}{9}$; $21.3\dot{1}\dot{7} = \frac{213 \times 99 + 17}{990} = \frac{21104}{990}$;

$3.142\dot{5}4\dot{3} = \frac{3142 \times 999 + 543}{999000} = \frac{3139401}{999000}$.

19. *Theorem* 7. Any mixt Circulate, in which the firſt circulating Figure begins in ſome integral Place, may have its finite Value found by finding the finite Value of the circulating Part by *Theorem* 5. and to it adding the finite Part; which Sum may be expreſſed in one Fraction, by multiplying the finite Part by the Denominator found by *Theorem* 5. and then adding the Numerators for a Numerator, and placing that Denominator under, for the required Denominator.

20. One Example will ſhew the Method of uſing this Theorem, and alſo the Reaſon of the Thing. Let the Example be, to find the finite Value of $26\dot{3}\dot{5}3.5$, Here the finite Part is 26000; for

26

$\frac{r}{n,\ 9\text{'s} \times 1 \text{ with } b,\ 0\text{'s}}$; and ∵ the whole Value $= \frac{f}{1 \text{ with } b,\ 0\text{'s}}$

$+ \frac{r}{n,\ 9\text{'s} \times 1 \text{ with } b,\ 0\text{'s}}$ = (by Reduction of Fractions and Addition) $\frac{f \times n,\ 9\text{'s} \times 1 \text{ with } b,\ 0\text{'s} + r \times 1 \text{ with } b,\ 0\text{'s}}{n,\ 9\text{'s} \times 1 \text{ with } b,\ 0\text{'s} \times 1 \text{ with } b,\ 0\text{'s}}$ = (by abbreviating, by dividing both the Numerator and Denominator by 1 with b, 0's) $\frac{f \times n,\ 9\text{'s} + r}{n,\ 9\text{'s} \times 1 \text{ with } b,\ 0\text{'s}}$ Q. E. D.

$26\dot{3}\dot{5}3.5$ is $= 26000 + \dot{3}\dot{5}3.5$; now $\dot{3}\dot{5}3.5 = \frac{35000}{99}$, and $\because 26\dot{3}\dot{5}3.5 = 26000 + \frac{35000}{99}$; 15.

but $26000 = \frac{26000 \times 99}{99}$ (for any Number being both multiplied and divided by the ſame Number, muſt ſtill continue of the ſame Value;) $\because 26\dot{3}\dot{5}3.5 = 26000 + \frac{35000}{99}$ is $=$ ($\frac{26000 \times 99}{99} + \frac{35000}{99} =$) $\frac{26000 \times 99 + 35000}{99} = \frac{2609000}{99}$.

21. It may not be improper to remind the Reader, that in the firſt Eſſay, Art. 75 and 126 of the MATHEMATICAL ESSAYS, are very ready Methods of multiplying, or dividing, by any Number of Nines.

22. *Theorem* 8. A Series of 9's ſuppoſed to be infinitely continued, is equal to an Unit in the next Place on the Left-hand, thus, $.\dot{9} = 1$, $.0\dot{9} = .1$, $.00\dot{9} = .01$, &c. This might have been given by Way of Corollary to *Theorem* 4. for from that Theorem it is manifeſt, that $.\dot{9} = \frac{9}{9} = 1$, $.0\dot{9} = \frac{9}{90} = \frac{1}{10} = .1$, $.00\dot{9} = \frac{9}{900} = \frac{1}{100} = .01$; and hence the Truth of the above Theorem plainly appears. Or the Reaſon of it may be eaſily ſhewn, independently of that Theorem; for, that $.\dot{9}$ (or .9999, *&c. ad infinitum*) is $= 1$, may be proved thus. It is well known that $.9 = \frac{9}{10}$ wants but $\frac{1}{10}$ of Unity, and if we take in two Figures $.99 = \frac{99}{100}$, it wants but $\frac{1}{100}$ of Unity, and if three Figures .999 the Error is but $\frac{1}{1000}$ of an Unit, &c. hence it follows, that if the Number of 9's be infinitely continued, the Difference betwixt the Series and Unity will be 1 divided by

an infinite Number of Nines, and ∵ the Quotient muſt be infinitely little, or nothing: Therefore, .999, *&c. ad infinitum*, is = 1.

23. *Theorem* 9. Any Vulgar Fraction, whoſe Denominator conſiſts of 9's, or of 9's with 0's on the Right-hand, and whoſe Numerator is a Number not having more Figures than the Number of 9's in the Denominator, is equal to that pure decimal Circulate, whoſe Repetend is the Numerator of the given Fraction, with as many 0's on the Left-hand as there are 9's in the Denominator more than Figures in the Numerator; and the firſt Figure of the circulating Part muſt have as many 0's on the Left-hand of it, as there are 0's in the Denominator. Thus, $\frac{4}{9} = .\dot{4}$, $\frac{3}{9} = .\dot{3}$; $\frac{6}{90} = .0\dot{6}$; $\frac{3}{900} = .00\dot{3}$; $\frac{37}{99} = .\dot{3}\dot{7}$; $\frac{342}{999} = .\dot{3}4\dot{2}$; $\frac{31}{990} = .0\dot{3}\dot{1}$; $\frac{1231}{9999000} = .000\dot{1}23\dot{1}$; $\frac{341}{9999000} = .0000\dot{3}4\dot{1}$. The Reaſon of this is evident from *Theorem* 4. it being only the Reverſe of that Theorem; for which Reaſon, it might have been writ by Way of Corollary.

24. *Corollary*. Hence it follows, that, *cæteris paribus*, when the Denominator is 9's without any 0's, the Repetend begins immediately after the decimal Point.

25. Hence, any Vulgar Fraction, whoſe Denominator is ſuch as mentioned in this Theorem, and whoſe Numerator conſiſts of more Places of Figures than there are 9's in the Denominator, and the Figures, which the Numerator has more than the Denominator be 0's, may have its Value readily expreſſed as a pure Circulate, without Diviſion, thus, $\frac{40}{9} = \dot{4}.4$, for by Theorem, $\frac{4}{9} = .\dot{4}4$ and ∵ $\frac{40}{9}$ being $= \frac{4}{9} \times 10$, muſt be $= .\dot{4}4 \times 10 = \dot{4}.4$; and $\frac{34200}{999} = \dot{3}4.\dot{2}$ for $\frac{342}{999} = .\dot{3}4\dot{2}$ by the Theorem, and ∵ $\frac{34200}{999}$ being $= \frac{342}{999} \times 100$, $\frac{34200}{999}$ muſt be $= .\dot{3}4\dot{2} \times 100 = \dot{3}4.\dot{2}$.

26. It may here be remarked, that if the Numerator of a Vulgar Fraction, whoſe Denominator is 9's, or 9's with 0's on the Right-hand, conſiſts of ſome Figures compleatly repeated, and the Number of Figures in the Numerator equal to the Number of 9's in the Denominator, ſuch Fraction is equivalent to one whoſe Numerator conſiſts of but one Period of the repeated Figures, and its Denominator of as many 9's as there are Figures in that Period; thus; $\frac{3434}{9999} = \frac{34}{99}$; for by the laſt Theorem, $\frac{3434}{9999} = .\dot{3}43\dot{4} = .\dot{3}\dot{4}$, and $\frac{34}{99} = .\dot{3}\dot{4}$, and conſequently, $\frac{3434}{9999} =$ * $\frac{34}{99}$; after the ſame Manner, $\frac{66}{99} = \frac{6}{9}$, for $\frac{66}{99} = .\dot{6}\dot{6} = \dot{6}$, and $\frac{6}{9} = .\dot{6}$ alſo; ∴ $\frac{66}{99} = \frac{6}{9}$. Wherefore, when ſuch Fractions occur, we may take but one Period of Figures, to find the Circulate by.

* Art. 23. of 1ſt of Math. Eſſays.

27. *Theorem* 10. Every Vulgar (finite) Fraction may be changed, either into a determinate, or circulating Decimal.

Demonſtration. To reduce a Vulgar Fraction to a decimal one, we * divide the Numerator by the Denominator, by Diviſion of Decimals; which is done by annexing an 0 each Time we go, to each Remainder for a partial Dividend; and therefore, ſince in Diviſion each Remainder muſt be leſs than the Diviſor, (for were any Remainder equal to the Diviſor, we could have gone once more in the Quotient,) it follows, that we can never have ſo many Steps in any Diviſion, as there are Units in the Diviſor, before we either come to 0 remaining, or have two Remainders the ſame; (for otherwiſe there muſt be as many Numbers leſs than the Diviſor, as there are Units in the Diviſor; which is abſurd.) And if the Remainder be 0, then the Diviſion is evidently finiſhed, and the Quotient a finite determinate Decimal; but if two Remainders in the Work are found equal, it is plain there muſt be a Repetend, for the Work may be continued at Pleaſure, and the ſame Remainders muſt come out, and conſequently,

* 34.

ſequently, a Repetend of the ſame Figures. Hence the Truth of the Theorem.

28. From this Theorem it evidently follows, that the Repetend in any Circulate can never have more Places of Figures (but may have fewer) than the Number expreſſed by the Denominator of the equivalent Vulgar Fraction leſs 1; the Vulgar Fraction being alſo in its leaſt Terms; thus, for Example Sake, $\frac{2}{13} = .\dot{1}5384\dot{6}$; and that this is limited to the Vulgar Fraction in its leaſt Terms, is clear; for the Fraction $\frac{2}{13}$ in any other Terms may be expreſſed generally by $\frac{2d}{13d}$; but $\frac{2d}{13d} = \frac{2}{13}$, and therefore $\frac{2d}{13d}$ * $= .\dot{1}5384\dot{6}$, and conſequently, any Vulgar Fraction, which in Decimals becomes a Circulate, muſt have its Repetend limited by the Denominator of that Fraction, in its leaſt Terms.

* Art. 23. of 1ſt. of Math. Eſſays.

CHAP. II.

ADDITION *of* CIRCULATES.

29. A *General Rule.* Make the ſeveral Repetends ſimilar, then add up theſe ſimilar Repetends, and divide their Sum by a Number conſiſting of as many 9's as there are Places of Figures in the Repetend; the Remainder of the Diviſion is the Repetend of the Sum; and if it does not conſiſt of as many Figures as the Places in each ſimilar Repetend, 0's muſt be annexed on the Left-hand, to ſupply that Defect of Places: As for the Quotient, it is ſo many Units of the next ſuperior Column, and muſt therefore be carried to that Column, which, with the others remaining, muſt be added up as uſual.

30. *Example.* Add up $.\dot{4}$; $.58\dot{3}$; $2.7\dot{1}$; and $36.43\dot{2}\dot{1}$ together.

Diſſimilar.	*Made ſimilar.*
$.\dot{4}$	$.44\dot{4}\dot{4}$
$.58\dot{3}$	$.58\dot{3}\dot{3}$
$2.7\dot{1}$	$2.71\dot{1}\dot{1}$
$36.43\dot{2}\dot{1}$	$36.43\dot{2}\dot{1}$

Here we have firſt 44 + 33 + 11 + 21 = 109, the Sum of the Repetends = $\frac{109}{99}$ of the next Column, = 1 and $\frac{10}{99}$ ∵ under the Repetends put 10 and carry 1, which gives the Sum $40.17\dot{1}\dot{0}$

Here, the beſt Method of dividing by the 9's is that ſhewn in the 1ſt. of *Mathematical Eſſays*, Art. 126. and the Work of 109 ÷ 99 will ſtand thus,

1|09
 |1
——
1|10

31. *Example* 2. Add .25716 and $.\dot{3}\dot{4}$ together; here to make the finite Expreſſion .25716 a Circulate, whoſe Repetend ſhall conſiſt of two Places, we muſt annex two o's, thus, .25716 = $.25716\dot{0}\dot{0}$; and to make $.\dot{3}\dot{4}$ ſimilar to this laſt, that is, to conſiſt of 7 decimal Places, for $.\dot{3}\dot{4}$ we write its equal $.34343\dot{4}\dot{3}$; and now we are to find the Sum of $.25716\dot{0}\dot{0}$ and $.34343\dot{4}\dot{3}$; in Order to which, we firſt add the Repetends together, and their Sum being 43, which is leſs than 99, we have nothing to carry to the next ſuperior Place; and ∵ the Sum is $.60059\dot{4}\dot{3}$.

32. *Example* 3. Add $9.\dot{8}1\dot{4}$; 1.5; $872\dot{6}$; $.08\dot{3}$ and $124.\dot{0}\dot{9}$ together.

Here it appears, that in the ſimilar Expreſſions the Repetends muſt begin in the third decimal Place, and conſiſt of ſix Places of Figures, becauſe the above given Repetends conſiſt of 1, 2, and 3 Places, and $1 \times 2 \times 3 = 6$, is the leaſt common Multiple; therefore, we ſhall have,

Diſſimilar.		*Similar.*
$9.\dot{8}1\dot{4}$	$=$	$9.81\dot{4}8148\dot{1}$
1.5	$=$	$1.50\dot{0}0000\dot{0}$
$87.2\dot{6}$	$=$	$87.26\dot{6}6666\dot{6}$
$.08\dot{3}$	$=$	$.08\dot{3}3333\dot{3}$
$124.\dot{0}\dot{9}$	$=$	$124.09\dot{0}9090\dot{9}$

Now the Sum of theſe Repetends, $481481 + 000000 + 666666 + 333333 + 090909 = 1572389$; and $\frac{1572389}{999999} = 1$ and 572390 for the Repetend. Now the Sum of the other Columns, *viz.* $9.81 + 1.50 + 87.26 + .08 + 124.09 + 1$, the Carriage from the Repetends $= 222.75$, and ∴ the whole Sum $= 222.75\dot{5}7239\dot{0}$.

33. As to the Reaſon of dividing the Sum of the Repetends by a Number conſiſting of as many Nines as there are Places of Figures in the Repetend, ſuch Perſons as underſtand what has been already ſaid concerning finding the equivalent Vulgar Fraction to any circulating Expreſſion, cannot be ignorant of it; for it is manifeſt by *Theorem* 4, that if r repreſents any Repetend, and n the Number of Figures of which it conſiſts, that ſuch Repetend, as a Fraction of a Unit in the next left-hand Place, is $= \frac{r}{n, 9\text{'s}}$; and ∴ the Sum of any Number of ſimilar Repetends, in the Fraction of an Unit of the next left-hand Place, will be repreſented by, $\frac{r}{n, 9\text{'s}} + \frac{R}{n, 9\text{'s}} + \&\text{c.}$ or $\frac{r + R + \&\text{c.}}{n, 9\text{'s}}$.

CHAP.

CHAP. III.

SUBTRACTION *of* CIRCULATES.

34. THE *Rule.* If the Repetends are not ſimilar, they muſt firſt be made ſimilar; then ſee, which is the greater, the Repetend of the Subducend or Minorand; and if the Repetend of the Minorand is the greater, add 1 to that Figure in the Minorand, which is at the greateſt Diſtance from the decimal Point on the Right-hand; then ſubtract as uſual.

35. *Examples.*

Subducend	$3.416\dot{3}$	From	$12.1\dot{4}$	Here the Repetend of the Minorand is the greater, and ∵ we muſt call the 6, ſeven
Minorand	$2.106\dot{2}$	Subtract	$3.2\dot{6}$	
Remainder	$1.310\dot{1}$	Remains	$8.8\dot{7}$	

Diſſimilar.	*Made ſimilar.*	*Diſſimilar.*	*Made ſimilar.*
From 8.314	$8.314\dot{0}$	From 10.5	$10.5\dot{0}\dot{0}$
Subtract $2.\dot{6}$	$2.666\dot{6}$	Subtract $3.\dot{4}\dot{5}$	$3.4\dot{5}\dot{4}$
	Remains $5.647\dot{3}$		Remains $70\dot{4}\dot{5}$

Diſſimilar.	*Made Similar.*
From $1.2\dot{2}\dot{7}$	$1.2\dot{2}727272727\dot{7}$
Subtr. $0.\dot{3}462\dot{1}$	$0.\dot{3}462134621\dot{3}$
	Remains $0.8\dot{8}10592651\dot{3}$

Diſſimilar.

	Diſſimilar.	*Made ſimilar.*
From	$.\dot{8}43\dot{1}$	$.843\dot{1}84\dot{3}$
Take	$.325$	$.325\dot{0}00\dot{0}$
Remains		$.518\dot{1}84\dot{3}$

36. The only Thing which is neceſſary to be explained concerning the above Rule, is, the Reaſon of adding 1 to the Right-hand Place of the Minorand, when the Repetend of the Minorand is greater than that of the Subducend; and this is eaſily ſhewn, for in ſubtracting, when we cannot take any Part of the Minorand out of the correſponding Part of the Subducend, we always add an Unit of the next Place, to that Part of the Subducend, that Subtraction may be made, remembering when we go to the next Place of the Minorand, to add 1, (for the Unit of that Place which was added to the Subducend,) that their Difference may continue the ſame; the Reaſon of which Method of ſubtracting has been already ſhewn *; and that the above Method of ſubtracting Circulates is agreeable to this general Rule, will plainly appear; for in the laſt Article it was required to ſubtract $.3\dot{4}62134621\dot{3}$ from $1.2\dot{2}72727272\dot{7}$: Now, we cannot take $\dot{4}62134621\dot{3}$ from $\dot{2}72727272\dot{7}$, or, which is the ſame, $\frac{4621346213}{9999999999}$ from $\frac{2727272727}{9999999999}$, (expreſſed as Fractions of an Unit in the next ſuperior Place;) ∵ by the general Method of Subtraction juſt above mentioned, we muſt add an Unit of the next ſuperior Place to $\frac{2727272727}{9999999999}$, and then ſubtract; that is, we muſt

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* Art. 47. of 1ſt of Math. Eſſays.

ſubtract from $\frac{2727272727}{9999999999} + \frac{9999999999}{9999999999}$, or, which is the ſame, from $\frac{2727272727+9999999999}{9999999999}$, or from its equal, $\frac{12727272726}{9999999999}$; where it may be obſerved, that adding the 9's only makes the firſt Right-hand Place 1 leſs, the reſt of the Figures continuing the ſame; therefore, we muſt ſubtract 4621346213 from 12727272726, for the Repetend; whence it appears, that we may ſuppoſe the Right-hand Place of the Repetend of the Subducend to be 1 leſs, or, which is the ſame in Effect, that of the Right-hand Place of the Minorand to be 1 more than it really is; and then ſubtract as uſual.

CHAP. IV.

MULTIPLICATION *of* CIRCULATING DECIMALS.

37. THE Product of any decimal Expreſſions, whether determinate, or circulate, may be found, by finding their equivalent Vulgar Fractions, and then finding the Product of theſe equivalent Fractions, by the Method of multiplying Fractions, taught in the firſt of the *Mathematical Eſſays*; (which Product muſt be equal to that of the given decimal Factors; for the Vulgar Fractions being by the Suppoſition equal to the decimal Expreſſions, it is manifeſt their Products muſt alſo be equal:) And (therefore,) the Vulgar Fraction expreſſing the Product, being turned into Decimals, will give the Product of the given Decimals, in a

decimal Expreſſion; and the Diviſion may be carried on at Pleaſure, till either 0 remain, or the Repetend be found, or as many decimal Places as ſhall be thought ſufficiently exact for the intended Purpoſe.

38. *Example* 1. Multiply $3.\dot{4}$ by $.75$.

Firſt, $3.\dot{4} =$ (by *Theorem* 6.) $\frac{3\times9+4}{9}=\frac{31}{9}$; and by the Nature of Decimals, $.75=\frac{75}{100}$ ∴ the required Product is $\frac{31}{9}\times\frac{75}{100}=$ (by *Multiplication of Vulgar Fractions*) $\frac{31\times75}{9\times100}=\frac{2325}{9\times100}$. Now 2325 may be divided by 9 × 100 by * Cultellation, or Piece-Meal, thus, $2325\div9=258\frac{3}{9}=258.\dot{3}$, found as in the Margin; and $258.\dot{3}\div100=2.58\dot{3}$, found by cutting off two Places more for Decimals.

$$\begin{array}{r|l} 232 & 5 \\ 23 & 2 \\ 2 & 3 \\ & 2 \\ \hline 258 & 2 \quad +\ 1 \text{ Carr.} \end{array}$$

39. *Example* 2. Multiply $32.\dot{7}$ by $3.\dot{2}$.

Here, $32.\dot{7}=\frac{32\times9+7}{9}=\frac{295}{9}$; and $3.\dot{2}=\frac{3\times9+2}{9}=\frac{29}{9}$; now $\frac{295}{9}\times\frac{29}{9}=\frac{295\times29}{9\times9}=\frac{8555}{9\times9}$; and $\frac{8555}{9\times9}$, or its equal $\frac{8555}{81}$ may be put into a decimal Expreſſion by common Diviſion; or by the Method ſhewn in the Margin, by taking the $\frac{1}{9}$ and then the $\frac{1}{9}$ of that again; and ſo the Product is 105.617283, very nearly true, but not exact; for we have not continued the Product till it circulates.

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* A Term uſed by ſome Decimal Writers, and perhaps derived from the *Latin* Word *cultello*, to divide as with a Knife.

$$\begin{array}{l} 8555. \\ \frac{1}{9}\;\overline{} \\ 950.\dot{5}55. \\ \frac{1}{9}\;\overline{} \\ 105.617283. \end{array}$$

40. Before we proceed any further, it may not be improper to lay down the following Rule, for the more ready dividing decimal Expreſſions by any Number of 9's.

The Rule. Take as many Places of Figures for the Dividend as will give ſo many decimal Places in the Quotient as you ſhall think to be ſufficiently exact, though there ſhould be no Circulation diſcovered in that Number of Places; then divide, as if the Dividend was a whole Number, by the Method ſhewn in *Art.* 126 of the firſt of *Mathematical Eſſays*; and after the Operation the decimal Places may be cut off.

For an Illuſtration, let it be required to divide $950.\dot{5}$ by 9, (ſee the laſt Article;) here, as $950.\dot{5}$ is very nearly = 950.555555555, being ſufficiently exact for any Purpoſe, we divide as is here annexed.

95055555555	5
9505555555	5
950555555	5
95055555	5
9505555	5
950555	5
95055	5
9505	5
950	5
95	0
9	5
	9
105617283949	$\frac{9+5}{9}$ Carriage

or, 105617283950$\frac{5}{9}$, and it being manifeſt that the Quotient can have but three Places of Figures, the Anſwer muſt be 105.61728395O5, very near the Truth.

Otherwiſe. By the Expence of fewer Figures, by this Rule. Separate the Dividend by vertical Lines, into diſtinct Columns, each containing as many Places of Figures as there are Places of 9's in the Diviſor; and in ſeparating theſe Columns, begin at the Left-hand, and proceed towards the Right; and if it happens that there are not Places enough on the Right-hand for a compleat Column, that Defect muſt be made up by annexing Cyphers on the Right-hand, if the Dividend be a determinate Number, or by continuing the Repetend, if it be a Circulate. This being done, take the Figures in the firſt Column, (on the Left-hand) and place them under thoſe in the next, and find their Sum, and place it under the Figures in the third Column, and ſo proceed to the laſt Column; then add up the ſeveral Columns, and their Sum will be the required Quotient.

Note. When the Diviſor is a whole Number, the Figures under the laſt, or Right-hand Column, are the Remainder of the Diviſion, or a Repetend.

For an Illuſtration, let us divide $950.\dot{5}$ by 9; according to this Method the Work will ſtand thus.

9	5	0	5	5	5	5	5	5	5	5	5
	9	4	4	9	4	9	4	9	4	9	4
	1	1	1	2	2	3	3	4	4	5	6
10	5	6	1	7	2	8	3	9	5	0	5

Hence the Anſwer as before.

Explanation. Having parted out the Columns, under the ſecond we put 9, the Figure which is in the firſt Column; then we have in the ſecond $5 + 9 = 14$, to be carried to the third Column; but as that has but one Place, we put the 4 in that Column, and the 1 in the ſecond; then in the third Column there is 4, to which prefixing the 1 placed in the ſecond, we have 14 to be carried to the fourth Column, which containing but one Place, we can only write the 4 under that Column, and the 1 in the third. Now, in the fourth Column we have $4 + 5 = 9$, to which prefixing the 1 placed in the third, it is 19 to be carried to the fifth; therefore, in the fifth Column we write down the 9, and the 1 in the fourth, becauſe the fifth Column contains but one Figure: Again, in the fifth we now have, $9 + 5 = 14$, and the 1 in the fourth Column, (being = 10 Units of the fifth) which makes $14 + 10 = 24$, to be carried to the ſixth; which Column having but one Place, we can only write the 4 in that Column, and place the 2 in the fifth: After this Manner we proceed to the laſt Column, in which it may be obſerved that we have written a 6, which, according to this Method of adding the Sums of the ſeveral Columns, would have been but a 5; but we added 1 to it, to allow for the Carriage which would have been, if we had continued one Column more. Having thus compleated the ſeveral Columns, we add them up as in common Addition, beginning with that on the Right-hand, ſaying $6 + 4 + 5 = 15$, put down 5, and carry one, then 1 (carried) $+ 5 + 9 + 5 = 20$, put down 0 and carry 2, &c.

The Reaſon of this Method of dividing by 9's, may be ſhewn by dividing 34133622 by 999, thus:

$$\begin{array}{r|r|r}
341 & 336 & 322 \\
 & 341 & 677 \\
\hline
\multicolumn{2}{r|}{341677} & 999 \\
\hline
\end{array}$$

Anſ. $341677\frac{222}{999}$,
or, 341678.

and multiplying 341678 by 999, by the Method ſhewn in the firſt *Eſſay*, *Article* 75, thus:

$$\left\{\begin{array}{lr}
 & 341678000 \\
 & 341678 \\
\cline{2-2}
\text{The Product} & 341336322 \\
\cline{2-2}
\end{array}\right.$$

for by comparing theſe two Operations together, it will plainly appear, that one is only the Reverſe of the other. —— But to proceed with our Examples of multiplying decimal Expreſſions.

41. *Example* 3. Multiply $4.\dot{3}$ by $.18$.

Here $4.\dot{3} = \frac{4\times 9+3}{9} = \frac{39}{9}$; and $.18 = \frac{18}{100}$;

$\because$ we have the required Product $= \frac{39}{9} \times \frac{18}{100} = \frac{702}{9\times 100} = .78$; ſee the Operation in the Margin.

$$\begin{array}{r|l} \frac{1}{9} & 702 \\ \hline \frac{1}{100} & 78 \\ \hline & .78 \\ \hline \end{array}$$

42. *Example* 4. Multiply $.\dot{1}4285\dot{7}$ by $.0\dot{7}1428\dot{5}$.

Here we have $.\dot{1}4285\dot{7} = \frac{142857}{999999}$; and $.0\dot{7}1428\dot{5} = \frac{714285}{9999990}$; hence the required Product is $= \frac{142857}{999999} \times \frac{714285}{9999990} = \frac{102040612245}{999999 \times 9999990}$; which we turn into Decimals, by two Diviſions, thus:

The firſt Diviſion. $\left\{ \begin{array}{r|l} 102040 & 612245 \\ & 102040 \\ \hline 102040 & 714285 \end{array} \right.$

Hence $102040612245 \div 999999 = 102040\frac{714285}{999999}$, or $= 102040.\dot{7}1428\dot{5}$ $\because$ the ſecond Diviſion, by the laſt Method ſhewn in *Article* 40, will ſtand as follows.

$\Big\}\ 102040$

102040	714285	714285	714285	714285	714285	714285	714285	&c.
	102040	816325	530610	244895	959180	673465	387750	
		1	2	2	3	4	5	
102040	816326	530612	244897	959183	673469	387755	102040	

and, by barely looking on the given Multiplier and Multiplicand, it is manifeſt the Anſwer muſt be about .01, and ∵ the true Product is $= .0120\dot{4}081 6326530612244897959183673469387755$.

43. We are not ignorant that there are ſeveral other Methods of multiplying Circulates; but as they are not any Thing better, if ſo good; and a Multiplicity of different Rules, ſerving only to confuſe the Learner, we have thought proper to omit them.

CHAP. V.

DIVISION of CIRCULATING DECIMALS.

44. A *General Rule.* Change both the Diviſor and Dividend into their equivalent Vulgar Fractions, and then find the Quotient, as ſhewn in *Diviſion of Vulgar Fractions*; and the Vulgar Fraction, expreſſing the Quotient, may be changed into a decimal Expreſſion, by the Methods already treated of.

45. An Example, or two, will illuſtrate this Rule. *Example* 1. Divide $2.58\dot{3}$ by .75.

Here we have $2.58\dot{3} = \frac{258\times9+3}{900} = \frac{2325}{900}$, and $.75 = \frac{75}{100}$; hence, by Diviſion of Vulgar Fractions, the Quotient is $= \frac{2325\times100}{75\times900} = \frac{2325}{75\times9}$

$$= 3.\dot{4}\text{, found thus, }\quad \begin{array}{r} 75)2325(31 \\ \hline 75 \\ \hline \end{array} \qquad \tfrac{1}{9}\,\begin{array}{l} 31 \\ \hline 3.\dot{4}4\text{, \&c.} \end{array}$$

46. *Example* 2. Divide $0.\dot{3}$ by $\dot{1}4285\dot{7}$.

Here, their equivalent Vulgar Fractions are $\frac{1}{3}$ and $\frac{1}{7}$, but $\frac{1}{3} \div \frac{1}{7} = \frac{1\times 7}{3\times 1} = \frac{7}{3} = 2.\dot{3}$.

47. As to particular Rules, we ſhall only hint, that when the Diviſor is a finite Expreſſion, the Quotient may be found as in common Diviſion.

Example, Divide $2.58\dot{3}$ by .75; which is the ſame as the firſt Example in this Chapter. The Operation will ſtand as here annexed; where it is manifeſt that 33 would always remain, and $\dot{4}$ would always repeat in the Circulate.

$$\begin{array}{r} .75)2.58\dot{3}(3.44\text{, \&c.} \\ 225 \\ \hline 333 \\ 300 \\ \hline 333 \\ 300 \\ \hline 33 \end{array}$$

Here we have taken down another 3, becauſe 3 is the Repetend.

48. There are many other Methods of dividing Circulates, but for the Reaſon already given in Multiplication of circulating Decimals, we have omitted them, believing, that though we have not treated this Subject ſo largely as ſome Authors have done, yet we have treated it ſo fully as its Uſefulneſs deſerves; and have been ſufficiently copious to enable any Perſon, of a tolerable Capacity, to manage the Operations of circulating Decimals, and ſhall therefore conclude this Eſſay, with the following uſeful Tables.

49. A TABLE, for the more expeditious finding the equivalent decimal Expreſſion of any Vulgar Fraction, whoſe Denominator is not greater than 99.

NUMERATORS.	DENOMINATORS.							
	2	3	4	5	10	11	12	13
1	,5	$,\dot{3}$	,25	,2	,1	$,\dot{0}\dot{9}$	$,08\dot{3}$	$,\dot{0}7692\dot{3}$
2		$,\dot{6}$	,5	,4	,2	$,\dot{1}\dot{8}$	$,1\dot{6}$	$,\dot{1}5384\dot{6}$
3			,75	,6	,3	$,\dot{2}\dot{7}$	,25	$,\dot{2}3076\dot{9}$
4				,8	,4	$,\dot{3}\dot{6}$	$,\dot{3}$	$,\dot{3}0769\dot{2}$
5					,5	$,\dot{4}\dot{5}$	$,41\dot{6}$	$,\dot{3}8461\dot{5}$
6					,6	$,\dot{5}\dot{4}$	,5	$,\dot{4}6153\dot{8}$
7					,7	$,\dot{6}\dot{3}$	$,58\dot{3}$	$,\dot{5}3846\dot{1}$
8					,8	$,\dot{7}\dot{2}$	$,\dot{6}$	$,\dot{6}1538\dot{4}$
9					,9	$,\dot{8}\dot{1}$	,75	$,\dot{6}9230\dot{7}$
10						$,\dot{9}\dot{0}$	$,8\dot{3}$	$,\dot{7}6923\dot{0}$
11							$,91\dot{6}$	$,\dot{8}4615\dot{3}$
12								$,\dot{9}2307\dot{6}$

DENOMINATORS.

NUMERATORS.	6	7	8	9
1	,16	,142857	,125	,1
2	,3	,285714	,25	,2
3	,5	,428571	,375	,3
4	,6	,571428	,5	,4
5	,83	,714285	,625	,5
6		,857142	,75	,6
7			,875	,7
8				,8

NUMERATORS.	14	15	16	17
1	,0714285	,06	,0625	,0588235294117647
2	,142857	,13	,125	
3	,2142857	,2	,1875	
4	,285714	,26	,25	
5	,3571428	,3	,3125	
6	,428571	,4	,375	
7	,5	,46	,4375	
8	,571428	,53	,5	
9	,6428571	,6	,5625	
10	,714285	,6	,625	
11	,7857142	,73	,6875	
12	,857142	,8	,75	

NUME-

DENOMINATORS.

NUMERATORS.

	10	11	12	13
13				
14				
15				

	18	19	20	21
1	$,0\dot{5}$	$,\dot{0}5263157894736842\dot{1}$	,05	$,\dot{0}4761\dot{9}$
2	$,\dot{1}$		,1	$,\dot{0}9523\dot{8}$
3	$,1\dot{6}$		,15	$,\dot{1}4285\dot{7}$
4	$,\dot{2}$		,2	$,\dot{1}9047\dot{6}$
5	$,2\dot{7}$		,25	$,\dot{2}3809\dot{5}$
6	$,\dot{3}$		,3	$,\dot{2}8571\dot{4}$
7	$,3\dot{8}$		,35	$,\dot{3}$
8	$,\dot{4}$		,4	$,\dot{3}8095\dot{2}$
9	,5		,45	$,\dot{4}2857\dot{1}$
10	$,\dot{5}$		,5	$,\dot{4}7619\dot{0}$
11	$,6\dot{1}$		,55	$,\dot{5}2380\dot{9}$
12	$,\dot{6}$		,6	$,\dot{5}7142\dot{8}$
13	$,7\dot{2}$		,65	$,\dot{6}1904\dot{7}$
14	$,\dot{7}$		,7	$,\dot{6}$
15	$,8\dot{3}$		,75	$,\dot{7}1428\dot{5}$
16	$,\dot{8}$		,8	$,\dot{7}6190\dot{4}$
17	$,9\dot{4}$		,85	$,\dot{8}0952\dot{3}$

DENOMINATORS.

NUMERATORS.

	14	15	16	17
13	$,9\dot{2}8571\dot{4}$	$,8\dot{6}$	,8125	
14		$,9\dot{3}$	,875	
15			,9375	
	22	23	24	25
1	$,0\dot{4}\dot{5}$	$,\dot{0}43478260869565217391\dot{3}$	$,041\dot{6}$	,04
2	$,\dot{0}\dot{9}$		$,08\dot{3}$	,08
3	$,1\dot{3}\dot{6}$		,125	,12
4	$,\dot{1}\dot{8}$		$,1\dot{6}$	,16
5	$,2\dot{2}\dot{7}$		$,208\dot{3}$	,2
6	$,\dot{2}\dot{7}$		,25	,24
7	$,3\dot{1}\dot{8}$		$,291\dot{6}$	,27
8	$,\dot{3}\dot{6}$		$,\dot{3}$	,32
9	$,4\dot{0}\dot{9}$		,375	,36
10	$,\dot{4}\dot{5}$		$,41\dot{6}$	,4
11	,5		$,458\dot{3}$	,44
12	$,\dot{5}\dot{4}$		,5	,48
13	$,5\dot{9}\dot{0}$		$,541\dot{6}$	,52
14	$,\dot{6}\dot{3}$		$,58\dot{3}$	,56
15	$,6\dot{8}\dot{1}$		,625	,6
16	$,\dot{7}\dot{2}$		$,\dot{6}$	,64
17	$,7\dot{7}\dot{2}$		$,708\dot{3}$	,68

DENOMINATORS.

NUMERATORS.	18	19	20	21
18			,9	,857142
19			,95	,904761
20				,952380
21				
22				
23				
24				

NUMERATORS.	26	27	28	29
1	,0384615	,037	,03571428	,0344827586206896551724137931
2	,076923	,074	,0714285	
3	,1153846	,1	,10714285	
4	,153846	,148	,142857	
5	,1923076	,185	,17857142	
6	,230769	,2	,2142857	
7	,2692307	,259	,25	
8	,307692	,296	,285714	
9	,3461538	,3	,32142857	
10	,384615	,370	,3571428	
11	,4230769	,407	,39285714	
12	,461538	,4	,428571	
13	,5	,481	,46428571	

NUME-

DENOMINATORS.

NUMERATORS.

	22	23	24	25
18	$,\dot{8}\dot{1}$		,75	,72
19	$,8\dot{6}\dot{3}$		$,791\dot{6}$	,76
20	$,\dot{9}\dot{0}$		$,8\dot{3}$	,8
21	$,9\dot{5}\dot{4}$		,875	,84
22			$,91\dot{6}$	,88
23			$,958\dot{3}$	,92
24				,96

	30	31	32	33
1	$,0\dot{3}$	$,\dot{0}3225806451612\dot{9}$	,03125	$,\dot{0}\dot{3}$
2	$,0\dot{6}$		,0625	$,\dot{0}\dot{6}$
3	,1		,09375	$,\dot{0}\dot{9}$
4	$,1\dot{3}$		,125	$,\dot{1}\dot{2}$
5	$,1\dot{6}$		,15625	$,\dot{1}\dot{5}$
6	,2		,1875	$,\dot{1}\dot{8}$
7	$,2\dot{3}$		,21875	$,\dot{2}\dot{1}$
8	$,2\dot{6}$		,25	$,\dot{2}\dot{4}$
9	,3		,28125	$,\dot{2}\dot{7}$
10	$,\dot{3}$		,3125	$,\dot{3}\dot{0}$
11	$,3\dot{6}$		,34375	$,\dot{3}\dot{3}$
12	,4		,375	$,\dot{3}\dot{6}$
13	$,4\dot{3}$		,40625	$,\dot{3}\dot{9}$

DENOMINATORS.

NUMERATORS.	26	27	28	29
14	,538461	,518	,5	
15	,5769230	,5	,53571428	
16	,615384	,592	,571428	
17	,6538461	,629	,60714285	
18	,692307	,6	,6428571	
19	,7307692	,703	,67857142	
20	,769230	,740	,714285	
21	,8076923	,7	,75	
22	,846153	,814	,7857142	
23	,8846153	,851	,82142857	
24	,923076	,8	,857142	
25	,9615384	,925	,89285714	
26		,962	,9285714	
27			,96428571	
28				
29				
30				
31				
32				

DENOMINATORS.

NUMERATORS.

	30	31	32	33
14	,4$\dot{6}$		,4375	,$\dot{4}\dot{2}$
15	,$\dot{5}$		,46875	,$\dot{4}\dot{5}$
16	,5$\dot{2}$		,5	,$\dot{4}\dot{8}$
17	,5$\dot{6}$		,53125	,$\dot{5}\dot{1}$
18	,6		,5625	,$\dot{5}\dot{4}$
19	,6$\dot{3}$		,59375	,$\dot{5}\dot{7}$
20	,$\dot{6}$		,625	,$\dot{6}\dot{0}$
21	,7		,65625	,$\dot{6}\dot{3}$
22	,7$\dot{3}$		,6875	,$\dot{6}\dot{6}$
23	,7$\dot{6}$		,71875	,$\dot{6}\dot{9}$
24	,8		,75	,$\dot{7}\dot{2}$
25	,8$\dot{3}$		,78125	,$\dot{7}\dot{5}$
26	,8$\dot{6}$		,8125	,$\dot{7}\dot{8}$
27	,9		,84375	,$\dot{8}\dot{1}$
28	,9$\dot{3}$		,875	,$\dot{8}\dot{4}$
29	,9$\dot{6}$		,90625	,$\dot{8}\dot{7}$
30			,9375	,$\dot{9}\dot{0}$
31			,96875	,$\dot{9}\dot{3}$
32				,$\dot{9}\dot{6}$

DENOMINATORS.

NUMERATORS.	34	35	36	37	38
1	,02941176470588235	,0285714	,027	,027	,0263157894736842105
2		,0571428	,05	,054	
3		,0857142	,083	,081	
4		,1142857	,1	,108	
5		,142857	,138	,135	
6		,1714285	,16	,162	
7		,2	,194	,189	
8		,2285714	,2	,216	
9		,2571428	,25	,243	
10		,285714	,27	,270	
11		,3142857	,305	,297	
12		,3428571	,3	,324	
13		,3714285	,361	,351	
14		,4	,38	,378	
15		,428571	,416	,405	
16		,4571428	,4	,432	
17		,4857142	,472	,459	
18		,5142857	,5	,486	
19		,5428571	,527	,513	
20		,571428	,5	,540	
21		,6	,583	,567	

DENOMINATORS.

NUMERATORS.	39	40	41	42
1	,025641	,025	,02439	,0238095
2	,051282	,05	,04878	,047619
3	,076923	,075	,07317	,0714285
4	,102564	,1	,09756	,095238
5	,128205	,125	,12195	,1190476
6	,153846	,15	,14634	,142857
7	,179487	,175	,17073	,16
8	,205128	,2	,19512	,190476
9	,230769	,225	,21951	,2142857
10	,256410	,25	,24390	,238095
11	,282051	,275	,26829	,2619047
12	,307692	,3	,29268	,285714
13	,3	,325	,31707	,3095238
14	,358974	,35	,34146	,3
15	,384615	,375	,36585	,3571428
16	,410256	,4	,39024	,380952
17	,435897	,425	,41463	,4047619
18	,461538	,45	,43902	,428571
19	,487179	,475	,46341	,4523809
20	,512820	,5	,48780	,476190
21	,538461	,525	,51219	,5

DENOMINATORS.

NUMERATORS.	34	35	36	37	38
22		$,6\dot{2}8571\dot{4}$	$,6\dot{1}$	$,\dot{5}9\dot{4}$	
23		$,6\dot{5}7142\dot{8}$	$,63\dot{8}$	$,\dot{6}2\dot{1}$	
24		$,6\dot{8}5714\dot{2}$	$,\dot{6}$	$,\dot{6}4\dot{8}$	
25		$,\dot{7}1428\dot{5}$	$,69\dot{4}$	$,\dot{6}7\dot{5}$	
26		$,7\dot{4}2857\dot{1}$	$,7\dot{2}$	$,\dot{7}0\dot{2}$	
27		$,7\dot{7}1428\dot{5}$	$,75$	$,\dot{7}2\dot{1}$	
28		8	$,\dot{7}$	$,\dot{7}5\dot{6}$	
29		$,8\dot{2}8571\dot{4}$	$,80\dot{5}$	$,\dot{7}8\dot{3}$	
30		$,\dot{8}5714\dot{2}$	$,8\dot{3}$	$,\dot{8}1\dot{0}$	
31		$,8\dot{8}5714\dot{2}$	$,86\dot{1}$	$,\dot{8}3\dot{7}$	
32		$,9\dot{1}4285\dot{7}$	$,\dot{8}$	$,\dot{8}6\dot{4}$	
33		$,9\dot{4}2857\dot{1}$	$,11\dot{6}$	$,\dot{8}9\dot{1}$	
34		$,9\dot{7}1428\dot{5}$	$,9\dot{4}$	$,\dot{9}1\dot{8}$	
35			$,97\dot{2}$	$,\dot{9}4\dot{5}$	
36				$,\dot{9}7\dot{2}$	
37					
38					
39					
40					
41					

DENOMINATORS.

NUMERATORS.	39	40	41	42
22	,564102	,55	,53658	,523809
23	,589743	,575	,56097	,5476190
24	,615384	,6	,58536	,571428
25	,641025	,625	,60975	,5952380
26	,6	,65	,63414	,619047
27	,692307	,675	,65853	,6428571
28	,717948	,7	,68292	,6
29	,743589	,725	,70731	,690476
30	,769230	,75	,73170	,714285
31	,794871	,775	,75609	,7380952
32	,820512	,8	,78048	,761904
33	,846153	,825	,80487	,7857142
34	,871794	,85	,82926	,809523
35	,897435	,875	,85365	,83
36	,923076	,9	,87804	,857142
37	,948717	,925	,90243	,8809523
38	,974358	,95	,92682	,904761
39		,975	,95121	,9285714
40			,97560	,952380
41				,9761904

DENOMINATORS.

NUMERATORS.	43	44	45	46
1	,023255813953488372093	,0227	,02	,02173913043478260869565
2		,045	,04	
3		,0681	,06	
4		,09	,08	
5		,1136	,1	
6		,136	,13	
7		,1590	,15	
8		,18	,17	
9		,2045	,19	
10		,227	,2	
11		,25	,24	
12		,27	,26	
13		,2954	,28	
14		,318	,31	
15		,3409	,3	
16		,36	,35	
17		,3863	,37	
18		,409	,39	
19		,4318	,42	
20		,45	,4	

DENOMINATORS.

NUMERATORS.	47	48	49	50
1	$,\dot{0}21276595744680851063829787234042553191489361\dot{7}$	$,0208\dot{3}$	$,\dot{0}2040816326530612244897959183673469387755\dot{1}$	,02
2		$,041\dot{6}$		,04
3		,0625		,06
4		$,08\dot{3}$		,08
5		$,1041\dot{6}$		,1
6		,125		,12
7		$,1458\dot{3}$		,14
8		$,1\dot{6}$		,16
9		,1875		,18
10		$,208\dot{3}$		,2
11		$,2291\dot{6}$		,22
12		,25		,24
13		$,2708\dot{3}$		,26
14		$,291\dot{6}$		,28
15		,3125		,3
16		$,\dot{3}$		,32
17		$,3541\dot{6}$		,34
18		,375		,36
19		$,3958\dot{3}$		,38
20		$,41\dot{6}$		,4

DENOMINATORS.

		43	44	45	40
NUMERATORS.	21		,47$\dot{7}\dot{2}$	,4$\dot{6}$	
	22		,5	,4$\dot{8}$	
	23		,52$\dot{2}\dot{7}$	,5$\dot{1}$	
	24		,$\dot{5}\dot{4}$	,5$\dot{3}$	
	25		,56$\dot{8}\dot{1}$	,$\dot{5}$	
	26		,5$\dot{9}\dot{0}$	,5$\dot{7}$	
	27		,61$\dot{3}\dot{6}$	,$\dot{6}$	
	28		,$\dot{6}\dot{3}$	,6$\dot{2}$	
	29		,65$\dot{9}\dot{0}$	,6$\dot{4}$	
	30		,6$\dot{8}\dot{1}$	,$\dot{6}$	
	31		,70$\dot{4}\dot{5}$	,6$\dot{8}$	
	32		,$\dot{7}\dot{2}$	,7$\dot{1}$	
	33		,75	,7$\dot{3}$	
	34		,7$\dot{7}\dot{2}$	,7$\dot{5}$	
	35		,79$\dot{5}\dot{4}$	,$\dot{7}$	
	36		,$\dot{8}\dot{1}$	,$\dot{8}$	
	37		,84$\dot{0}\dot{9}$	,8$\dot{2}$	
	38		,8$\dot{6}\dot{0}$	,8$\dot{4}$	
	39		,88$\dot{6}\dot{3}$	,8$\dot{6}$	
	40		,$\dot{9}\dot{0}$	,$\dot{8}$	

DENOMINATORS.

NUMERATORS.

	47	48	49	50
21		,4375		,42
22		,458$\dot{3}$		,44
23		,4791$\dot{6}$		,46
24		,5		,48
25		,5208$\dot{3}$		,5
26		,541$\dot{6}$		,52
27		,5625		,54
28		,58$\dot{3}$		,56
29		,6041$\dot{6}$		,58
30		,625		,6
31		,64583		,62
32		,$\dot{6}$		,64
33		,6875		,66
34		,708$\dot{3}$		,68
35		,7291$\dot{6}$		,7
36		,75		,72
37		,7708$\dot{3}$		,74
38		,791$\dot{6}$		,76
39		,8125		,78
40		,8$\dot{3}$		,8

DENOMINATORS.

NUMERATORS.	43	44	45	46
41		$,93\dot{1}\dot{8}$	$,9\dot{1}$	
42		$,9\dot{5}\dot{4}$	$,9\dot{3}$	
43		$,97\dot{7}\dot{2}$	$,9\dot{5}$	
44			$,9\dot{7}$	
45				
46				
47				
48				
49				

DENOMINATORS.

NUMERATORS.	47	48	49	50
41		$,8541\dot{6}$		,82
42		,875		,84
43		$,8958\dot{3}$		,86
44		$,91\dot{6}$		,88
45		,9375		,9
46		$,958\dot{3}$		,92
47		$,9791\dot{6}$		,94
48				,96
49				,98

PART II.

	51	52	53	54
1	,0196078431372549	,01923076	,0188679245283	,185
	59	**60**	**61**	**62**
1	,0169491525423728813559322033898305084745762711864406779661	,016	,016393442622950819672131147540983606557377049180327868852459	,0161290322580645

55

	55	56	57	58
1	$,0\dot{1}\dot{8}$	$,017\dot{8}5714\dot{2}$	$,\dot{0}1754385964912280\dot{7}$	$,0\dot{1}724137931034482758620689655\dot{}$
	63	64	65	66
1	$,\dot{0}1587\dot{3}$	$,015625$	$,0\dot{1}5384\dot{6}$	$,0\dot{1}\dot{5}$

	67	68	69	70
1	,014925373134328358208955223880597	,014705882352941176	,0144927536231884057971	,0142857
	75	**76**	**77**	**78**
1	,013	,01315789473684210526	,012987	,0128205
	83	**84**	**85**	**86**
1	,01204819277108433734939759036144578313253	,01190476	,01176470588235294	,0116279069767441860465

71

	71	72	73	74
1	,01408450704225352112676056338028169	,0138	,01369863	,0135
	79	**80**	**81**	**82**
1	,0126582278481	,0125	,012345679	,012195
	87	**88**	**89**	**90**
1	,0114942528735632183908045977	,01136	,01123595505617977528089887640449438202247191	,01

NUME-

NUMERATOR.	DENOMINATORS.				
	91	92	93	94	95
1	,010989	,010869565217391304347826	,010752688172043	,01063829787234042553191489361702127659574468085	,0105263157894736842

For this Table, we are indebted to the Labour of the ingenious Mr. *Marſh*, of *Saliſbury*. —— Its Uſe may be ſhewn by a few Examples. 1ſt, What is the decimal Expreſſion for $\frac{3}{7}$? *Anſwer*. By a bare Inſpection of this Table, againſt 3, the Numerator in the Side, and under 7, the Denominator on the Head of the Table, we find $\frac{3}{7}$ is $=$,428571.

Example 2. What Decimal is $= \frac{90}{120}$? This by Abbreviation is $= \frac{9}{12}$ or $\frac{3}{4}$; by either of which in this Table we ſhall find the required Decimal is ,75.

Example 3. What Decimal is $= \frac{75}{125}$? As this has a greater Denominator than is to be found in the Table, and is not in its loweſt Terms, we reduce it lower, thus, $\frac{75}{125}$ is $= \frac{3}{5}$, which in the Table gives ,6.

NUME-

DENOMINATORS.

		96	97	98	99
NUMERATOR.	1	$,01041\dot{6}$	$,\dot{0}1030927835051546391752 5$ 7731958762886597938144329 8969072164948453608247422 $680412371134020618556\dot{7}$	$,0\dot{1}02040816326530612244$ $89795918367346938775\dot{5}$	$,\dot{0}\dot{1}$

Example 4. What is the decimal Expreſſion of $\frac{14}{420}$? By this Table $\frac{14}{42} = ,\dot{3}$. But as $\frac{14}{420}$ is but $\frac{1}{10}$ of $\frac{14}{42}$, it is manifeſt its decimal Expreſſion muſt be $\frac{1}{10}$ of $,\dot{3}$ or $,0\dot{3}$.

Example 5. Turn $\frac{3}{52}$ into a decimal Expreſſion. Here by the Table we have only the Value of $\frac{1}{52} = ,01\dot{9}2307\dot{6}$, and ∵ muſt multiply that Decimal by 3, in Order to have that required; which may be done in the common Manner of multiplying, only obſerving what Carriage would have been, if the Repetend had been once more written down; that is, if the Repetend in this Example had been twice wrote, we ſhould have a 9 on the right Hand Place of the 6; and ∵ as 9 × 3 = 27, ſhould have had 2 to carry to the 6; wherefore, we begin the Operation with ſaying, $6 \times 3 = 18 +$ (the Carriage) $2 = 20$, which is 0 and go 2, then $7 \times 3 = 21 +$ (the Carriage) $2 = 23$, that is 3 and go 2, *&c.* as in common Multiplication.

$$\begin{array}{r} ,01\dot{9}2307\dot{6} \\ 3 \\ \hline ,05\dot{7}6923\dot{0} \end{array}$$

50. *A* TABLE *of* FOREIGN MONEY.

ACCOUNTS *are kept in*

DENMARK *by* Farthings.

Rixdollar =	216
Hor	54
Schelling	2.25

FRANCE.

Livre	44
Sol	2.2
Denier	.18

GERMANY.

Florin	144
Kreuxer	2.40
Pfenin [a]	.30

Rixdollar	220
Kreuxer	2.29
Pfenin [b]	.29

Rixdollar	216
Marc-lub	72
Schelling	4.50
Pfenin [c]	.37

Dollar	216
Kreuxer [d]	2.40

Rixdollar	216
Groch	6
Pfenin [e]	.5

Rixdollar	216
Florin	120
Kreuxer	2
Hellert [f]	.25

IRELAND.

Pound	720
Shilling	36
Penny	3
Farthing	.75

ITALY.

Ducat	240
Gros di Banco [g]	10

Florin	156
Soldi [h]	26

Livre	28.80
Sol	1.44
Denier [i]	.12

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[a] In *Switzerland*, and moſt of the chief Cities.

[b] In *Baſil*, *Francfort*, *Nuremberg*, *Straſburg*, *&c.*

[c] In *Berlin*, *Hamburg*, *&c.*

[d] In *Augsbourg* and *Bolzamont*.

[e] In *Leipſic* and *Naumburg*.

[f] In *Zurich*.

[g] In *Candia*, *Venice*, *&c.*

[h] In *Geneva*.

[i] In *Bergamo*, *Bologna*, *Genoa*, *Mantua*, *Milan*, *Modena*, *Novi*, and *Placentia*.

Crown	244
Julio	24.4
Grain	3.05
Quartrine [k]	.61

Ducat de Banco	162
Tarin	32.4
Grain [l]	1.62

Ounce	600
Tarin	20
Carlin	10
Grain	1
Piccoli [m] (fere)	.17

MUSCOVY.

Ducat	408
Ruble	204
Grif	20.4
Altin	6.12

NETHERLANDS.

Pundt	518.40
Schelling	25.92
Penning	2.16

Pundt	540
Schelling	27
Penning [n]	2.25

Florin	86.62
Stuyver	4.33
Penning [o]	2.16

POLAND.

Rixdollar	216
Groch	2.4
Pfenin	.12

PORTUGAL.

A Mill-Ree	286
Ree	.286

PRUSSIA.

Florin	72
Groch	2.40
Pfenin	.12

SCOTLAND.

Pound	80
Shilling	4
Penny	.33

SPAIN.

A Rea or Rial of Plate	21.6
A Maravedi	.63

Note, 2 Rials of Velloon = 1 Rial of Plate.

SWEDEN.

Rixdollar	216
Copper Dollar	36
Rouſtique	1.12

TURKEY.

Abouquel	49.5
Meidein	1.5
Aſpre	.5
Purſe	112*l.* 10*s.*
—— of Gold	6750*l.*

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[k] In *Ancona*, *Ferrara*, *Rome*.

[l] In *Naples*.

[m] In *Malta* and *Sicily*.

[n] In *Flanders*.

[o] In *Holland*, *&c.*

This Table will ſerve the Purpoſes of much larger Tables; for the real Value of the Money being expreſſed in Farthings of our Money, the *Difference* in Value of any two is found by *Subtraction*; and how many of a leſſer Species are equal to one of a greater, by *Diviſion*.

This Table was firſt printed in the *Gentleman's Magazine*, and from thence, with ſome Corrections, was publiſhed in *Lowe's Arithmetick*; from which laſt-mentioned Author we have here given it, with a Number of Corrections of our own: However, if it ſhould be yet imperfect, we ſhould be glad to receive the Corrections from our mercantile Contributors.

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51. *A* TABLE *of* FOREIGN MEASURES.[a]

I. APPLICATORY.

ARſcheen, Brace, Cane, Cavado Derſch, Ell, Pico, Ras, Shock, Toiſe, Vara, Veſhcove, Yard. *V. Proportions of Meaſures, p.* 65.

II. CUBIC.

1. *Dry*[b].

Almud 11. Alquie 8. Aſnée 3. Bochel 6. Boiſſeau 3. Charge 3. Chetweric 7. Chetwert 7.

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[a] The late ingenious Mr. *Lowe*, who gave this Form to the *Table of foreign Meaſures*, was ſenſible that in a Work of this Nature there muſt be ſome Imperfections; therefore, our ingenious Contributors are deſired to communicate the Corrections of ſuch Places as they know on proper Authority to be erroneous.

N. B. The Proportion of the foreign Meaſures to thoſe of *Britain*, when not ſpecified in the Notes, may be found under the Head of *Proportion of Meaſures*, *p.* 65. The Uſefulneſs of this Table is a ſufficient Apology for its being inſerted.

[b] The following alphabetical *Indexes* of Meaſures and Weights are here given (for the readier Recourſe) with References, by Figures, to the Account given of them in the ſubſequent Paragraphs; which are, therefore, diſtinguiſhed by Numbers prefixed.

Corbe 6. Emine 3. Fanega 11. Halſter 5. Hoedt 5. Killow 12. Laſt 4 5 9. Litron 3. Load 3. Meſurette 3. Mina 6. Mine 3. Minot 3. Moggio 6. Mouver 5. Moy 6. Mude 5. Muid 3. Muken 5. Picotin 3. Pipe 3. Quartier 3. Quartillo 11. Raiſer 5. Rugio 6. Sac 3 5 6. Salma 11. Salmo 6. Scheppel 4 5 9. Setier 3. Staro 6. Stax 6. Tun 3 5 10. Viertel 2. Voye 3.

1. **Denmark**, *V.* Scandinavia. — **Morea**, *V.* Italy. — **Norway**, *V.* Scandinavia. — **Sicily**, *V.* Italy. — **Sweden**, *V.* Scandinavia.

2. **Flanders**: Viertel. *V.* Proportions.

3. **France**: [*Corn*, *&c.*] Quarte 2 Litron[c] 4 Picotin 4 Boiſſeau[d] 3 Minot[e] 2 Mine[f] 2 Setier[g] 12 Muid[h]. —— **Differences** (1) *Char-coal:* Litron 2 Picotin 4 Boiſſeau 8 Minot 2 Mine 16[i] Muid. (2) *Lime:* Boiſſeau 3 Sac 36 Muid[k]. (3) *Oats:* Buſhel = double that of any other Grain. (4) *Plaſter:* Boiſſeau 12 Sac 36 Muid. (5) *Salt*[l]:

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[c] By Ordonnance, the *Litron* is divided into 2 Demi-litrons, and 4 Quarter-Litrons; and contains 36 cubic Inches of *Paris*.

[d] By Ordonnance, the *Paris Buſhel* is to be 8 Inches, 2 Lines, and a Half high; and 10 Inches broad (or, in Diameter) withinſide.—But it is different, in different Juriſdictions. *V. Oats.* (3)

[e] By Ordonnance, the *Minot* is to be 11 Inches 9 Lines high; and 14 Inches 8 Lines in Diameter. — It commonly weighs 240 lb. Marc-weight.

[f] The *Mine* is no real Veſſel; but an Eſtimation of ſeveral Others.

[g] A *Setier* of good Wheat weighs betwixt 244 and 248 lb. Marc-weight. — In *Berri* the Setier conſiſts of 16 Boiſſeaux.

[h] The *Muid* of *Paris* is to weigh 2640 lb. that of *Orleans* 600 lb. — That of *Berri* is 21 Boiſſeaux — That of *Rouen* is 12 Setiers, which makes 14 of *Paris*; and weighs 3360 lb. — And is divided into Mines: but thoſe Mines only contain $2\frac{1}{2}$ *Paris* Setiers.

[i] Inſtead of 16 Mines to the *Muid*, the City Meaſure gives 20.

[k] Or 3 Boiſſeaux is 1 Minot; and 48 Minots, 1 Muid.

[l] *Salt* is ſold, all over *France*, by the Muid; which is greater, or leſſer, according to the Cuſtom of the Provinces, where it is made, and where it is ſold. — In *Marenne* in the Iſle of *Rhé*, and other Places in *France*, where Salt is made, they ſell it by the *Hundred*; which they divide into 28 Muids, and every Muid into 24 Buſhels: and the Hundred commonly makes, in *Amſterdam*, $11\frac{1}{2}$ Laſts, or 23 Tuns.

Meſurette 16 Litron [m] 16 Quartier 4 Minot [n] 4 Setier 10 Muid. (6) *Sea-coal:* Litron 4 Picotin 4 Boiſſeau 6 Minot 15 Voye. (7) *Wood:* Boiſſeau 8 Minot 2 Mine 20 Muid. — VARIATIONS (1) *Aſnée* (of Maçon) = $1\frac{1}{3}$ Setier. (2) *Emine* (of Caſtres) = $\frac{1}{2}$ a Setier [o]. (3) *Load* of Tarſchoon = lb. 352.80: of Arles, 360: of Beaucaire, 367.20: of St. Gilles, 428. (4) *Pipe* (in Bretagne) = 10 Charges, each of 4 Boiſſeaux; and weighs 600 lb. (5) *Tun* [p] of Auray = lb. 2200: of Breſt, Port-louis, Quinpercorentin, 2240: of Audierne, 2300.

4. GERMANY: Scheppel [q] 90 Laſt.

5. HOLLAND: Scheppel 4 Mude 27 Laſt [r]. —— DIFFERENCES: *Oats:* Laſt (at Berg-op-zoom) 28.25 Viertels. (At Bruges) 14.50 Hoedts. (At Dixmuyde) 24 Raiſers. (At Ghent) 19 Sacs, or 38 Halſters. (At Gravelin) 18 Raiſers. (At Liſle) 30 Raiſers. (At Steenberge) 29 Viertels. (At Tongres) 14 Muids. —— VARIATIONS: (1) *Halſter* = [Muid] $\frac{1}{6}$ in Louvain. [Sac] $\frac{1}{2}$ at Ghent. (2) *Hoedt* = [Mouvers] 8, in Bois-le-duc, Cleves, and Guelderland. [Muids] 9 in Overyſſel, Zwell. [Raiſers] 18, at Dunkirk. [Sacs] 10.50, at Utrecht 10.66, at Delf, Rotterdam. [Viertels] 13 at Amſterdam. (3) *Laſt* = [Hoedts] 17.50, at Bruges. [Mouvers] 20.50, at Bois-le-duc. 22, in Guelderland. [Muids]

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[m] The *Litron* for Salt is larger than that for Corn; and is divided into 2 Halves, 4 Quarters, and 16 Meſurettes.

[n] The *Minot* of Salt commonly weighs 240 lb. Marc-weight.

[o] The *Setier* of *Caſtres* weighs about 200 lb. Weight of that Place; which is about 170 lb. Marc-weight.

[p] The *Tun* of *Nantes* conſiſts of 10 Setiers; the Setier, of 10 Buſhels. The Meaſure . . being heaped, it weighs between 2200 and 2250 lb. . . when ſtricken, it weighs 18 or 20 *per Cent.* leſs.

[q] Of the *Scheppels* of *Hamburgh*, 83 make about 10 Quarters in *London.*

[r] The *Laſt* commonly weighs 4000 lb. Weight — At *Amſterdam*, and in *North Holland*, a Laſt . . of Barley, commonly weighs between 3200 and 3400 lb. . . of Rye, 4000 and 4200 . . of Wheat, 4200 and 4800.

8, in Liege [s]. 15, at Tongres. 25, at Utrecht. 33, in Frieſland. 36, at Deventer. 37, at Louvain. [Raiſers] 22.58, at Gravelin, St. Omers. 30.50, at Dixmuyde, 41, at Liſle. [Sacs] 4.50, at Middleburg [t], in Zeeland. 25, at Bruſſels, Utrecht. 26, at Alcmaer. 28, at Tergou. 29, at Delf, Rotterdam, Schedam [u], and Ghent. 33, at Haerlem. 36, at Amſterdam, Montfort, Vianen, Yſelſtein [w]. 44 [x], at Enchuyſen, Hoorn, Muyden, Naarden, Weſop. [Scheppels] 33, at Thiel. 68, at Ruremond. 90, at Hamburg. [Viertels] 28, at Mechlin. 33.50, at Breda, Steenberge. 34, at Berg-op-zoom. 37.50, at Amſterdam. 38, at Antwerp. (4) *Mouver* = 4 Scheppel. (5) *Muken* = $\frac{1}{4}$ of a Viertel. (6) *Raiſer* = [Scheppels] 2, at St. Omers. (7) *Sac* [Muid] $\frac{1}{6}$, at Ghent. [Scheppels] 2, at Enchyſen. 2.25, in Zeeland. 2.50, at Brill, Fluſhing, Zurickſee. 3, at Amſterdam. 8, at Leyden. [Viertels] 32, at Antwerp.

6. ITALY: *Bologna:* Corbe [y]. *Florence:* Staro [z] 24 Moggio. *Genoa:* Mina 20 Tun [a]. *Leghorn:* Stax 3 Sac [b] 8 Moggio. *Morea:* Bochel [c]. *Rome:* Rugio [d]. *Sicily:* Salmo [e]. *Venice:* Staro [f].

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[s] Where they reckon 12 Setier to a *Muid*, and 8 Muids to a Laſt.

[t] Where the *Sac* is little more than 2 Scheppels.

[u] The *Laſt* of theſe Places is 2 *per Cent.* more than the Laſt of *Amſterdam.*

[w] Where they reckon 2 Sacs to a Muid.

[x] Each *Sac* containing 2 Scheppels.

[y] Of the *Corbes*, 100 make about 92 Setiers of *Paris.*

[z] The *Staro* weighs about 50 lb. — Salt is ſold by the Staro of 72 lb.

[a] The *Tun* of *Genoa* is about 40 Buſhels of Wincheſter Meaſure.

[b] At *Leghorn* 5 Sacs is accounted to make little leſs than 10 *Engliſh* Buſhels.

[c] Of the *Bochels*, 9 $\frac{2}{3}$ make about 8 Buſhels *Wincheſter* Meaſure.

[d] The *Rugio* weighs about 412 lb. and makes, in *Florence*, 2 $\frac{1}{2}$ Staros.

[e] Of the *Salmo*, they have 2 Sorts; *viz.* a great one, and a ſmall one; which is the common one; and is about 7 $\frac{1}{9}$ Buſhels, *Wincheſter* Meaſure; and about 17 *per Cent.* leſs than the great one.

[f] The *Staro* of *Venice* is about 1 Setier of *Paris.*

7.

7. MUSCOVY : Chetweric 8 Chetwert [g].

8. PORTUGAL : Alquie 60 Moy [h].

9. PRUSSIA : Scheppel 60 Laſt [i].

10. SCANDINAVIA : Tun [k].

11. SPAIN : *Biſcany:* Fanega[l]. *Catalonia:* Quartillo [m] 4 Salma. *Granada :* Almude [n] 2 Fanega. *Valencia :* Fanega [o].

12. TURKEY : Killow [p].

2. *Liquid.*

Aem 1 3 4. Achtelin 3. Açumbré 9. Amphora 5. Anker 4. Arroba 9. Aſnée 2. Barile 5. Barique 2. Baſſee 5. Beſon 3. Bigorza 5. Boccale 5. Botte 2. Brenta 5. Buſſard 2. Cantar 9. Cavada 6. Chopine 2. Cogno 5. Demi-ſetier 2. Driolinck 3. Fat 7. Feoder 3 4. Feuillette 2. Fiaſco 5. Heemer 3. Je 3. Kan 8. Lera 5. Levor 5. Loder 5. Maſſem 3. Maſtilli 5. Mero 5. Meter 20. Millerole 2. Miſerole 5. Mittidel 5. Mizaro 5. Muid 2. Muſſie 4. Oke 10. Oxio 5. Pignatoli 5. Pinte 2 4. Pipe 1 2 9. Poinçon 2. Poiſſon 2. Pot 2 6 8. Quarta 5. Quartillo 9. Queuë 2. Reoder 3. Rubbia 5.

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[g] The *Chetwert* holds about 5 $\frac{3}{20}$ Buſhels of *Wincheſter* Meaſure.

[h] A *Moy* contains about 3 *Engliſh* Quarters.

[i] Of the *Pruſſia* Scheppels, 4 make a Muid ; which is a Stone of 34 lb.

[k] The *Tun* of *Denmark* and *Norway* is about 4 *London* Buſhels.

[l] Of the *Fanegas* of *Biſcany* and *Valencia*, 5 make an *Engliſh* Quarter.

[m] The *Quarter* of *Catalonia* is the ſame as the *Engliſh* Quarter : and 2½ of them is a Cargo, or Load, of 360 lb. Weight.

[n] An *Almude* is about 1½ *Engliſh* Buſhels : in Weight . . by Heap, 144 lb. . . by Strike, 99 lb. *Engliſh*. — *N. B.* 100 *Sacs* of *Granada* make . . 3 Laſts 10 Muids of *Amſterdam* . . 64 Setiers of *Paris* . . 128 Buſhels of *Bourdeaux*.

[o] The *Fanega* of *Valencia* makes about 2 *Engliſh* Buſhels.

[p] The *Killow* weighs about 20 Okes : and 8 ⅔ of them make about 1 *Engliſh* Quarter.

Sachie 5. Salma 5. Scandale 2. Schreve 3. Sechys 5. Seiltin 3. Setier 2. Somar 9. Staio 5. Steckan 4. Stoop 1 4 7. Tomolo 5. Tun 2 4 5. Viertel 3 4. Urna 5. Yune 3.

1. **Flanders**: Stoop [q] 50 Aem [r] 3.04 Pipe. —— **Differences**: *Beer* is ſold by the Barrel of 52 Stoops. The Stoop contains 7 Pints: and the Aem is about 42 Gallons Wine-meaſure, in London.

2. **France**: Poiſſon [s] 2 Demiſetier 2 Chopine [t] 2 Pinte [u] 2 Pot 4 Setier [w] 36 Muid [x] 3 Tonneau. —— **Differences**: *Brandy* (at Montpelier, Tholouſe, *&c.*) is ſold by the Quintal of 100 *lb.* Weight with the Caſk. —— **Variations**: (1) *Aſnée* (at Lion) 80 Pots. (2) *Barique* (at Bourdeaux) $\frac{1}{4}$th of a Tonneau, or 3 Paris Muids. (3) *Botte* (in the Southern Provinces) a Veſſel containing about a Muid. (4) *Buſſard* (in Anjou and Poitou) half a Pipe. (5) *Feuillette* (in Burgundy) 120 Pintes (in ſome Provinces 1½ a Pinte.) (6) *Millerole* (in Provence) about 66 Pintes of Paris. (7) *Pipe* [y] (in Anjou and Poitou) a Muid and half of Paris, or 432 Pintes. (8) *Poinçon* (at Paris) a Demi-queue, or half a queue. (In Blaiſois and Touraine) half the Orleans-tun, or 288 Pintes. (At Rouën) 13 Boiſſeaux. (9) *Queuë* (in Champagne, and at Orleans) 54 Setiers. (10)

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[q] The *Stoop*, they reckon, weighs 6 lb. of their Weight.

[r] Six *Aems* make a Tun of 252 Gallons, *Wincheſter* Meaſure.

[s] The *Poiſſon* contains 6 cubic Inches.

[t] The *Chopine* of common Water weighs 1 lb. of *Paris*.

[u] The *Pinte* . , of *Paris* is about the 6th Part of the *Roman* Congius; and contains 2 lb. of common Water. That . . of *St. Denis* is bigger — In ſeveral Places it is called a *Pot*.

[w] The *Setier*, among Gaugers, is an Eſtimation of 8 Pintes.

[x] The *Muid* — at *Bourdeaux*, contains 110 Pots with the Lee, or 100 Pots clear Meaſure; the Pot containing about 2 Mingles of *Amſterdam* — of *Champaign* = 48 Setiers — of *Montpelier* and ſeveral other Places in *Languedoc*, = 18 Setiers; the Setier 32 Pots, equal to 35 Steckans of *Amſterdam* — of *Paris*, according to an Ordonnance of *Henry* IV. 300 Pintes; according to the Regulation of *Louis* XIII. 280 Pintes.

[y] The *Pipe* goes by the Name of *Tun* in the Provinces beyond the *Loire*.

Scandale (in Provence) $4\frac{1}{2}$ Gallons Wincheſter-Meaſure. (11) *Tonneau* (at Bayonne and Bourdeaux) 3 Paris Muids, or 864 Chopines. (At Berri, and Orleans) near 2 Paris Muids, or 576 Chopines.

3. GERMANY: Maſſem 4 Viertel 20 Aem 6 Feoder[z] 2.50 Reoder. —— VARIATIONS . . *Augsburg*: Maſſem 8 Beſon 12 Je[a] 8 Feoder . . *Heidelberg*: Maſſem 4 Viertel 12 Aem 10 Feoder . . *Nuremberg*: Maſſem 64 Heemer 12 Feoder . . *Vienna*: Seiltin 4 Achtelin 32 Heemer 32 Feoder . . *Wirtemberg*: Maſſem 10 Yune 16 Aem 6 Feoder. — *N. B.* (1) *Driolinck* (of Vienna) = 24 Heemers. (2) *Schreve* (of Vienna) = 4 Maſſems.

4. HOLLAND: Muſſie 4 Pint 2 Mingle 2 Stoop 8 Steckan 2 Anker 4 Aem 14 Feoder. —— DIFFERENCES: (1) *Hony* (at Amſterdam) is ſold at ſo many Pounds Flemiſh *per* Tun; conſiſting of ſix Aems . . or by ſo many Florins *per* Barrel . . or by the Hundred-weight. (2) *Oils* are ſold by the Tun; which contains 6 Aems, equal to 1600 Paris Pintes. . . Coarſe Fiſh-oil they commonly keep in Barrels, containing from 15 to 20 Steckans. —— VARIATIONS: (1) *Viertel*, or *Virge*[b]: $5\frac{1}{6}$ Mingles . . that of Wine, 6 Mingles . . that for Brandy, $6\frac{1}{4}$. (2) Tun, *V.* Differences: Hony, Oil.

5. ITALY: *Calabria*: Pignatoli[c] 32 Staio 10 Salma. *Ferrara*: Sechys 8 Maſtilli. *Florence*, and *Leghorn*: Mittidel 2 Fiaſco 20 Barile 10 Cogno.

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[z] The *Feoder* is ſuppoſed the Load of a Waggon with 2 Horſes.

[a] The *Je* is 12 Beſons, or 3 Muids.

[b] At *Bruges* they call the *Virges* Seſtiers; reckoning 16 Stoops to a Seſtier. — *N. B.* It is common to put *French*, *Spaniſh*, and *Portugal* Wines into Pipes, Butts, and other Pieces; ſome of which will contain, at *Amſterdam*, 60 to 90 Virges. It is uſual, therefore, with the *Hollanders*, to reduce thoſe Meaſures into Butts by the following Reckoning: *For one Butt*, Virges . . 27, at *Cognac*, *Embden*, *Monguion*, *Rhé*, *Rochelle* . . 29, at *Nantes*, and other Places in *Bretagne* and *Anjou* . . 30, at *Hamburgh*, and *Lubeck* . . 32, at *Amſterdam*, and other Places in *Holland*; and at *Bourdeaux*, and other Places in *Guienne*.

[c] The *Pignatoli* is equal to the *French Pinte*.

Genoa: Barile 2 Miſerole 5 Botte [d]. *Iſtria:* Sechys 6 Urna. *Morea:* Lodere. *Rome:* Boccale [e] 7.50 Rubbia 13.50 Brenta. *Venice:* Lera 4 Sachie 4 Quart 4 Bigorza 4 Amphora [ee]. *Verona:* Baſſee 16 Brenta. —— Differences: (1) *Corn* (at Venice) is ſold by the Staio; and is, in Quantity, about 1 Setier of Paris. (2) *Oil* (at Genoa) Barile 14 Tun [f]. (At Leghorn) Mittidel 32 Oxio. (In Morea) Levor [g] 10 Barile. (At Naples) Tomolo 16 Salma [h]. (At Venice) Mero [i] 40 Mizaro.

6. Portugal: Cavada 12 Almuda [k]. —— Variations: *Pots* are alſo uſed, holding . . ſome ½ a Gallon . . others ¼ of a Gallon.

7. Prussia: Stoop 180 Fat [l].

8. Scandinavia: Pot 4 Kan [m].

9. Spain: Quartillo 4 Açumbre [n] 8 Arroba [o]. *Andaluſia:* Quartillo 4 Somar 8 Arroba. *Valencia:* Cantar [p]. —— Differences: *Oil* [q]: Pipe 40.50 Arroba.

10. Turkey: Oke 8 Meter [r].

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[d] The *Botte* is about 100 Pints — The *Venetian* Botte is divided into Muſtaches; 76 whereof make the Amphora.

[e] The *Boccale* contains ſomewhat more than a *Paris* Pinte.

[ee] By wholeſale, the *Amphora* is 14 Quarts; and the *Bigorza*, 3 ½ Quarts

[f] The *Tun* of *Genoa* makes 236 *London* Gallons.

[g] The *Levor* weighs 7 ½ lb. 10 whereof fill a Candia-barile; which ſhould hold 15 *Engliſh* Gallons, and weigh 112 ½ lb. Avoirdupois.

[h] 5 ½ *Salmas* are accounted to a Tun of 236 Gallons of Oil in *London*.

[i] The *Mero* is about 3 ½ Pints by Meaſure: but, by Weight, it is more.

[k] The *Almuda* holds 4 ½ Gallons *Wincheſter* Meaſure.

[l] The *Fat* is about 80 Stoops *Antwerp* Meaſure.

[m] The *Kan* holds near 1 Gallon *Wincheſter* Meaſure.

[n] An *Açumbre* is about 3 Pints *Engliſh*.

[o] 81 *Arrobas* in *Andaluſia*, make 252 Gallons *Engliſh* . . but, upon the *Seville* Gauge, 236.

[p] The *Cantar* holds about 2 Gallons *Wincheſter* Meaſure.

[q] The Meaſure for *Oil* is the Arroba of 25 lb. *Hayes, Negotiator's Magazine*, P. 350.

[r] The *Meter* is about 2 thirds of a *Wincheſter* Gallon.

WEIGHTS[*].

Ace 4. Arrata 7. Arroba 5, 7, 10. Baleman 11. Barquit 6. Cantar 5, 11. Carat 2, 4. Cargo 10. Caſtillan 10. Centner 3, 8, 9. Denier 5. Dram 2, 4, 5, 7, 10, 11. Engel 4. Grain 1, 2, 4, 5, 10, 11. Gros 4, 5. Killet 11. Laſt 8, 9. Liſpound 3, 4, 8, 9. Livre 2. Load 4. Loodt 3, 4. Marc 2, 4, 5, 10. Medical 11. Millier 2. Numile 3. Obolo 5. Ochavo 7. Octave 5. Oke 5, 11. Ounce 1, 2, 3, 5, 7, 10, 11. Penny-weight 1. Pood 6. Pound 1, 3, 4, 5, 7, 8. Primi 5. Quarta 5. Quint 3. Quintal 1, 2, 5, 7, 10. Quirat 11. Rota 11. Rotello 5. Rugio 5. Saliqua 5. Schippound 3, 4, 8, 9. Seſtertia 3. Scruple 2, 5. Shock 8. Stone 3, 4, 8. Tomine 10. Tun 3. Vicomté 2. Wage 4. Zollotnic 6.

1. FLANDERS: Ounce 16 Pound 100 Quintal. — TROY: Grain 32 Penny-weight 20 Ounce 8 Pound[t].

2. FRANCE: Livre 100 Quintal[u] 10 Millier. — TROY[w]: Karat 24 Grain 24 Scrupule 3 Drachme 8

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[*] In moſt Parts of *Europe* the Weights for coarſer Commodities are *different* from thoſe, with which they weigh Gold, Silver, Jewels, &c. Theſe latter I ſhall take Notice of under the Title of *Troy-weight.* — In the mean Time it muſt be obſerved that there are, in ſome Places, ſtill farther Diſtinctions of Weights: Thus . . at *Genoa* they have 5 Kinds of Weights: (1) Large Weights, by which all Merchandizes are weighed at the Cuſtom-houſe. (2) Caſh-weights, for Piaſtres, and other Species. (3) The Cantara or Quintal, for the coarſeſt Commodities. (4) The Large-balance, for raw Silks; and (5) The Small-balance for finer Commodities.

[t] 112 *Flemiſh Pounds* make about 100 lb. Troy-weight of *London*.

[u] In ſome Places a Difference is made between a *Quintal* and a *Hundred-pound Weight:* ſo that the Buyer and Seller are obliged to explain themſelves on this Head in making of Bargains.

[w] Beſide (1) The *Poids de Marc*, generally uſed by Merchants for Things in little Compaſs; (2) In *Provence* and *Languedoc* they have the *Poids de Table*; which is found to differ, in ſeveral Places, from the Marc-weight, 18, 20, or 25 *per Cent.* 16 Ounces of

8 Once[x] 8 Marc 2 Livre[y]. —— DIFFERENCES . . . *At Calais* they have three Sorts of Weights: (1) The firſt is the *Town-weight*; 100 lb. of which makes about 92 lb. in London. (2) The ſecond is the *Merchant's Weight*; of which 100 lb. makes 113 in London. (3) The third is called the *Engliſh Wool-weight*; and is about 3 *per Cent.* lighter than the Town-weight . . . *At Lions*, the Weights are of two Sorts: (1) The *City-weight*, which weighs 14 Ounces of the Pound *de Marc*, for common Uſe; and (2) The *other* of 15 Ounces, for Silk. *N. B.* 100 lb. Silk-weight makes 108 of the City-weight: and 100 lb. of the City-weight is about 94 lb. Avoirdupois Engliſh . . . *At Rouen*, they have two Sorts of Weight: *viz. de Vicomté*, and *de Marc*, 108 lb. of the former makes 100 of the latter, or 110 lb. of Paris-weight[z].

3. GERMANY: Loodt 2 Ounce 16 Pound[a] 14 Liſpound[b]. —— TROY: Seſtertia 4 Numile 4 Quint 4 Loodt 16 Marc[c]. —— VARIATIONS: (1) *Cent-*

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of the Table being ſcarce 14 of the Marc. (3) At *Rouën* they have another Sort of Weight, called the *Poids de Vicomté*; 100 lb. of which is reckoned to make about 108, or 110 lb. Marc, or *Paris* Weight; eſpecially in weighing of Wool: but, in weighing other Things, the ſaid 100 lb. is counted 104 lb. Marc-weight[*]. (4) In *Languedoc*, they have alſo another Sort of Weight, called the *Roman* or *Statera-weight*; which is much the ſame as the Table-weight.

[*] Of the Vicount-weight, they have no leſs a Weight than 13 lb. ſo that all Goods, weighing under that Weight, are weighed by the Marc weight.

[x] The *Ounce* — with Goldſmiths and Monyers, is divided as follows, *viz.* Grain 7.20 Felin 2 Maille 2 Eſtelin 20 Once —— with Phyſicians, is the 12th Part of the Livre. *V. Livre* [y].

[y] The *Livre* — of *Avignon*, *Lions*, *Montpelier*, and *Toulouſe*, is 13 Ounces: that — of *Rouen*, 14: that — of *Marſeille*, and *Rochelle*, 19, *V. Ounce* [x].

[z] This laſt is only to be underſtood in the weighing of Wool: or elſe 100 lb. Marc-weight weighs but 104 lb. of the Viſcount-weight. *V. Note* [*]. — And 100 lb. *de Vicomté* is about $113\frac{1}{3}$ lb. Avoirdupois *Engliſh*.

[a] The *Pound* of *Vienna* . . in ſome Commodities, is divided into 32 Loodts . . in others, into 28 Pints.

[b] A *Liſpound* of Feathers, or Wool, is 16 lb.

[c] 116 Marcs make 100 lb. Troy-weight in *London*.

ner[d] = 8 Liſpound. (2) *Schippound* of Feathers and Wool is 20 Liſpound. (3) *Stone* . . of Flax, 20 lb. . . of Wool and Feathers, 10 lb. (4) *Tun* of Butter and Tallow is 16 Liſpound.

4. HOLLAND: Grain 4 Dram 3 Gros 2.50 Engel 10 Loodt 16 Marc 2 Pound 8 Stone. —— TROY: (1) *For weighing Groſs Gold:* Ace 32 Engel 20 Ounce 8 Marc, (2) *For Fining of Gold:* Parts 24 Grain 12 Carats 24 Marc[e]. —— DIFFERENCES: *Salt* (at Amſterdam) is ſold by a great Hundred of 404 Scheppels; which is reckoned 7 Laſts, or 14 Tuns, or 28000 lb. which is alſo counted 208 Sacs; and is ſold by the Pound Flemiſh: and 11 ½ Laſts of Amſterdam make about the great Hundred in the Iſle of Rhé in France. —— VARIATIONS: (1) *Liſpound* = 15 Pounds. (2) *Load* = 400 lb. (3) *Schippound* = 20 Liſpound. (4) *Wage* = 165 lb.

5. ITALY: Ounce 12 Pound 25 Arroba 6 Quintal. *Morea:* Dram 11.11 Ounce[f] 12 Pound 3 Oke 45 Quintal. *Sicily:* Ounce 12 Pound 2.50 Rotello 100 Cantar[g]. *Venice:* Pound 100 Quintal[h]. —— TROY: *Genoa*, *Florence*, and *Leghorn:* Grain 24 Denier 3 Groſs 8 Ounce 8 Marc[i]. *Naples:* Octave

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[d] A *Centner* makes about 120 lb. Avoirdupois in *London*. — The Centner of Tin, at *Dantzic* (which conſiſts of 120 lb.) makes, in *London*, about 112 lb.

[e] The *Marc-weights* of *Holland* are about 1 *per Cent.* lighter than the Troy-weight of *London*.

[f] In weighing *raw Silk*, they reckon 15 Ounces to the Pound.

[g] The *Cantar* is about 176 lb. Avoirdupois in *London*.

[h] The *Venice Weights* are diſtinguiſhed into groſs and ſuttle. (1) By the *Groſs* they weigh coarſer Metals, Feathers, and other lumbering Commodities. (2) By the *Suttle* they weigh Silk, Spices, Drugs, &c. — And (1) 100 lb. Groſs-weight, make 158 lb. Suttle; or 106 lb. Avoirdupois in *London*. (2) 100 lb. Suttle, make 63.66 Groſs; or about 65.75 Avoirdupois in *London*.

[i] Of the *Italian Marcs*, 100 lb. Troy-weight, in *London*, make about . . 130, at *Genoa* . . 158.50, at *Florence*. — *N. B.* 100 lb. Troy-weight, is equal to 86.50 lb. of their Weight for Silver; wherein they reckon 12 Ounces to the Pound.

8 Ounce 12 Pound[k]. *Rome:* Primi 4 Saliqua 3 Obolo 2 Scruple 3 Dram 8 Ounce 8 Marc. *Venice:* Grain 4 Saliqua 9 Quarta 4 Ounce 8 Marc[l]. —— DIFFERENCES: *Corn*, or Grain (at Florence) is ſold by the Moggio of 24 Staios, of 50 lb. Weight, each. —— VARIATIONS: (1) *Quintal* (at Leghorn) is of 4 Sorts: that of . . 100 lb. for Alum, 150 lb. for Sugar . . 151 lb. for Fiſh . . 160 lb. for Wool. (At Rome) of 2 Sorts: that of . . 160 lb. for Spices, and other choice Goods . . 250 lb. for heavy bulky Goods; about 200 lb. Avoirdupois in London. (2) *Rotello* (at Genoa) is 18 Ounces. (3) *Rugio* (at Rome) is 412 lb. of their Weight; and makes (in Florence) 3 ½ Staios.

6. MUSCOVY: Zollotnic 96 Pound[m] 40 Pood 10 Barquit.

7. PORTUGAL: Dram 2 Ochavo 8 Ounce[n] 16 Arrata 32 Arroba 4 Quintal[o].

8. PRUSSIA: (1) *Groſs or bulky Goods:* Pound 34 Stone[p] 10 Schippound. (2) *Fine Goods:* Pound 16 Liſpound 20 Schippound. —— VARIATIONS: (1) *Centner* of Tin (at Dantzic, 120 lb. at Koningſberg, 128 lb.) makes, in London, 112 lb. (2) *Laſt* . . of Hemp, Flax, and ſuch light Goods is 6 Schippound, or 60 Stone, *viz.* at Dantzic, 2040 lb; at Koningſberg, 2400 lb.[q]. But . . for Pot-aſhes, the Laſtage is reckoned double; *viz.* 12 Schippound of

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[k] 86.50 *Naples Pounds* make about 100 lb. Troy-weight, in *London.*

[l] 116.50 *Roman, Venetian,* and *Verona Marcs* make 100 lb. Troy-weight, in *London.*

[m] The *Ruſſia* Pound is counted equal to 13 oz. 3 dw. 6 gr. Troy-weight; or 14.15 oz. Avoirdupois.

[n] The *Portugal Ounce* is alſo divided into Penny-weights and Grains; the ſame Parts the Ounce Troy is divided into, in *London.*

[o] The *Portugal Weights* are between 3 and 4 *per Cent.* heavier than the *London* Avoirdupois.

[p] The little *Pruſſia,* and *Dantzic Stone* (for Pepper, Spices, and other fine Goods) is 24 lb.

[q] The *Laſt* . . of *Dantzic,* makes, in *London,* about 17 cw. 7 lb. . . of *Koningſberg,* a ſmall Matter above 18 cw.

them take up no more Room, in a Ship, than 6 Schippound of Hemp or Flax. (3) *Shock* is 60 Pieces of any Commodity ſold by Tale[r].

9. SCANDINAVIA: Pound 20 Liſpound 20 Schippound[s]. —— VARIATIONS: (1) *Centner* of Gunpowder is 120 lb. (2) *Laſt* of Flax, Hemp, Cordage, and Tallow, is 6 Schippound.

10. SPAIN: Dram 8 Ounce 8 Marc 2 Pound 25 Arroba 4 Quintal. *Caſtile:* Grain 28 Dram 16 Ounce 16 Pound 25 Arroba 4 Quintal. *Catalonia:* Quintal[t] 3 Cargo. *Valentia:* Pound 24 Arroba 4 Quintal 2.50 Cargo. —— TROY: *Gold:* Grain 12 Tomine 8 Caſtillan 6.25 Ounce 8 Marc. *Silver:* Grain 75 Dram 8 Ounce 8 Marc[u]. —— DIFFERENCES: *Wool* is ſold by the Arroba of 30 lb. And 100 lb. of London makes about 102 lb. in Catalonia: but it makes about 92 lb. of their Wool-weight.

11. TURKEY: Grain 4 Quirat 16 Dram 12 Ounce 12 Rota 150 Cantar[w]. —— TROY: Killet 24 Medical[x]. —— DIFFERENCES: *Silk* is ſold by the Baleman, containing 6 Okes[y].

SYNONYMS.

Aſchnee, *V. Aſnée.* Aum, *Aem.* Barrel, *Brenta.* Bottle, *Boccale*, *Mingle.* Brante, *Brenta.* Buſhel, *Boiſſeau*, *Staio.* Butt, *Pipe.* Carat, *Saliqua.* Denier, *Scrupule.* Dram, *Medical.* Fat, *Pipe.* Feillette, *Feuillette.* Fertel, *Viertel.* Flaſk, *Fiaſco.* Fuillet, *Feuillette.* Gallon, *Setier.* Grain, *Primi.* Gros, *Drachme.* Hemine, *Chopine.* Hogſhead, *Muid.* Karat, *Carat.* Letcht, Leth, *Laſt.* Load, *Cargo, Salma, Voye.* Mil-roe, Owl, *Millerole.* Mingeeble, *Mingle.* Mui, *Muid.* Oak, *Oke.* Peché, *Pot.* Penny, *Numile.* Pinte, *Pot.* Poſſon, *Poiſſon.* Punchion, *Poinçon.* Pound, *Arrata.* Quarte, *Pot.* Robe, Rove, *Arroba.* Roquille, *Poiſſon.* Schreve, *Viertel.* Septier, *Setier.* Setier, *Chopine.* Somme, *Somar.* Staro, *Staio.* Stekamen, *Steckan.* Tierce, *Aem.* Tiſchauſera, *Sachie.* Toiſe, *Fathom.* Tun, *Tonneau.* Verge, *Viertel.* Virge, *Viertel.* Virtule, *Viertel.* Voedar, *Feder.* Zambre, *Açumbre.*

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[r] Thus, a *Shock* . . of Wood is 60 Pieces; . . of Linen, 60 Auns. — But, when Linen is very narrow, and not creaſed or folded double; 2 Auns of ſuch Linens is reckoned but 1 Aun.

[s] 7.50 of the *Swediſh* Schippound make, in *London*, 20 cw.

[t] *Wool* is ſold by the Arroba of 30 lb.

[u] Their Weight *for Gold and Silver, in Duſt, or in Bars, in the Indies*, is as follows; a Tomine is worth 2 Rials; and the Tomine weighs 16 Grains: a Caſtillan is 16 Tomine 4 Grains: 6 Caſtillan and 2 Tomines make 1 Ounce Marc-weight: and 8 Ounces to 1 Marc.

[w] The *Turkey Cantar* makes about 120 lb. in *London*.

[x] 20 *Medical* of Gold is 3 Ounces, Troy-weight, in *London*.

[y] If the *Baleman* be weighed with the Lodera; it will weigh 13 Loderas, and 112 Drams.

PROPORTIONS* *of* MEASURES.

(APPLICATORY.)

Feet.

AN Engliſh Foot divided into 1000 Parts, Others will be as follows: At Amſterdam, 942. Antwerp, 946. Bononia, 1204. Bremen, 964. Cairo[a], 1824. Cologn, 954. Conſtantinople[b], 2314. Copenhagen, 965. Dantzic, 944. Dort, 1184. Florence[c], 1913. Francfort, 948. Greek, 1007.

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* The *Proportions* being fixed as underneath, it will be eaſy (thereby) to find an Anſwer to other Queſtions relating to the following Articles: *e. g.* — What is the Value of 1 *Vara* of *Spain?* (*Anſw.*) 3.70 qrs. of a Yard. For, As 108 Varas, to 100 Yards; So 1 Vara, to 3.70 qrs. yd. — What Number of *Engliſh Yards* anſwers to 100 Varas of *Spain?* (*Anſw.*) 92.59 Yards. For, As 108 Varas, to 100 Vards; So 100 Varas, to 92.59 Yards. — What is the Value of one *Ell* of *England?* (*Anſw.*) 1.25 Yards. For, As 80 Ells, to 100 Yards; So 1 Ell, to 1.25 Yards.

[a] Called the *Derſch.*

[b] Called the *Pike,* or *Pico.*

[c] Called the *Brace,* or *Braccio,*

Leyden, 1033. Lorrain, 958. Mantua, 1569. Mechlin, 919. Middleburg, 991. Naples, 2100. Paris[d], 1068. Prague, 1026. Rhinland[e], 1033. Riga, 1831. Roman: *Pes Colotianus*, 967: *on the Monument of Coſſutius*, 970: *on that of Statilius*, 972: *on a Congius of Veſpaſian*, 986. Spain, 1001. Straſburg, 920. Toledo, 899. Turin, 1062. Venice, 1162.

YARDS.

100 *Yards of England* make

Arſcheens —— 140.35, *of* Ruſſia.

Braces — 80, *of* Ferrara — 104.33, *of* Bergamo, Bologna — 130.90, *of* Ancona — 136, *of* Mantua, Modena, Venice; *for Silk*: — 136.26, *of* Milan; *for Cloth.* —— 154.75, *of* Florence, Leghorn, Lucca — 171.33, *of* Milan: *for Silk.*

Canes — 38.69, *of* Leghorn — 40, *of* Naples, Sicily — 40.66, *of* Genoa — 44, *of* Rome — 46.66, *of* Marſeilles, Montpelier — 50, *of* Guienne, Thoulouſe — 55.66, *of* Barcelona — 57.33, *of* Andaluſia — 61.19, *of* Avignon, Dauphiné, Provence — 97.33, *of* Valencia.

Cavados — 133.33, *of* Portugal.

Ells — 77.22, *of* Lions — 78, *of* France — 80, *of* Ferrara, Geneva, London, Oſnabrug — 114.60, *of* St. Gall *for Woollen* — 125, *of* Cambray, Douay — 126, *of* Bruges — 130, *of* Ypres — 131, *of* Artois — 131.66, *of* Brabant, Narva — 132, *of* Ruremond — 133.33, *of* Holland, Norimberg — 134, *of* Antwerp — 135, *of* Dunkirk, Middleburg — 139, *of* Guelderland, Maeſtricht — 144, *of* Biſcany (for Silk) Tournay — 146, *of* Denmark — 148.66, *of* Norway — 149.33, *of* St. Gall *for Linen* — 150, *of* Dantzick — 151, *of* Liege — 153.85, *of* Pruſſia — 156, *of* Sweden — 156.25, *of* Naumberg — 160, *of* Baſil, Bern, Cologn, Francfort, Hamburg, Koningſberg, Leipſic, Lubec, Riga. — 166.60, *of* Breſlaw.

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[d] A *Toiſe* is 6 Feet.

[e] Or *Leyden* Foot; by which moſt of the Northern Nations go.

Picos — *of* Candia (*for Silk*) 162.68 (*for Cloth*) 152.68 — *of* Turkey (*for Camlets and Grograms*) 150 (*for Linen*) 66.66 (*for Woolen*) 133.33.

Ras — 16.30, *of* Piedmont.

Shocks — 1.40, *of* Poland.

Varas — 81.33, *of* Portugal — 108, *of* Spain — 109, *of* Granada.

Veſhcoves — 8.77, *of* Ruſſia.

(CUBIC.)

Corn.

10.25 *Quarters of England* make

Ackſings — 27, *of* Delf and Rotterdam — 88, *of* Schoonhoven.

Aſnées — 14.72, *of* Lions.

Buſhels — 38, *of* Bourdeaux.

Fanegas — 157, *of* Spain.

Hoedts — 7.50, *of* Bruges.

Laſt — 1, *of* Amſterdam.

Muids — 10.50, *of* Bruxelles — 4.58, *of* Ghent — 27, *of* Amſterdam — 33, *of* Groeningen.

Raiſers — 18, *of* Dunkirk.

Sacks — 28, *of* Dort — 40, *of* Middleburg.

Scheppels — 30.75, *of* Dantzic — 56, *of* Pruſſia — 83, *of* Hamburg.

Setiers — 17.53, *of* Calais — 19, *of* Paris.

Viertels — 37,50, *of* Antwerp.

(ITINERARY.)

A Mile[f] *contains Rhinland Feet*

In Arabia, 6187. Britain, 5454. Burgundy, 6000. Egypt, 25000. Flanders, 6666. France, 5250. Germany, *ſmall*, 20000: *middle*, 22500: *largeſt*, 25000. Holland, 8000. Lithuania, 18500. Muſcovy, 3750. Perſia, 18750. Poland, 19850. Scotland, 6000. Spain, 7090. Sweden, 30000.

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[f] Or, ſuch Diſtinction of Diſtances, as has been made by ſeveral Nations, for the Uſe of travelling; by whatever Names they are called; *Leagues*, *Paraſangs*, *Verſts*, &c.

PROPORTIONS *of* WEIGHTS.

100 lb. *Avoirdupois of England* make

At Amſterdam, 91 lb. 8 oz. Ancona, 136. Andaluſia, 102 : *of Wool Weight* 92. Antwerp, 96 : 8. Biſcany, 90 (of Iron) 78 (of other Things.) Bologna, 125. Bremen 94 : 4. Candia (Groſs) 84 : 12. (Suttle) 131 : 9. Caſtile, 103 : 8. Catalonia, 102. Dantzic, 106 : 12. Denmark, 92. Ferrara, 135 : 5. Florence, 138 : 14. Francfort, 89 : 7. Genoa, 137 : 4. Geneva, 81 : 7. Granada, 104 : 2. Hamburgh, 93 : 5. Leghorn, 132 : 11. Leipſic, 96 : 1. Liege, 96 : 5. Lions, 106. Mantua, 138 : 12. Milan, 153 : 11. Modena, 138 : 12. Naples, 154 : 10. Norway, 92. Paris, 90 : 8. Poland, 114 : 5. Portugal, 104 : 13. Provence, 113. Pruſſia, 116. Rochelle, 90 : 9. Rouen, 88. Ruſſia, 90. Savoy, 121 : 13. Spain, 97 : 4. Tholouſe, 107 : 11. Valencia, 92 : 12. Venice, 152. Vienna, 81 : 7.

For Brabant, *V. Antwerp.* Hall, Leipſic, Holland, *Amſterdam.* Marſeilles, *Provence.* Naumberg, *Leipſic.* Norimberg, *Francfort.* Seville, *Spain.*

The END.

AN ESSAY ON THE NATURE, INVESTIGATION, and APPLICATION, OF LOGARITHMS.

By BENJAMIN DONN.

Si numeris in ratione geometrica progredientibus ſubſcribantur totidem alii æquidifferentes; dicuntur hi illorum Logarithmi.

Wolfius, Elem. Arith.

BRITISH MUSEUM

PREFACE.

THE Invention of *Logarithms* is univerſally allowed to be one of the moſt uſeful Diſcoveries in the Art of Numbers: For, as Dr. *Keil* juſtly obſerves, "It is by their Means "that Numbers almoſt infinite, and ſuch as "are otherwiſe impracticable, are managed "with eaſe and expedition. By their Aſſiſtance "the Mariner ſteers his Veſſel, the Geometrician inveſtigates the Nature of the higher "Curves, the Aſtronomer determines the Places of the Stars, and the Philoſopher accounts "for other Phenomena of Nature; and laſtly, "the Uſurer computes the Intereſt of his Money."

The Honour of the Invention of Logarithms is ſufficiently ſecured to Lord *John Neper*, Baron of *Merchiſton* in *Scotland*, by the Teſtimonies of all who have treated on the Subject, as well Foreigners, as Inhabitants of *Britain*.

In 1614, Lord *Neper* publiſhed at *Edinburgh* a Canon of Logarithms, in which 0 is taken for the Logarithm of Radius (or 10000000000,) and the Logarithms of the Sines are affirmative.

This Book being in *Latin*, was tranſlated into *Engliſh* by Mr. *Edward Wright*, and after his Deceaſe publiſhed by his Son *Samuel*, in the Year 1716.

The great Uſefulneſs of Logarithms was immediately perceived by the learned Mathematicians of the Age; and amongſt them Mr.

Briggs (then Profeſſor of Geometry at *Greſham* College, *London*,) was ſo charmed with the noble Invention, that he went to *Scotland* on Purpoſe to conſult with the Inventor, and aſſiſt him in perfecting his Deſign: The Reſult of their Conſultation was, that the Inventor and Mr. *Briggs* agreed to change the Form of the Logarithms into a more compendious one; in which the Logarithm of 1 was to be 0; and 1000000, *&c.* the Logarithm of Radius.

In the Year 1619, the noble Inventor being dead, his Son Lord *Robert* publiſhed the Conſtruction of the *admirable Canon*; in the Preface to which he takes Occaſion to mention the great Friendſhip his Father had for Mr. *Briggs*; on whom now the whole Labour of calculating Tables entirely devolved. This did not diſcourage Mr. *Briggs*, for applying himſelf to the Work with a Degree of Induſtry that ſurprized the whole World, he publiſhed at *London*, in 1624, his *Arithmetica Logarithmica*, agreeable to the new Form, for the firſt 20 Chiliads of Logarithms, (or from 1 to 20000,) and for 11 Chiliads more, *viz.* from 90000 to 101000, calculated to 14 Places of Figures. He alſo gave Directions for ſupplying the intermediate Chiliads.

In the Year 1627, *Adrian Vlac* (or *Flack*) publiſhed a Canon to 10 Places of Figures, in which the intermediate Chiliads were filled up, according to Mr. *Briggs*'s Directions.

In 1633, *Vlac* publiſhed *Brigg*'s Logarithms of Sines and Tangents to every 10 Seconds, and of Numbers from 1 to 20000.

In the Year 1658, Dr. *John Newton* publiſhed at *London* *Trigonometria Britannica*, in which are Tables of Logarithms of Numbers from 1 to 100000, and alſo the Logarithms of Sines and Tangents to $\frac{1}{100}$ Parts of every Degree.

The Editions of the Tables of Logarithms now moſt eſteemed are *Sherwin*'s, containing the Logarithms taken from Dr. *John Newton*; and *Gardener*'s, containing the Logarithms of Numbers taken alſo from Dr. *John Newton*, and the Logarithms of Sines and Tangents to every 10 Seconds, taken from *Vlac's Canon Magnus*.

We muſt not however forget Mr. *Dodſon*'s *Antilogarithmic Canon*, publiſhed in the Year 1742; being a Table of Numbers, conſiſting of 11 Places of Figures, correſponding to all Logarithms under 100000; which indeed is a laborious and ingenious Work.

Other Authors who have laboured in compleating Tables, *&c.* are, in the Year 1619, *Speidell* and *Urſinus*; in 1620, *Gunter*; in 1624, *Wingate*; in 1625, *Urſinus* again, and *Kepler*; in 1626, *Henrion*; in 1627, *Speidell*, and *Kepler* again in his *Rudolphine Tables*; alſo ſometime before this, *Gellibrand*, as in *Vlac*'s *Trigonometria Artificialis*, publiſhed this Year, the explanatory Part is an Abridgement of *Gellibrand*'s Trigonometry. In the Year 1631, *Norwood*; in 1632, *Cavalerius*; in 1633, *Rowe*; in 1634, *Frobenius*; in 1670, *Caramuel:* Since which Time, Dr. *Gregory*, *Mercator*, Sir *Iſaac Newton*, Dr. *Halley*, *William Jones*, Eſq. and a Number of other learned

Men, have publiſhed infinite converging Series, for computing Logarithms more expeditiouſly; ſome of which we may have Occaſion to explain hereafter.

It would be endleſs to give a Liſt of all the Authors who have touched on Logarithms; becauſe, almoſt all who have treated on Trigonometry, on Navigation, &c. have given compendious Tables of Logarithms, &c.

By this compendious ESSAY, it is hoped the Learner may attain as clear Ideas of the Nature and Application of Logarithms, as he could by much larger Treatiſes.

Proper Tables of Logarithms will be given in the Eſſay on *Trigonometry*, or *Navigation*, as they will there be of more immediate Uſe, &c. as the Tables of Sines, Tangents, &c. could not with any Propriety be given in this Place.

Bideford, February 7,
1759.

B. DONN.

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CHAP. I.

Of DEFINITIONS, *and the Nature and Inveſtigation of* LOGARITHMS.

1. LOGARITHMS (λόγος and ἀριθμὸς) are Numbers ſo contrived and adapted to other Numbers, that by their Addition, or Subtraction, the Product, or Quotient, of the Numbers to which they are adapted, may be found.

This Definition expreſſes the principal Deſign and Uſe of thoſe artificial Numbers, which Mathematicians diſtinguiſh by the Name of *Logarithms.* But here, (as it frequently happens in other Sciences,) a clear, accurate, etymological Senſe, cannot be underſtood, till we have made a particular Enquiry into the Principles of the Science itſelf.

2. To proceed then, let us conſider the Nature of a geometrical Progreſſion, with the Indices ſet over the reſpective Terms.

For Example:

Indices,	0.	1.	2.	3.	4.	5.	6.	*&c.*
Terms,	1.	2.	4.	8.	16.	32.	64.	*&c.*

Here it is evident, that if we add any two Indices together, their Sum will be the Index of that Number, which is equal to the Product of the Numbers whoſe Indices are added together: Thus, *e. g.* The Indices 2 and 3 added together are = 5; the Numbers anſwering to the Indices 2 and 3, are 4 and 8; and 4 × 8 = 32 = the Number anſwering to the Index 5.

It alſo follows from the Nature of ſuch Progreſſions, that if we ſubtract one Index from another, their Difference will be the Index of that Number or Term, which is equal to the Quotient of the two Numbers or Terms, correſponding to the Indices whoſe Differences we found: *e. g.* The Indices $6 - 4 = 2$; the Numbers correſponding to theſe Indices are 64 and 16, and $64 \div 16 = 4$ = the Number correſponding to the Index 2.

3. Hence it follows, that (by Art. 1.) the Indices of a Series of Numbers in geometrical Progreſſion, are Logarithms of the Terms in that Progreſſion.

4. Hence, by the Nature of geometrical Progreſſions, it follows, that, if the Logarithm of any Number be multiplied by the Index of the Power, the Product will be equal to the Logarithm of the Root when involved to the Height denoted by the Index. Thus, for Inſtance: The Index or Logarithm of 4 in the above Series 2, being multiplied by 3 = 6 = the Index or Logarithm of 64; and 64 = 4 cubed = 4 × 4 × 4.

5. From hence alſo it follows, that the Index or Logarithm of any Number being divided by 2, the Quotient will be the Index or Logarithm of the ſquare Root; but if divided by 3, the Index or Logarithm of the cube Root; if divided by 4, the Quotient will be the Index or Logarithm of the 4th Power, *&c.*

Example. The Logarithm of 64 in the above Series is 6, which being divided by 3, the Quotient is 2, the Index or Logarithm of 4, which is the cube Root of 64; for 4 × 4 × 4 = 64.

6. It follows, from the Nature of geometrical Progreſſions, that the Ratio of the firſt, and any other Term, is compoſed of ſo many equal Ratios, as are expreſſed by the Index of that other Term: *e. g.* In the above Series, the Ratio of the firſt and 7th Term is made up of 6 equal Ratios, for the Ratio of the 1ſt to the 2d Term is as 1 to 2, and of the 2d to the 3d as 1 to 2, and of the 3d to the 4th as 1 to 2, of the 4th to the 5th as 1 to 2, of the 5th to the 6th as 1 to 2, and of the 6th to the 7th as 1 to 2; ∵ the Ratio of the 1ſt to the 7th is compoſed of 1×1×1×1×1×1 to 2×2×2×2×2×2; *i. e.* as 1 to 64. Hence plainly appears the Propriety of the Term *Logarithms*, it ſignifying, according to its Etymology, a *Number of Ratios*.

7. Logarithms may be of various Kinds; for inſtead of the Series 1, 2, 4, &c. above, we may ſubſtitute any other; thus *e. g.* if we write the geometrical Series 1, 10, 100, &c. inſtead of 1, 2, 4, &c. it will ſtand thus:

Indices or Logarithms	0.	1.	2.	3.	4.	&c.
Terms or natural N^{os}.	1.	10.	100.	1000.	10000,	&c.

This is the Form of *Brigs*'s Logarithms; which are thoſe we ſhall now make it our Buſineſs to diſcourſe on.

The greateſt Difficulty now remaining is to explain the Method made Uſe of by Mr. *Brigs*, for finding the Logarithms of the intermediate Terms, becauſe the natural Numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, &c. are not in a geometrical Progreſſion.

8. Before we proceed any farther, it may be proper to obſerve, that, if we ſuppoſe a geometrical Series beginning from Unity, whoſe common Multiplier is $1+m$, the Series will be 1, $1+m$, $\overline{1+m}|^{2}$, $\overline{1+m}|^{3}$, $\overline{1+m}|^{4}$, &c. Now if m be ſup-

poſed indefinitely little, it is evident that ſome of the Terms will approach indefinitely near to the natural Numbers 2, 3, 4, 5, *&c.* and, conſequently, it is poſſible to find the natural Number anſwering to any Logarithm, very nearly, though not exactly true.

9. The Method of doing which may be thus explained. —— By the Definition of Logarithms, if x and y repreſent any two Numbers, the Logarithm of $\overline{x \times y} =$ the Logarithm of $x +$ the Logarithm of y; and therefore, (by Art. 5.) the Logarithm of $\sqrt{x \times y} = \frac{\text{the Log. } x + \text{Log. } y}{2}$: Hence, the **arithmetical Mean* of any two Logarithms is equal to the Logarithm of the *geometrical Mean* of the two natural Numbers; conſequently, the Logarithm of any intermediate Number may be found, by finding the geometrical Mean between the two given Numbers, (whoſe Logarithms are given,) and the Logarithm anſwering thereto, (*viz.* the arithmetical Mean of the two given Logarithms:) Then, the geometrical Mean between the geometrical Mean juſt found and the neareſt Extreme, and the Logarithm anſwering thereto. After this Manner we proceed, till we find a geometrical Mean ſo very nearly equal to the Number whoſe Logarithm we want, as we ſhall think ſufficiently near the Truth; then the Logarithm of that geometrical Mean is ſufficiently near the Logarithm required.

10. To illuſtrate this, let it be required to find the Logarithm of 9. Here we have the Logarithms of 1 and 10 given.

---

* Half the Sum of any two Numbers is called an *arithmetical Mean*; and the ſquare Root of the Product of two Numbers is called the *geometrical Mean.*

(1.) The Log. of 1 = 0

Log. of 10 = 1

$$\begin{array}{r} \text{Sum} = 1 \\ \frac{1}{2} \overline{\qquad\qquad} \\ \text{Arithmetical Mean} \quad 0.5 \end{array}$$

And $\sqrt{1 \times 10} = \sqrt{10} = 3.1622777$, ∵ the Log. of 3.1622777 is = 0.5.

(2.) Hence we have the Logarithms of 3.1622777 and 10 given.

$$\begin{array}{lr} \text{Log. of } 10 \text{ - - } = & 1. \\ \text{Log. of } 3.1622777 = & 0.5 \\ \hline & 1.5 \\ \frac{1}{2} & \overline{\quad 0.75} \end{array}$$

Hence the Log. of $\sqrt{10 \times 3.1622777}$, *viz.* of 5.6234132 is 0.75.

(3.) We have now the Logarithms of 10 and 5.6234132 given.

$$\text{Numbers} \begin{cases} 10. \text{ - - - - its Log.} = 1. \\ 5.6234132 \text{ - its Log.} = 0.75 \end{cases}$$

$$\begin{array}{r} \overline{1.75} \\ \frac{1}{2} \overline{\qquad} \end{array}$$

$\sqrt{10 \times 5.6234132} = 7.4989421$ its Log. = 0.875

(4.) Here we have the Log. of 10 and 7.4989421 given.

$$\text{Numbers} \begin{Bmatrix} 10. \\ 7.4989421 \end{Bmatrix} \text{Their Logarithms} \begin{Bmatrix} 1. \\ 0.875 \end{Bmatrix}$$

$$\begin{array}{r} \overline{1.875} \\ \frac{1}{2} \overline{\qquad} \end{array}$$

$\sqrt{10 \times 7.4989421} = 8.6596431$ its Log. = 0.9375

(5.) Hence we have the Logarithms of 10 and 8.6596431 given.

$$\text{Numbers} \begin{Bmatrix} 10. \\ 8.6596431 \end{Bmatrix} \text{Their Logarithms} \begin{Bmatrix} 1. \\ 0.9375 \end{Bmatrix}$$

$$\begin{array}{r} \overline{1.9375} \\ \frac{1}{2} \overline{\qquad} \end{array}$$

$\sqrt{10 \times 8.6596431} = 9.3057204$ its Log. = 0.96875

(6.)

(6.) We have now the Log. of 8.6596431 and 9.3057204 given.

$$\left.\begin{array}{r}9.3057204\\8.6596431\end{array}\right\}\begin{array}{c}\text{Their}\\\text{Logarithms}\end{array}\left\{\begin{array}{l}0.96875\\0.9375\\\hline 1.90625\\\hline\end{array}\right.$$

$$\overline{\sqrt{9.3057204 \times 8.6596431}} \quad \text{its Log.} \quad \tfrac{1}{2}\ 0.953125$$

And proceeding after this Manner, after 25 Extractions the Logarithm of 8.9999998 will be found to be 0.9542425, which we take for the Logarithm of 9, becauſe 8.9999998 differs but $\frac{2}{100000000}$ or $\frac{1}{50000000}$ from 9. —— By the ſame Method the Logarithms of the prime Numbers were found.

11. The Logarithms of the prime Numbers being found, the Logarithms of the Numbers compoſed of them are found by adding the Logarithms of the component Parts; the Reaſon of which is evident from the Definition, in Art. 1. Take an Example. Given the Logarithm of 2 = 0.30103, and the Logarithm of 3 = 0.4771212, to find the Logarithm of 2 × 3 or 6.

$$\begin{array}{r}\text{The Log. of } 2 = 0.30103\\\text{Log. of } 3 = 0.4771212\\\hline\text{Log. of } 6 = 0.7781512\\\hline\end{array}$$

12. The Logarithm of any Number being given, the Logarithm of any Number equal to the Square, or Cube, &c. of that Number, may be found, by multiplying the Logarithm of the Root by 2, 3, &c. reſpectively, according as the Power, whoſe Logarithm is to be found, is a Square, or Cube, &c. This is manifeſt by Art. 4.

13. Laſtly, the Logarithm of any Power being given, the Logarithm of its Root is found by dividing that Logarithm by the Index of the given Power: By Art. 5.

14. After the Method juſt now deſcribed the Tables of Logarithms were at firſt made. And if we

conſider the great Labour and Patience neceſſary to the Accompliſhment of ſo great a Work, how much are we beholden to Lord *Neper* and Mr. *Brigs*? Certainly their Names will, and ought to be perpetuated from one Age to another, even to the End of the World!

There have lately been invented much eaſier Methods of conſtructing Logarithms than the above; but as they depend on Fluxions, Conic Sections, *&c.* they cannot be underſtood in this Place. And it is ſufficient to have ſhewn the Method by which the Tables were made, as we have now no Occaſion to conſtruct new ones.

15. Having explained the Nature of making a Table of Logarithms of Numbers greater than Unity, the next Thing to be done is to ſhew how the Logarithms of fractional Numbers may be found. In Order to which let it be obſerved, that, as we have hitherto ſuppoſed a geometrical Series to increaſe from an Unit on the right Hand, we may now ſuppoſe it to decrease from an Unit towards the left Hand, as here annexed.

Log.	—3.	—2.	—1.	0.	+1.	+2.	+3, *&c.*
	C.			A.			B.
Numbers	$\frac{1}{1000}$.	$\frac{1}{100}$.	$\frac{1}{10}$.	1.	10.	100.	1000, *&c.*

Here the Series from the Point A, or Unity, increaſes towards B, and decreaſes towards C.

It is evident by a bare Inſpection, that, if the Numbers are both Fractions, the Sum of their Logarithms is equal to the Logarithm of their Product. But if one Number is on the Right-hand of the Point A, and the other on the Left of it; that is, if one Number is greater than an Unit, and the other a Fraction, then the Difference of their Logarithms will be equal to the Logarithm of their Product; and the Logarithm of the Product will be on the ſame Side of the Point A, as is the greater of the two Logarithms.

16. In Order to make a proper Diſtinction, we may denote the Logarithm of any Number greater than an Unit by the Sign +, for they may be reckoned affirmative; and then the fractional Quantities muſt be conſidered as negative.

17. It is alſo evident from Inſpection of the above Series, that if a Number, having an affirmative Logarithm, is divided by another Number whoſe Logarithm is a greater affirmative Number, the Difference of their Logarithms taken negatively will be the Logarithm of their Quotient. Thus, *e. g.* the Logarithm of 100 = 2, and the Logarithm of 1000 = 3; their Difference taken negatively is — 1 = the Logarithm of $\frac{1}{10}$, the Quotient of 100 ÷ 1000. —— This might have been deduced from the Nature of Fractions. For the Numerator of any Fraction may be eſteemed as a Dividend, and the Denominator as a Diviſor: Therefore, by the Nature of Logarithms, (Art. 1.) the Logarithm of the Numerator *minus* the Logarithm of the Denominator is equal to the Logarithm of the Fraction: But ſince the Denominator of a proper Fraction is always greater than the Numerator, its Logarithm will be greater than that of the Numerator, and conſequently it will be impoſſible to take the Logarithm of the Denominator from that of the Numerator; therefore, we are obliged to take the Logarithm of the Numerator from that of the Denominator, and expreſs the Difference negatively, for the Logarithm of the Fraction.

18. The Logarithm of 1, 10, 100, 1000, *&c.* being 0, 1, 2, 3, *&c.* reſpectively, it is manifeſt, that the Logarithms of the intermediate Terms may be conſidered as a mixed Number, made up of an Integer and decimal Fraction; and that the integral Part, called by ſome Writers the *Index*, *(Lat.)* by Others the *Exponent*, (from *expono*, *Lat.)* and by Others the *Charaſteriſtick*, (from *Character*, *Lat.)* is one leſs than the Number which expreſſes how many Figures the natural Number conſiſts of; *e. g.* if the

natural Number has 4 Places, the Characteriſtick of the Logarithm will be 3; if of 3 Places, 2; if of 2 Places, 1, *&c.*

19. Diviſion being performed by Subtraction of the Logarithms, and the Logarithm of 10 being 1, it follows, that the Logarithm of $\frac{1}{10}$ of any Number, muſt be one leſs than the Logarithm of that Number. Thus, *e. g.* the Logarithm of 7851 being + 3.894925,

$$\text{The Log. of}\left\{\left[\begin{array}{l} \dfrac{7851}{10} = 785.1 \\ \dfrac{785.1}{10} = 78.51 \\ \dfrac{78.51}{10} = 7.851 \\ \dfrac{7.851}{10} = .7851 \\ \dfrac{.7851}{10} = .07851 \\ \dfrac{.07851}{10} = .007851 \\ \&c. \end{array}\right]\right\}\text{is}\left\{\left[\begin{array}{l} \left.\begin{array}{l} +\,2.894925 \\ +\,1.894925 \\ +\,0.894925 \end{array}\right\}\text{Affirmative Logarithms.} \\ \left.\begin{array}{l} -\,0.105075 \\ -\,1.105075 \\ -\,2.105075 \end{array}\right\}\text{Negative Logarithms.} \end{array}\right]\right.$$

The above affirmative Logarithms were found by ſucceſſively ſubtracting the Logarithm of 10, *viz.* + 1 from the preceding Logarithm; then, to find the firſt negative Logarithm, + 1 (the Log. of 10) was to be taken from the Logarithm + 0.894925; but as the Log. to be ſubtracted is greater than that from which it was to be taken, we are obliged to ſubtract the leſſer 0.894925 from the greater 1, and expreſs their Difference negatively, for the required Logarithm, (as directed in Art. 17,) which gives — 0.105075. Then to find the other negative Logarithms, the Logarithm + 1 being to be ſubtracted from a negative Logarithm, we have only to add 1 ſucceſſively, and expreſs the Sum negatively; for by Art. 46, *Eſſay on Arithmetick,* to ſubtract an Affirmative is the ſame as to add a Negative.

20. The Logarithms of Fractions may be expreſſed in ſuch a Manner, that the fractional Part may be affirmative, and the Characteriſtick negative. Logarithms thus expreſſed are called *binomial Logarithms* (from *binomius*, *Lat.*) and are better adapted for Practice than the negative ones.

21. To find the binomial Logarithm of a proper Fraction.

* From the Logarithm of the Numerator ſubtract that of the Denominator; remembering, that when we come to the Characteriſticks, the Characteriſtick of the Numerator is to be taken from that of the Denominator; (after having added the one that was borrowed, if any, to the Characteriſtick of the Denominator.)

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* That the binomial Logarithms found by this Rule, are, in Effect, the ſame as the negative Logarithms afore deſcribed, may be thus ſhewn. Let $\frac{n}{d}$ be any proper Fraction; then, putting c for the integral, and r for the fractional Part, of the Logarithm of n; we have $c + r =$ Logarithm of n; and putting t for the Characteriſtick, and $a =$ the decimal Part of the Logarithm of d, then $t + a =$ the Logarithm of d. Hence, by Art. 17, $\overline{t+a} - \overline{c+r} = t + a - c - r$ taken negatively, (that is, changing the Signs,) *viz.* $- t - a + c + r =$ the Logarithm of $\frac{n}{d}$.

But for the binomial Logarithm we have two Caſes. 1ſt, When r is greater than a, then $t - c$ taken negatively is $- t + c$, ∴ the binomial Logarithm, according to this Rule, is $- t + c + r - a$, which may ſtand thus, $- t - a + c + r$, which is the ſame as the above Expreſſion for the negative Logarithm. 2dly, When r is leſs than a. Firſt $c + r = \overline{c - 1} + \overline{1 + r} =$ the Log. of n; now ſubtracting a from $1 + r$, there remains $1 + r - a$; and taking $c - 1$ from t, (according to this Rule,) or, which is the ſame Thing, c from $t + 1$, there remains $t + 1 - c$, which taken negatively is $- t - 1 + c$ for the Characteriſtick; ∴ $- t - 1 + c +$ (the decimal Part of the Logarithm) $1 + r - a =$ the binomial Logarithm $= - t + c + r - a = - t - a + c + r$, the ſame Expreſſion as that above, for the negative Logarithm. *Q. E. D.*

Example. The binomial Logarithm of $\frac{7.851}{10}$ is $= -1 + .894925$.

For the Logarithm of $7.851 = 0.894925$ by Art. 19.

The Logarithm of $10 = 1.000000$

The Log. of $\frac{7.851}{10}$ by this Rule $= -1 + .894925$.

But this Logarithm is more commodiouſly wrote thus, $\bar{1}.8942925$, the Daſh over the Characteriſtick ſhewing it is a negative Quantity, and the remaining Part affirmative.

Hence the Log. of $\frac{7.851}{10} = .7851$ is $= \bar{1}.894925$

of $\frac{.7851}{10} = .07851$ is $= \bar{2}.894925$.

of $\frac{.07851}{10} = .007851$ is $= \bar{3}.894925$.

&c. *&c.*

Hence it is evident, that the decimal Part of the binomial Logarithm of a decimal Fraction, is the ſame as that of an integral Number of the ſame Figures; and it is alſo manifeſt, that if the ſignificant Figure of the decimal Number is in the firſt Place, the Characteriſtick is -1; if the ſignificant Figure be in the ſecond Place, the integral Part of the Logarithm is -2, *&c.* And this is the Kind (of Logarithms) we ſhall make Uſe of in the remaining Part of this ESSAY, to expreſs fractional Quantities.

22. In taking out of a Table the Logarithm of any Integer not exceeding 10000, we have the fractional Part by Inſpection; to which we prefix for a Characteriſtick 3, if the Number conſiſts of 4 Figures; 2, if of 3 Figures, *&c.* that is, the Characteriſtick muſt be one leſs than the Number of Places: But if the Number, whoſe Logarithm is required, be above 10000; then, find the Logarithm of the two neareſt Numbers to it that can be found

in the Table, and ſay, As their Difference : the Difference of their Logarithms :: the Difference between the neareſt Number and that whoſe Logarithm is required : the Difference of their Logarithms nearly; which being either added to, or ſubtracted from, the neareſt Logarithm, according as it is greater or leſs than the required one, will give nearly the Logarithm required. Take an Example.

Let it be required to find the Logarithm of 367182.

The fractional Part of the Logarithm of 3671 is by the Table .564784, and of 3672 is .564903, ∵

$$\begin{array}{lll}
\text{The Log. of}\left\{\begin{array}{l}367100\\367200\end{array}\right. & \begin{array}{l}\text{is } 5.564784\\ \text{is } 5.564903\end{array} & \begin{array}{r}\text{Neareſt N°. } 367200\\ \text{The N°. } 367182\end{array}\\
\hline
\text{Their Dif. } 100 \text{ \& of Log. } .00119 & & \text{Diff. } 18\\
\hline
\end{array}$$

∵ As 100 : .00119 :: 18 : .00021 nearly; ∵ 5.564903 — .00021 = 5.564882 nearly = the Logarithm of 367182.

This Method being grounded on Suppoſition, that the Logarithms of Numbers between 367100 and 367200 increaſe or decreaſe equally, according to their Diſtance from 367100 or 367200, is not ſtrictly true, but nearly ſo; and the greater any Numbers are with Reſpect to their Difference, the nearer will thoſe Differences be proportional: And therefore, though this will not give the exact Logarithm, yet it will be nearly ſo, and ſufficiently near the Truth for moſt Purpoſes either of Buſineſs or Pleaſure.

If the Number does conſiſt of both Integers and Fractions, find the fractional Part of the Logarithm as if all its Figures were Integers, and to that fractional Part prefix the Index belonging to the integral Part; *viz.* make the Characteriſtick one leſs than the Number of integral Figures, and you will have the Logarithm ſought.

Laſtly, if the Number whoſe Logarithm is required, be entirely fractional; find the fractional Part of the Logarithm, as if it was a whole Number, and to it prefix the Characteriſtick —1, if the firſt Figure to the left Hand be *Tenths*; but, if the firſt Figure be in the ſecond Place, or Place of *Hundreds*, the Characteriſtick muſt be —2, *&c.*

23. When two or more Logarithms are to be added together, they may be added by the Rules for adding affirmative and negative Quantities in Algebra. That is, 1. If they are all *affirmative*, add them as in common Addition, and the Sum will be the affirmative Logarithm required.

24. *Caſe* 2. If the Characteriſticks of the Logarithms to be added are all *negative*, add the decimal Parts together by common Addition; and if there is any Thing to be carried from the decimal Part to the integral Parts, it is *affirmative*, (by the Nature of binomial Logarithms,) which reſerve: Then find the *Sum* of the negative Characteriſticks; which, being negative, the Difference betwixt this *Sum*, and the Carriage (above reſerved,) with the Sign of the greater, will be the Characteriſtick required.

25. *Caſe* 3. When the Characteriſticks of the Logarithms to be added, are ſome *affirmative* others *negative.* — Add the fractional Parts as uſual, only the Carriage from the decimal Part to the integral being affirmative, muſt be added to affirmative Characteriſticks; whoſe Sum, including the *Carriage*, being found, and alſo the Sum of the negative Characteriſticks, find the Difference of theſe Sums, and to it prefix the Sign of the greater, for the required Characteriſtick.

26. When one Logarithm is to be ſubtracted from another, the Work may be done as in Subtraction of Algebra; by having due Regard to the affirmative and negative Quantities. But to be more particular.

Caſe 1. To ſubtract a leſs affirmative, from a greater affirmative Logarithm; find their Difference, as in common Arithmetick.

27. *Caſe* 2. When it is required to ſubtract a greater affirmative, from a leſs affirmative Logarithm; it is the ſame as to find the Logarithm of a

proper Fraction, and therefore may be found as directed in Article 21.

28. *Caſe* 3. When a *binomial* Logarithm is to be ſubtracted from an affirmative Logarithm, if the decimal Part of the affirmative Logarithm is greater than that of the Binomial, ſubtract the decimal Part of the *Binomial* from the decimal Part of the Affirmative; and prefix the Sum of their Characteriſticks, with the affirmative Sign. But if the *fractional* Part of the binomial Logarithm be greater than that of the Affirmative, add, or ſuppoſe in your Mind 1 to to be added to the fractional Part of the Affirmative; then ſubtract the fractional Part of tha Binomial from that Sum; (but, becauſe we added 1 for Conveniency, we muſt alſo to continue the Equality) add + 1 to the binomial Characteriſtick; but the Characteriſtick of the binomial Logarithm being negative, the adding + 1 to it is diminiſhing the negative Characteriſtick by Unity; which Characteriſtick ſo diminiſhed, being added to the affirmative Characteriſtick, will give the affirmative Characteriſtick of the required Logarithm.

29. *Caſe* 4. When an *affirmative* Logarithm is to be ſubtracted from a *Binomial*; if the fractional Part of the Binomial be greater than that of the affirmative Logarithm; to the Difference of their fractional Parts, place for a Characteriſtick the Sum of their Characteriſticks with the negative Sign: But if the *decimal* Part of the Binomial is leſs than that of the affirmative Logarithm, add 1 (or ſuppoſe it to be added in your Mind) to the fractional Part of the Binomial; and then ſubtract the decimal Part of the Affirmative from that Sum, and (to continue the Equality) add 1 to the Characteriſtick of the affirmative Logarithm; then the Sum of this, and the Characteriſtick of the Binomial, will, with the negative Sign, be the Characteriſtick of the required Logarithm.

30. *Caſe* 5. When a *binomial* Logarithm is to be ſubtracted from a *Binomial*; if the *fractional* Part of

that to be ſubtracted is leſs than that from which it is to be taken, find the Difference of their fractional Parts, to which prefix the Difference of their Characteriſticks, with the affirmative Sign, if the Characteriſtick of the Logarithm to be ſubtracted is the *greater*; otherwiſe it muſt have a negative Sign. But if the *fractional* Part of the Logarithm to be ſubtracted, be greater than the *fractional* Part of the Logarithm from which it is to be taken, add 1 to the fractional Part of this laſt, that Subtraction may be made; and then, (to continue the Equality, add 1 to the Characteriſtick of the *Minorand*, which Characteriſtick being negative, is done by Subtraction, *viz.* ſubtract 1 from the Characteriſtick of the *Minorand*; then the Difference between the Characteriſtick of the Minorand ſo diminiſhed, and the Characteriſtick of the Subducend, will give the required Characteriſtick; which muſt be *affirmative*, if the Characteriſtick of the *Minorand*, after being diminiſhed as above, is greater than that of the *Subducend*: Otherwiſe *negative*.

31. Before we proceed to the Application of Logarithms, it will be proper to ſhew how to find the *natural* Number anſwering to any given Logarithm, by Help of a Table of Logarithms. And as this is only the Reverſe of Article 22, it will be ſufficient to hint, 1ſt, That the Index, or Characteriſtick, of the Logarithm, if *affirmative*, will ſhew how many integral Places are in the required Number: And if *negative*, in what Place of Decimals the firſt, or ſignificant Figure, ſtands. Hence, if the Logarithm can be found in the Table, the anſwering Number is found by a bare Inſpection: But if it cannot be exactly found, find the next greater, and next leſſer, and ſay, As the Difference of theſe two Logarithms : the Difference of the anſwering Numbers :: the Difference between the given Logarithm and the neareſt tabular Logarithm : a fourth Number; which added to, or ſubtracted from, the natural Number anſwering to the neareſt tabular Logarithm,

(according as that Logarithm was leſs or greater than the given one,) will give the required Number very near the Truth.

Example. Let it be required to find the natural Number anſwering to the Logarithm 5.564882.

This in the Tables gives the next leſſer and

greater Logarithms,	5.564784 5.564903	The anſwering Numbers	367100 367200
Their Diff.	.000119		100

And 5.564903 — 5.564882 = .000021. Hence, As .000119 : 100 :: .000021; or, which is the ſame in Effect, As 119 : 100 :: 21 : 17.7 nearly. Hence, 367200 — 17.7 = 367182.3, nearly = the required Number.

CHAP. II.

MUTTIPLICATION *by* LOGARITHMS.

32. WE now come to ſhew the Application of Logarithms, having explained their Theory at large; and the firſt Thing is to perform Multiplication by them; which is done by adding the Logarithms of the Factors together, as directed in Art. 23, 24, and 25.

33. *Example* 1. Multiply 14 by 4.

Log. of 14 is 1.1461280
Log. of 4 is 0.6020600

Their Sum 1.74818880 anſwers in the Table to 56, the required Product.

34.

34. *Example* 2. Multiply 203 by 0.25.

Log. of 203 is 2.3074960

Log. of 0.25 is 1.3979400

This Log. 1.7054360 anſwers in the Table to 50.75.

35. *Example* 3. Multiply 7.677 by $\frac{5}{8}$.

To Log. of 7.677 = 0.8851915

Add Log. of 5 = 0.6989700

From 1.5841615

Subtract Log. of 8 = 0.9030900

Gives Log. of 4.798 nearly, 0.6810715

Or thus: Put $\frac{5}{8}$ into Decimals, *viz.* $\frac{5}{8}$ = .625.

Then, Log 7.677 = 0.8851915

Log. of $\frac{5}{8}$ or of .625 = $\bar{1}$.7958800

Sum is 0.6810715 as above.

36. *Example* 4. Multiply 0.55 by .023.

Log. .55 is $\bar{1}$.7403627

Log. .023 is $\bar{2}$.3617278

Log. of .01265 $\bar{2}$.1020905

37. *Example* 5. Multiply 80 by .003.

Log. 80 = 1.9030894

Log. .003 = $\bar{3}$.4771212

Log. .24 = $\bar{1}$.3802116

38. *Example* 6. Multiply 10, 80, 72, .5, .4, and .12, together.

Log. 10 = 1.0000000
Log. 80 = 1.9030894
Log. 72 = 1.8573325

To 4.7604219
Add $\bar{2}$.3802112

Log. .5 = $\bar{1}$.6989700
Log. .4 = $\bar{1}$.6020600
Log. .12 = $\bar{1}$.0791812

$\bar{2}$.3802112

This Log. 3.1406331 anſwers in the Table to 1382.4 nearly.

CHAP. III.

DIVISION *by* LOGARITHMS.

39. THE Subtraction of Logarithms anſwering to the Diviſion of the natural Numbers, from the Logarithm of the Dividend ſubtract the Logarithm of the Diviſor, by the Rules in Art. 26, 27, 28, 29, 30; and the Remainder will be the Logarithm of the required Quotient.

40. *Example* 1. Divide 56 by 4.

Log. 56 = 1.7481880
Log. 4 = 0.6020600

Log. 14 = 1.1461280

41. *Example* 2. Divide 5 by 8.

Log. 5 = 0.6989700
Log. 8 = 0.9030900

Log. .625 = $\bar{1}$.7958800

42. *Example* 3. Divide 50.75 by .25.

Logarithm of 50.75 = 1.7054360

Log. of .25 = $\bar{1}$.3979400

Log. of 203 = 2.3074960

43. *Example* 4. Divide 4.718 by .625.

Log. of 4.718 = 0.6737579

Log. of .625 = $\bar{1}$.7958800

Log. of 7.549 ferè = 0.8778779

44. *Example* 5. Divide .75 by 100.

Log. of .75 = 1.8750613

Log. of 100 = 2.0000000

Log. of .0075 = $\bar{3}$.8750613

45. *Example* 6. Divide .24 by 80.

Log. of .24 = $\bar{1}$.3802112

Log. of 80 = 1.9030900

Log. of .003 = $\bar{3}$.4771212

46. *Example* 7. Divide .75 by .25.

Log. of .75 = $\bar{1}$.8750613

Log. of .25 = $\bar{1}$.3979400

Log. of 3 = 0.4771213

47. *Example* 8. Divide .01265 by .55.

Log. of .01265 = $\bar{2}$.1020905

Log. of .55 = $\bar{1}$.7403627

Log. of .023 = $\bar{2}$.3617278

CHAP. IV.

Of the APPLICATION *of* LOGARITHMS *to* CIRCULATING DECIMALS.

48. AS in ſome Calculations, in which we propoſe to be very accurate, we may have Occaſion to find the Logarithms of *circulating Decimals*; we ſhall explain the Method by which they may be found, as follows.

The RULE. Firſt turn the *circulating* Decimal into its equivalent vulgar Fraction, (as ſhewn in Theorems 4, 5, 6, and 7, in Articles 13, 14, 15, 16, 17, 18, 19, 20, and 21, of the *Eſſay on Circulating Decimals*,) and then find the Logarithm of that vulgar Fraction, by Art. 21 of this ESSAY.

49. *Example* 1. What is the Logarithm of $.\dot{4}$?

By Art. 14. of the *Eſſay on Circulating Decimals*, $.\dot{4} = \frac{4}{9}$.

Hence from Log. of 4 = 0.6020600

Subtract Log. of 9 = 0.9542425

Gives Log. of $\frac{4}{9}$ or of $.\dot{4}$ = $\bar{1}$.6478175

50. *Example* 2. What is the Logarithm of $\dot{3}.\dot{5}$?

Solution. By Art. 16 of the *Eſſay on Circulating Decimals*, $\dot{3}.\dot{5} = \frac{350}{99}$.

From Log. of 350 = 2.5440680
Subtract Log. of 99 = 1.9956352

Gives Log. of $\frac{350}{99}$ = 0.5484328.

51. *Example* 3. What is the Logarithm of $.2\dot{3}$?

Solution. By Art. 18 of the *Eſſay on Circulating Decimals*, $.2\dot{3} = \frac{21}{90}$.

From Log. of 21 = 1.3222193
Subtract Log. of 90 = 1.9542425

Gives Log. of $\frac{21}{90}$ = $\bar{1}.3679768$

52. *Example* 4. What is the Log. of $26\dot{3}\dot{5}3.5$?

Solution. By Art. 20 of the *Eſſay on Circulating Decimals*, $26\dot{3}\dot{5}3.5 = \frac{2609000}{99}$.

Therefore, from Log. of 2609000 = 6.4164741
Subtract the Log. of 99 = 1.9956352

The Remainder is the Log. of $\frac{2609000}{99}$ 4.4208389

53. The Logarithms of Circulating Decimals being found by the above Method; Multiplication, and Diviſion, are performed by them, after the ſame Manner as is ſhewn of finite Numbers in Chap. 2. and 3. of this ESSAY.

CHAP.

CHAP. V.

The GOLDEN RULE *by* LOGARITHMS.

54. FROM the Nature of the *Golden Rule*, and Logarithms, we have this Rule. Add the Logarithms of the ſecond and third Terms together; from their Sum ſubtract the Logarithm of the firſt Term; and the Remainder will be the Logarithm of the fourth, or required Term.

55. Suppoſe in the Rule of Three Direct we have this Stating, As 4.1 : 5 :: 65 : a fourth Number.

To the Log. of 5 = 0.6989700
Add the Log. of 65 = 1.8129134

2.5118834
Sub. Log. of 4.1 = 0.6127838

Log. of 79.26 nearly 1.8990996

55. If from the Logarithm of 1 be ſubtracted the Logarithm of any Number, the Remainder is called the *Arithmetical Complement* of that Logarithm. Hence, the *Arithmetical Complement* of the Logarithm of any Number, is equal to the Logarithm of the Quotient of 1 divided by that Number.

56. When it is required to divide one Number by another, it is in Effect the ſame, if we divide 1 by the Diviſor, and multiply that Quotient by the given Dividend. (For $\frac{a}{m} = \frac{1}{m} \times a$.) Which is done by Logarithms, by adding the Logarithm of the Dividend to the *Arithmetical Complement* of the Logarithm of the Diviſor.

57. Hence, the Operation of the Example in Art. 55, may ſtand thus:

$$\begin{array}{lrl} \text{From Log. of } 1 & = & 0.0000000 \\ \text{Sub. Log. of } 4.1 & = & 0.6127839 \\ \hline \text{Gives } \textit{Arith. Comp.} & = & \bar{1}.3872161 \end{array} \qquad \begin{array}{r} \bar{1}.3872161 \\ 0.6989700 \\ 1.8129134 \\ \hline 1.8990995 \end{array}$$

Gives Log. as above 1.8990995

CHAP. VI.

INVOLUTION *by* LOGARITHMS.

58. TO raiſe any Number to a given Power by Logarithms, multiply the Logarithm of the Root by the Index of the given Power. Thus, for Inſtance, if we are to ſquare any Number, we muſt multiply its Logarithm by 2; if to cube a Number, by 3, *&c.* and the Product (by Art. 4.) will be the Logarithm of the required Power.

Note, If the Root be a Fraction, its Logarithm will be a *Binomial*; in ſuch Caſe, the Carriage from the decimal Part in multiplying being affirmative, and the Product of the Characteriſtick negative, the Carriage muſt be ſubtracted from that negative Product, for the required negative Characteriſtick.

59. *Example* 1. What is the Square of 15?

$$\begin{array}{lrl} \text{Log. of } 15 & = & 1.1760913 \\ & & 2 \\ \hline \text{Log. of } 225 & = & 2.3521826 \end{array}$$

60. *Example* 2. Cube .55.

$$\begin{array}{lrl} \text{Log. of } .55 & = & \bar{1}.7403627 \\ & & 3 \\ \hline \text{Log. of } .166375 & = & \bar{1}.2210881 \end{array}$$

CHAP.

CHAP. VII.

EVOLUTION *by* LOGARITHMS.

61. TO extract the Root of any Number, divide the Logarithm of the Power by its Index, *viz.* To extract the ſquare Root, divide by 2; for the Cube Root, by 3, *&c.* See Art. 5.

Note. In extracting the Root of a fractional Number, if we cannot divide the Characteriſtick exactly by the Index of the Power, it will be proper to increaſe the Characteriſtick by the Addition of ſuch a Number as will make it exactly diviſible by the Index of the Power, and then divide the Characteriſtick ſo increaſed; but (ſince increaſing the negative Characteriſtick by any Number is really diminiſhing the Logarithm by that Number, we muſt, in Order to continue the Equality, add it to the Logarithm again; therefore, the next Place of the Logarithm being *Tenths*, each Unit in the Characteriſtick is equal to Ten in the Place of *Tenths*; therefore,) for every Unit which we added to the Characteriſtick we muſt now add 10's to the Place of *Tenths*; and divide as uſual.

62. *Example* 1. Extract the ſquare Root of 225.

Log. of 225 is = 2.3521825

$\frac{1}{2}$

Log. of 15 = 1.1760912

63. *Example* 2. Extract the Cube Root of .166375.

Log. of .166375 = $\bar{1}.2210881$

$\frac{1}{3}$

Log. of .55 = $\bar{1}.7403627$

Here, to the Characteriſtick $\bar{1}$, to make it diviſible by 3, we add $\bar{2}$, and the Sum is $\bar{3}$; $\frac{1}{3}$ of which is $\bar{1}$, for the Characteriſtick of the Log. of the Root: Then, to continue the Equality, we add 2 to the Place of Tenths, which makes how many Times 3 in 2.2, which gives .7, *&c.*

C H A P. VIII.

The APPLICATION *of* LOGARITHMS *to* FELLOWSHIP.

64. AS in Fellowſhip, the firſt and ſecond Terms continue the ſame in all the Statings: From the Logarithm of the ſecond Term ſubtract the Logarithm of the firſt Term, and call the Remainder the *reſerved Logarithm:* Then, this *reſerved Logarithm* being added to the Logarithm of the third Term of each reſpective Stating, will give the Logarithm of the fourth, or required Term of that Stating. The Reaſon of this is manifeſt from the Nature of Logarithms, and Chap. VIII. of the Second of the *Mathematical Eſſays.*

65. *Example.* Suppoſe 4 Men, A, B, C, and D, trade in Company; A put in 50*l.* B, 16*l.* C, 25*l.* and D, 18*l.* 10*s.* they gained 20*l.* 15*s.* What was each Man's Part?

Here the Statings would be,

£. £. £.
As 109.5 : 20.75 :: 50 : A's Share.
109.5 : 20.75 :: 16 : B's Share.
109.5 : 20.75 :: 25 : C's Share.
109.5 : 20.75 :: 18.5 : D's Share.

The Operation by Logarithms will be as follows.

From Log. of £. 20.75 = 1.3170181
Sub. Log. of 109.5 = 2.0394141

The Reſerved Log. = $\bar{1}$.2776040

Log. 50 = 1.6989700

Reſerved Log. = $\bar{1}$.2776040

Sum = 0.9765740 the Log. of A's Part anſwering in the Table to 9.47485 *l.*

Log. of 16 = 1.2041200

Reſerved Log. = $\bar{1}$.2776040

Log. of B's = 0.4817249 anſwers [to £. 3.031952.

Log. 25 = 1.3979400

Reſerved Log. = $\bar{1}$.2776040

Log. of C's Part = 0.6755440 anſwering [to 4.737425.

Log. of 18.5 = 1.2671717

Reſerved Log. = $\bar{1}$.2776040

Log. of D's Part = 0.5447757 anſwering [to 3.5056945.

C H A P. IX.

SIMPLE INTEREST *by* LOGARITHMS.

66. ADD the Logarithms of the Principal, Time. and Rate, together; the Sum will be the Logarithm of the Intereſt.

67. *Example*, What is the Intereſt of 410*l.* 10*s.* for 1 Year and 40 Days, at 5*l. per Cent. per Annum?*

The Time 1 Yr. 40 Ds. = $\frac{40}{365} = \frac{81}{73}$. Hence, The Logarithm of the Time is 0.0451622

Log. of 410.5 the Principal = 2.6133132

Log. of .05 the Rate = $\bar{2}$.6989700

Sum = 1.3574454 the Log. of 22.77743 *l.* = 22 *l.* 15 *s.* 5 *d.* $\frac{1}{4}$ for the Intereſt. Q. E. I.

CHAP.

CHAP. X.

COMPOUND INTEREST *by* LOGARITHMS.

68. THE RULE. Multiply the Logarithm of the Amount of 1*l.* for one Year, by the Time; and to the Product add the Logarithm of the Principal; the Sum will be the Logarithm of the Amount.

69. *Example.* What will 210*l.* 7*s.* 6*d.* amount to in 3 Years, at 5 *per Cent. per Annum?*

Solution. Firſt, 210*l.* 7*s.* 6*d.* = 210.375 *l.* and the Amount of 1*l.* for one Year at 5 *per Cent. per Annum* = 1.05.

Hence multiply the Log. of 1.05 = 0.0211893
By the Time = 3

0.0635679
Add Log. of 210.375 = 2.3229940

Log. of Amount 243.5353 *l.* = 2.3865619

Hence the Amount is 243*l.* 10*s.* 8*d.* ½ nearly.

70. *Scholium.* Mr. *Townley*, and a few others, inſtead of the negative Characteriſticks write their arithmetical Complements: Thus, for the Log. of .05, they would write $\overline{8}$.6989700, and for the Log. of .005 place down $\overline{7}$.6989700, *&c.* And the Rule for adding ſuch Logarithms is, If your Characteriſticks be negative, add them as in common Arithmetick, only noting, that if the Sum of the Characteriſticks be leſs than 10, add 10; if juſt 10, add Unity; if more than 10, caſt 10 away, the Sum, or Remainder, will be the required Logarithm, according to their Method. But if the Characteriſticks are of different Names, *viz.* one affirmative, the other negative, add them together as in common Addition; but if the Sum be 10, or more than 10, caſt 10 away, the Remainder will be affirmative; but if leſs than 10, negative.

Example. Multiply 203 by 25.

Log. of 203 is 2.3074960

Their Log. of 25 is $\overline{9}.3979400$

Log. of 50.75 = 1.7054360

As for Diviſion by Logarithms, they perform it after this Manner, *viz.* If one or both be negative, and the Characteriſtick of the Dividend is leaſt, add 10 to it; but if the Characteriſtick of the Dividend be the greater, ſubtract as uſual, and the Characteriſtick of the Remainder in the firſt Caſe will be affirmative, in the latter, negative.

Example. Divide .01265 by .55.

Log. of .01265 with their Index $\overline{8}.1020905$

Log. of .55 with their Index $\overline{9}.7403627$

Log. of .023 with their Index $\overline{8}.3617278$

Though this laſt Method, of writing the arithmetical Complements for the negative Characteriſticks, may be ſomething ſooner learnt, by ſuch Perſons as are ignorant of the Nature of affirmative and negative Quantities: Yet we would recommend the former Method as the moſt rational, and worthy the Notice of the more ingenious Reader.

Thus much is ſufficient at preſent concerning *Logarithms.* More expeditious Ways of conſtructing Tables, *&c.* will be ſhewn hereafter.

The **END.**

BRITISH MUSEUM 12OC76

AN

ESSAY

ON

Book-keeping by Double Entry:

Commonly called

Italian Book-keeping, or *Merchants Accounts.*

In which the *Reaſons* of the practical Rules are ſhewn from the Nature of Things, and the whole illuſtrated by proper Examples in two Sets of Books.

To which is prefixed, by Way of Introduction, the Nature of *Bills of Exchange, &c.*

By BENJAMIN DONN.

Magna eſt Veritas et prævalebit.

BRITISH MUSEUM

PREFACE.

EVERY Perſon the leaſt acquainted with Commerce muſt be ſenſible, that it would be impoſſible to carry on even a domeſtic Trade without ſome Method of Accounts: And were not their Excellence and Uſe ſufficiently known already, perhaps the following Words of Sir *Richard Steel* (in the *Spectator*, Number 174) would ſhew their Value; *viz.* "When a Man happens to break in *Holland*, "they ſay of him, he has not kept his Ac-"counts. This Phraſe perhaps amongſt us "would appear a ſoft or humourous Way of "ſpeaking, but with that exact Nation it bears "the higheſt Reproach; for a Man to be "miſtaken in the Calculation of his Expence, "in his Ability to anſwer future Demands, or "to be impertinently ſanguine in putting his "Credit to too great an Adventure, are all "Inſtances of as much Infamy, as with gayer "Nations to be failing in Courage, or common "Honeſty."

Various are the Methods that have been, and are at this Time made Uſe of, in keeping Accounts; but we may venture to affirm, that if any impartial Perſon, who rightly underſtands the Method of Double Entry, will but compare it with any other, he will preſently give the Preference to this. Books kept according to this Method will ſhew the Owner of them, by a bare Inſpection, what Quantity of Goods, and Caſh, he had at the Beginning of the Books; what he has ſince ſold, or loſt, or any otherwiſe diſpoſed of; what he has received; what Debts he owes, and what is due to him; and what he has gained or loſt upon each Parcel of Goods; in a Word, this Method of keeping Accounts will ſhew him at any Time a juſt State of his Affairs.

Hence, it appears that this Method is the moſt compleat, and therefore ought to be made Uſe of by all wholeſale Dealers at leaſt; but then thoſe that retail Things in very ſmall Quantities may demand, What Method ſhall we uſe? To whom it may be ſufficient to anſwer, that though they cannot be ſuppoſed to make Entries for every Farthing-worth ſold, yet they ought to approach the Method of Double Entry as near as poſſible, (at leaſt ſo near as the *Modern Method of Debtor and Creditor*,) for that being the beſt Method, the nearer they approach it the better muſt their's be: But not, upon any Account whatever, to ſpoil the Beauty and Correctneſs of their Books, by the ignorant and idle Faſhion of Croſſing, when Payment has been made.

I ſhall only add one Thing more in Praiſe of this Method, *viz.* That every Perſon, who has any Regard or Reſpect for his Family or Relations, muſt certainly be deſirous, that after his Deceaſe they may not be any Ways embarraſſed by not knowing the true State of his Affairs. Now there are many Perſons that have a particular Way of keeping their Accounts, by which they may perhaps know nearly the State of their own Affairs; but as this Method is ſuppoſed to be peculiar to the Owner of the Books, when he dies, his Executors, nor any other Perſons being perfectly acquainted with his Method, they muſt find it difficult (and perhaps in ſome Caſes impoſſible) to know the true Extent of his perſonal and real Eſtates; and hence the Executors may be involved in Diſputes and Troubles that could not happen, was his Method of Book-keeping generally known. Conſequently, all Perſons ought to make Uſe of ſuch a Method of Book-keeping, as is generally known, but none is ſo univerſally underſtood, and approved of, as the *Method of Double Entry*, and the *Modern Method* of Debtor and Creditor. Therefore every Perſon ought to keep his Accounts by one of theſe Methods; of which the firſt is undoubtedly the beſt in wholeſale Buſineſs. The Neceſſity of one common Method of Accounts might be illuſtrated by many real Inſtances, but for Brevity ſhall only take one from Mr. *Wier's* Preface to Mr. *Stevens's* Book-keeping; *viz.* "One *Nicholls* having been ſome Years in "Partnerſhip with Mr. *Lane*, a Merchant in "*Smyrna*, the former brought his Bill in the "Court of Chancery againſt the other; upon "the Balance whereof, he apprehended that "a conſiderable Balance would be coming due "to him: To this Bill the Defendant pleaded "the Statute of Limitations, which being "over-ruled, an Account was directed; the "final Reſult of it was, that inſtead of the "Defendant being indebted to the Plaintiff, "the Plaintiff was indebted to the Defendant "in 10000*l.* and upwards; an Event which

“ very much ſurpriſed the Court; but as Matters were laid before them then, not to be “ avoided, though at the ſame Time my Lord “ Chancellor declared, that it was highly probable, that the whole, in Relation to their “ Tranſactions, were not properly repreſented; “ for what could be more ſtrange, than to ſee “ two Partners, who were both apprized of “ all their Tranſactions, ſo widely miſtaken in “ Relation to their own Accounts, which “ thoſe Gentlemen certainly muſt have been, “ if preciſe Juſtice was done them in this “ Caſe!”

It is natural to ſuppoſe that Commerce as well as Numbers muſt be nearly coæval with the World; for every Man not having ſufficient of all Things, he will want Something of his Fellow-creatures: Hence, they found it neceſſary to ſupply each other with ſuch Neceſſaries as ſome had, and others ſtood in Need of; thus we find in the ſacred Scriptures that *Abel* was a *Keeper of Sheep*, and *Cain* a *Tiller of the Ground*, whence it may be reaſonably ſuppoſed, that *Abel* ſupplied *Cain* with Skins, *&c.* for Cloaths, and Fleſh for Meat; and that *Cain* in Exchange ſupplied *Abel* with Fruits, Corn, *&c.* for Food.

As the Number of Mankind increaſed, *Commerce* muſt alſo be increaſed, and as Neceſſity gave Riſe to it, the Deſire of having all Things convenient improved it; and Avarice, Vanity, and Luxury, carried it to its preſent Extent. It is plain that during the Management of Commerce by exchanging of Commodities,

dities, Trade muſt labour under great Difficulties; for many Times the Nature of the Commodities to be exchanged was ſuch that they could not be divided, or cut, without diminiſhing their Value; hence, the Neceſſity of Metals by which they might be valued. And as there might be great Frauds in the Weight and Quality of the Metal, Men at length came to ſtamp Names, Effigies of Princes, *&c.* on their current Pieces; whence came our Money. Gold and Silver (if not Copper) were uſed in Exchange in *Abraham*'s Time, if not before, as appears by the 23d and 24th Chapters of *Geneſis*, (the laſt of which Chapters is concerning *Iſaac*'s Marriage, about 1856 Years before *Chriſt.)*

As to the Origin of Books we have Nothing certain; all that we know is, that there were Books before thoſe of *Moſes*, he having mentioned ſome in his Writings. The Book of the Tables of the Commandments was written by GOD himſelf, according to the ſacred Writings, about 1490 Years before *Chriſt*.

Soon after the Invention of Money and Books, it is reaſonable to ſuppoſe that Men began to form ſome Method of keeping Accounts; but as to the Method of *Double Entry*, it is certainly of modern Invention, and from being frequently called *Italian Book-keeping*, it is natural to ſuppoſe had its Origin in *Italy*; but we have not been able to diſcover either the Author, or Time of the Invention. The moſt ancient Author that has fallen in our Hands is *Monteage*, in the Year 1682. Other Authors on this Subject are, *Snell*, 1701; *Hatton*,

Hatton, ; *Malcolm*, 1731; *Stephens*, *Webſter*, 1740; *Miers*, 1735; *Croſby*, 1749; *Dodſon*, 1750; *London*, 1750; *Shortland*, 1754.

This being a Subject not admitting any great Improvement, unleſs in the Method of handling it, all that ſeems neceſſary in this Place, is to give an Account of our Method of treating this Subject; *viz.* We firſt ſuppoſe a Perſon intending to Trade, deſirous of underſtanding a juſt Method of Accounts; not by committing a great Number of Rules to Memory, which has been too much the Method of teaching this, and other Arts; but is deſirous of deducing from the Nature of Things a few general Principles, which being rightly underſtood, may be of Service to him in his future Commerce. In Conſequence of this his Reſolution, he is ſuppoſed to lay down the Definitions of ſome Terms, and then by reaſoning from the Nature of Things, to deduce a few general Rules, ſufficient for keeping domeſtic Accounts; then he is ſuppoſed to begin a domeſtic Trade, and to make ſome Remarks on ſuch Tranſactions as may ſeem to want a little Explanation: And by thus reflecting on ſuch Occurrences as may appear to have ſome Difficulty in them, he may be ſuppoſed to increaſe in the Knowledge of Accounts.

Having now more Skill in Accounts, he is inclined to enter on a foreign Trade and Company Accounts; and therefore, again applies himſelf to inveſtigate from the Nature of Things, ſuch general Rules as are neceſſary for underſtanding this Kind of Accounts; which having done, he is ſuppoſed to enter into a foreign Trade upon his own Account, and alſo in Company, both employing Factors, and acting as Factor; all along reaſoning upon ſuch Tranſactions as may ſeem to have ſome Difficulties in them.

This is the Method we have choſen, and we hope it will be allowed to be a good one. How well we have executed this Plan, muſt be left to the Determination of our candid Readers: All that we ſhall deſire of any Perſon, is only this reaſonable Requeſt; not to paſs a Judgment on this Work, till he has carefully peruſed it.

Bideford, February 20,
1759.

B. DONN.

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THE
INTRODUCTION.

1. BEFORE we enter on Book-keeping, it will be proper, for the Sake of Learners, to explain ſome few Things relating to Bills, &c. In which, however, we ſhall be deſignedly ſhort, it being not conſiſtent with our Plan to diſcourſe largely on Bills, Promiſſory Notes, and other Cuſtoms of Merchants; but refer thoſe, who have an Inclination to ſee more of this Nature, to the practical Writers; from ſome of whom we have ſelected this INTRODUCTION.

2. *A Bill of Exchange* is a Note uſed by Merchants and Traders; ordering Money to be received in one Place, for the Value paid in another: To which People of Credit pay a great Regard; for without ſuch Contrivance foreign Trade could not be carried on.

3. In a Bill of Exchange the following Particulars muſt be mentioned. 1. The Place where the Drawer lives. 2. The Time of Payment. 3. The Perſon to whom the Bill is to be paid. 4. The Sum to be paid. 5. If the Bill be a foreign one, the Rate of Exchange. 6. Of whom the Value was received, or to whoſe Account the Draught is to be placed. 7. The Name of the Drawer, and the Place where to be paid.

4. The Perſons chiefly concerned in a Bill of Exchange are, 1. The *Remitter* or Sender of the Bill, who has paid the Value mentioned in the Bill to 2. the *Drawer*, who has received it in one Place, and delivered the Bill to the Remitter: Drawn upon,

3. the *Acceptant*, or Perſon who is to pay the Bill, to, 4. the *Poſſeſſor* or *Preſenter*, the Perſon who is to receive the Contents in another Place.

5. In drawing Bills of Exchange, the Signing of one Partner for Self and Company obliges the others.

6. A Bill of Exchange directed to one, to pay ſo much for Value received, ſhall be a good Diſcharge of the Debt, if the Bill be not returned back to the Drawer in Time, although it be not paid; for keeping the Bill long, is Evidence that he agreed to take the Merchant as Debtor.

7. If a Man pays a Bill of Exchange before due, and the Perſon to whom paid fails before the Time of Payment, he ſhall be obliged to pay it again to the Deliverer; becauſe the Drawer might have countermanded the ſame, or ordered the Bill to be made payable to another Perſon.

8. If a Perſon gives a Bill of Exchange, *&c.* upon a third Perſon, to another in Payment, and he takes it abſolutely; if he knew the third Perſon to be breaking, or in a failing Condition, and the Receiver of the Bill uſes all Diligence to get Payment, but cannot, this is a Fraud and no Payment; though if a Man takes a Note or a Bill, and after it is payable makes no Demand, ſo that he might be paid if he had been diligent enough, then if the Party on whom the Bill is drawn fails, it is at the Peril of him who took it.

9. Intereſt upon a Bill of Exchange commences from the Time of Demand.

10. If a Poſſeſſor of a Bill of Exchange by any Accident loſes it, he muſt cauſe Intimation to be made by a public Notary, before Witneſſes, that the Bill is loſt or miſlaid, requiring that Payment be not made to any Perſon without his Privity. And if any Bill of Exchange drawn in, or dated at, or from, any Place of this Kingdom, ſhall be loſt, the Drawer of the Bill ſhall give another Bill of the ſame Tenor,

Security being given to indemnify him, in Caſe the Bill ſo loſt ſhould be found again.

11. When a Bill of Exchange comes to the Preſenter's Hands, he is immediately to tender it to the Acceptant, who muſt write that he accepts it under the Bill. — If the Bill be payable at Uſance or after Date, the Acceptant's writing his Name, or making any other Mark on the ſame, is a ſufficient and valid Acceptance: But if it be payable after Sight, the Day which it is tendered for Acceptance muſt be mentioned upon it, becauſe from that Date the Time of Payment muſt be computed. And the Bill being thus accepted, the Acceptor becomes abſolutely and irrevocably Debtor to the Preſenter for the Contents; otherwiſe, the Hazard and Uncertainties on Bills of Exchange would be too great.

12. But if the Party to whom the Bill is directed refuſes to accept it, after 24 Hours Deliberation, Proteſt muſt be made for Non-acceptance at the Place of his Abode by a Notary Public, who is to be a Witneſs of that Refuſal; which Proteſt ſhould, for the Security of the Preſenter, be returned the firſt Poſt to the Remitter, that he may furniſh a new Bill, or take his proper Meaſures with the Parties concerned.

13. The Drawer of a Bill ſhould always, the ſame Poſt, take Care to give his Correſpondent Notice by Letter, that he has drawn upon him for ſo much, payable as in the Bill, to prevent its being proteſted, and ſent back non-accepted for Want of due Advice; for in that Caſe his Correſpondent may refuſe to accept till Advice arrives, if the Bill mentions Advice to be expected.

14. To prevent Interruption of Buſineſs by Miſcarriage of Letters, or other Accident, Merchants always draw *two* and often *three Outland Bills*, all of the ſame Tenor and Date, excepting in the *ſecond* againſt the *firſt*, and if there be three drawn, in the *third* againſt the *firſt* and *ſecond*, to prevent the Acceptor's

ceptor's paying more than one of them by Miſtake; which Bills the Remitter takes Care to ſend his Correſpondent, to whom they are made payable, by different Poſts, one of which being anſwered, the Reſt are of no Force.

15. If an accepted Bill be not paid upon Demand the very Day it falls due, it muſt be put into the Hands of a Notary Publick by the Preſenter, in Order to have a Proteſt drawn up, under a Copy of the Bill, for Non-payment; which Proteſt, within 14 Days at the fartheſt, muſt be returned (but not the Bill itſelf unleſs for ſpecial Reaſons) to the Remitter who paid the Value, and who is to give Satisfaction for his Concern therein to the Preſenter, and who will procure Satiſfaction of the Drawer, not only with Reſpect to the principal Sum, and the Intereſt thereof, from the Day of Proteſt, but alſo may recover the Rechange of the ſaid Sum, with Charges of Proteſt, and whatever Damages ſhall be incurred, by the Default of his Correſpondent, the Acceptant; though Rechange is ſeldom inſiſted on.

16. The Neglect of Proteſt in due Time leaves the Preſenter or Poſſeſſor of a Bill no Security but that of the Acceptant, whereas he has otherwiſe the Drawer and every one of the Indorſers (if any) beſides the Acceptant to depend on for the Principal and Damages; and if but one of them prove ſufficient he will be no Loſer.

17. No Bill of Exchange can be proteſted, unleſs the Value be mentioned therein to be received, and ſuch Perſon named of whom, nor can you note an Inland Bill under 20*l.* Value; ſuch Bills are commonly returned without Charges.

18. If you diſcount, or pay a Debt with an Inland Bill, payable to yourſelf or Order, you write your Name on the Backſide of it, and deliver it into the Poſſeſſion of the Perſon you intend to make it over unto, which is called indorſing, whereby you

aſſign all your Property therein over to him; and, in Caſe of the Acceptant's Failure before it is paid, you are, by Virtue thereof, as reſponſible to ſuch your Aſſign for the Contents and Damages, upon Proteſt made, as the Remitter is to you, and the Drawer to him. In like Manner let a Bill be indorſed by ſeveral Perſons, the Poſſeſſor, or the Perſon to whom it was laſt aſſigned, in Caſe of Nonpayment, cauſes Proteſt to be made; which being returned to the laſt Indorſer, he is obliged to ſatisfy the laſt Poſſeſſor as to the Contents and Charges, and returns it in the ſame Manner to the ſecond; he to the third, &c. till at laſt it recurs upon the Drawer, who is obliged to anſwer all Damages as before: In the very ſame Manner ought *Promiſſory Notes* to be treated, they being in the Eye of the Law of the Nature of Inland Bills of Exchange. —— But on an Outland Bill, beſides the Indorſer's Name, it is uſual to fill up the Indorſement, by appointing Payment to his Order, naming the Perſon he aſſigns it over to, ſpecifying the Reaſons and Conditions that induced him to make ſuch Indorſement, as, *Pay Mr.* R. W. *or Order Value S. S.* and if R. W. ſhould aſſign it to another; Pay Mr. *P. D.* or Order, *Value of himſelf, R. W.* remembering always, that unleſs the Word *Order* be inſerted, no Bill of Exchange or promiſſory Note is indorſible to another.

If the Acceptant of a Bill ſhould chance to fail between the Time of Acceptance and that of Payment, Proteſt may be made in that Caſe, for better Security, before the Bill becomes due; not but, by the good Underſtanding among Dealers, the Damages, as Brokerage, Intereſt, &c. that would follow Proteſt, are frequently prevented, eſpecially if the Drawer, or any of the Indorſers, be ſufficient Perſons; for any of their Correſpondents, out of Reſpect, may re-accept the Bill for Honour of the Drawer or Indorſer, if applied to by the Poſſeſſor, after he has, for the Security of the Re-acceptor, cauſed the Bill to be proteſted, with a Declaration, that the Bill was re-accepted for Honour of the Drawer, *&c.* underwriting it, *Accepted, for Honour of the Drawer, per* L. L. Or, if the Reſpect was ſhewn to an Indorſer, *Accepted, for Honour of the Indorſer* W. W. *per* L. L. In the ſame Manner are to be ſerved, ſuch Bills as are drawn upon a Perſon, who has not equivalent Effects of the Drawer's in his Hands, or that he has not received due Advice of the Bill preſented to him for Acceptance; which entitles him to a legal Claim upon the Drawer, if he thinks he may venture, under thoſe Circumſtances, to honour his Bill. As ſoon as theſe Bills, accepted under Proteſt, become due, it is the Cuſtom of Merchants to have a ſecond Proteſt, for Non-payment, made by the Poſſeſſor, with a Declaration as before, that the Re-acceptor did pay the ſame, with Charges, for Honour as beforeſaid; which Proteſts the Re-acceptant, for his own Security, will take Care to return with the firſt Opportunity to his Friend and Correſpondent, in whoſe Favour he advanced the Money.

19. As foreign Bills are commonly drawn at ſingle, double, or treble *Uſance*, it is neceſſary to obſerve, that *Uſance* between *London* and any Part of *France* is 30 Days after the Date of the Bill. From *London* to *Amſterdam*, *Antwerp*, *Brabant*, *Bremen*, *Flanders*, *Hamburgh*, *Middleburg*, *Rotterdam*, and *Zealand*, is *one Calendar Month*, and the ſame from theſe Places to *London*. From *London* to *Spain* and *Portugal*, *et contra*, is 2 Calendar Months.

From *London* to *Genoa*, *Leghorn*, *Milan*, *Rome*, and *Venice*, *et contra*, is 3 Months.

Some Places date their Bills by *Old Stile*, as *Denmark*, *Eaſt-Frieſland*, *Geneva*, *Gueldres*, *Hamburgh*, *Holſtein*, *Ruſſia*, *Utrecht*, and all the Proteſtant Principalities of *Germany* and Cantons of *Switzerland*.

Bills are dated according to the *New Stile* in all Places belonging to the Crown of *Great Britain*; alſo in *Amſterdam*, *Rotterdam*, *Leyden*, *Haerlem*, *Middleburgh*, *Ghent*, *Bruſſels*, *Brabant*, and in all the *Netherlands* except *Utrecht* and *Gueldre*; in *France*, *Spain*, *Portugal*, *Sweden*, *Italy*, *Hungary*, *Poland*, and all the Roman Catholic Principalities of *Germany* and Cantons of *Switzerland*.

Note, the 1ſt of any Month Old Stile is the ſame as the 12th New Stile.

After Bills of Exchange become due according to their Dates, it is the Cuſtom of Merchants to allow certain *Days of Grace*, *viz.*

" The Days of Grace are *, in *London*, 3 Days, " Holidays and *Sundays* incluſive; in *Amſterdam*, " *Rotterdam*, *Middleburgh*, and *Antwerp*, 6 Days; " in *Cologn*, *Breſlaw*, *Nuremberg*, and *Venice*, 6 " Days, but the Holidays and *Sundays* are not in- " cluded; at *Frankfort*, out of the Fair, 4 Days; " at *Leipſick*, out of the Fair, 5 Days; and *Aug-* " *ſburg* 5 Days; and in all the Places above- " mentioned, Bills payable at Sight muſt be paid " within 24 Hours: *Naples*, 8 Days; *Dantzick* and " *Koningsberg* 10 Days; *Paris*, *Roan*, *Rochelle*, " *Nantz*, *Bourdeaux*, and all *France*, 10 Days, *Sun-* " *days* and Holidays included; *Hamburgh* 12 Days, " *Stockholm* 12 Days, *Portugal* 6 Days, *Spain* 14 " Days, *Genoa* 30 Days: But in *Italy* there are no " Days of Grace allowed, but the Poſſeſſor of the " Bill may proteſt or not proteſt, according to his " Diſcretion."

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* *Haye's Negociator's Magazine.*

FORMS *of* BILLS.

London, 2 *Novemb.* 1758. 100*l.*

Thirty Days after Date pay to Mr. *John Lovecaſh*, or Order, One Hundred Pounds, Value received, as *per* Advice from *James Merchant.*

To Mr. Cornelius Matthews,
Merchant in Briſtol.

Bideford, 3 *Oct.* 1758. 80*l.*

At Sight pay to Mr. *George White*, or Order, Eighty Pounds, Value received, as *per* Advice from *John Draw-well.*

To Mr. Abraham Ready-money,
Merchant in London.

Rotterdam, 10 *March*, 1758. 120*l.* Sterling.

At Uſance pay this my firſt of Exchange in *London*, to the Order of Mr. *Richard Vernam*, One Hundred and Twenty Pounds Sterling, Value in Account, as *per* Advice from *Jan Van-Culen.*

To Mr. John Miller,
Merchant in Briſtol.

London, *June* 10, 1758, for Crowns 612 at Uſance.

At Uſance, pay this firſt of Exchange to Mr. *James Can*, or Order, Six Hundred and Twelve Crowns, at Sixty Sols *Tournois per* Crown, for Value here received of Mr. *James Johnſon*, and place it to Account, as *per* Advice from *Godfrey* and *Gower.*

To Mr. John Feſteau,
Merchant in Calais.

Exchange 500 Crowns, at 35*d. per Crown.*
London, 17th *April*, 1758.

At Uſance pay this my firſt of Exchange, to Mr. *A B.* or Order, the Sum of Five Hundred Crowns, at Sixty Sols *Tournois per* Crown, Value received in Goods of the ſaid *A. B.* at Thirty-five Pence Sterling *per* Crown, as *per* Advice from

Your's, &c. P. M.

To C. D. *Merchant at* Roan.

A

A Paris, *ce* $\frac{1}{12}$ Sept. 1758.
Pour 500 △, *à* 33*d. Sterl. per* △.

A double Uſance, payez, par çette premiere de change, à M. *Pierre Touzine*, ou à ſon ordre, cinq cens écus, à trente trois deniers Sterlins pour écu, valeur reçue de M. *Jacques Rochet* et paſſez à compte, ſuivant l'avis *De votre tres humble Serviteur,*
Baudouin Demedis.

A Monſieur Monſieur Philippe Rogers,
Marchand à Londres.

21. *Of Promiſſory Notes.*

A *Promiſſory Note* is a Note of Hand, or Bill, promiſing Payment of a Sum of Money at an appointed Time.

22. A *Promiſſory Note* mentioning *Order* is indorſible from one Perſon to another, which is done by the preſent Poſſeſſor's writing his Name on the Back of it, and delivering it up to the Party to whom he intends to aſſign over his Property therein.

23. It is unneceſſary to have a Promiſſory Note payable to the Bearer, indorſed; if you are ſatisfied the Note is good.

24. The delivering up a Promiſſory Note to the Perſon who ſigned it, is a ſufficient Voucher of its being paid, nor is there any Need of writing a Receipt thereon.

25. Promiſſory Notes, and Book-debts, if not legally demanded in the Space of Six Years, cannot be recovered by Law.

26. If you keep a Promiſſory Note upon Demand in your own Hands above Three Days, and the Perſon it is upon ſhould fail, the Loſs will be your own; but if he fail within the Three Days, it will light in Equity on the Perſon that paid it you.

27. *Forms of Promiſſory Notes.*

I promiſe to pay *Paul Peters*, Eſq. or Order, on Demand, Three Hundred Ninety-four Pounds, Ten Shillings, Value received this 14th, *June* 1757.

£. 394 : 10. ꝑ *William Edwards.*

Borrowed and Received of Mr. *Aaron Goodfriend*, the Sum of One Hundred Pounds Sterling, which I promiſe to pay on Demand: Witneſs my Hand, this 16th of *May*, 1758.

John Needham.

Witneſs *John Brown.*
George Truſty.

28. A *Receipt* is an Acquittance for Money received: This being a Thing ſo eaſy to be made, we ſhall only give one Example.

November the 16th, 1758. Received of Mr. *John Paywell*, the Sum of Three Pounds, Ten Shillings, and Eleven Pence, in full of all Demands.

£ 3 : 10 : 11 *Thomas Lovemoney.*

29. An *Invoice* is an Abſtract of a Merchant's Books, giving an Account of the Quantity, Quality, and Price, *&c.* of any Parcel of Goods which he ſends to his Correſpondent.

30. When a Factor has ſold Goods for his Correſpondent, and completed the Affair, he ſends an Abſtract of his Accounts relating thereto to his Correſpondent or Proprietor, under the Title of *Account of Sales*, and *Account Current.*

If the Reader has a Mind to ſtudy the Laws and Cuſtoms of Merchants, with Caſes, *&c.* on Bills of Exchange, *&c.* he may read *Cunningham*'s *Law of Bills of Exchange, &c.* *Beawes*'s *Lex Mercatoria Rediviva, or Merchant's Directory*, and *Poſtlethwayt*'s *Dictionary of Trade and Commerce*; in which Books the Subjects are largely treated of, which cannot be expected in this Work.

AN
ESSAY
ON
BOOK-KEEPING.

CHAP. I.

Of the Nature of BOOK-KEEPING, *&c.*

1. BEING juſt going to enter on Trade, I am now retired to my Cloſet, with ſome of the moſt eſteemed Writers on Accounts, to improve what I learned at School, *&c.* and draw up ſome rational Rules for Book-keeping. For though perhaps I am as well acquainted with Accounts as moſt young Men are, yet, as it is the general Fault of moſt Maſters to teach the Rules without ſhewing how they ariſe, my Time will not be thrown away in attempting to deduce the fundamental Rules from the Nature of Things; and if the Succeſs is anſwerable to my Hopes, the Pleaſure will abundantly repay the Study of the Inveſtigation.

2. Firſt then. *Book-keeping* is the Art of keeping a regular and true State of the Tranſactions of a Perſon's Affairs; ſo that the Book-keeper may readily ſee, at any Time, what Changes are made in the

State of his Affairs. Various are the Methods which have been uſed for this Purpoſe: But that which is the moſt general, and in greateſt Repute, is the Method of *Double Entry*, commonly known by the Name of *Italian Book-keeping*, from the firſt Attempt of this Nature being made in *Italy*; and *Merchant's Accounts*, from its being in almoſt general Uſe amongſt Merchants; as it is very difficult to keep a true State of their Affairs without ſuch a Method.

3. The Books that are in a Manner neceſſary to be kept, are, the *Memorial-Book*, *Journal*, and *Leger*. The *Memorial-Book* is ſo called from its Uſe, being that in which every Tranſaction is entered immediately as it occurs, that Nothing may ſlip the Memory. This Book begins with an Inventory of the real, and perſonal Eſtate, of the Owner; it being abſolutely neceſſary that he ſhould have a true State of his Affairs at Beginning of Trade. This Book is many Times called a *Waſte Book*, being only intended as a Preparation for the *Journal*.

4. The *Journal* (*Journal, French*) is a Book, in which the *Waſte Book* Entries are written in a more regular Method, pointing out the *Debtors*, and *Creditors*, that the *Leger* may be formed with leſs Difficulty.

5. I find two different Methods of making Entries in the *Journal Book*, *viz.* 1. That which has been hitherto moſt commonly uſed, and which was firſt invented. 2. A new Method practiſed and allowed of (as *Malcolm* ſays) by ſeveral eminent Traders and Accountants to be preferable to the common Method.

An EXAMPLE *of each* METHOD.

A Waſte Book Entry.	*Journal Entry by the firſt Method.*	*Journal Entry by the new Method.*	
A has drawn a Bill for 100*l.* upon B, payable to me.	B. Debtor to A 100*l.* for a Bill drawn upon B by A, payable to me.	D^{r}. B. C^{r}. A. 100 *l.*	A has drawn a Bill for 100 *l.* upon B payable to me.

One of Mr. *Malcolm*'s Reaſons for preferring the new Method, is, " As any Perſon who writes fair " and correct, is capable to make a right Copy of " the Waſte Book, who yet could not make up the " Journal; the Book-keeper, whoſe principal Taſk " is the Journal (in ſo far as it differs from the " Waſte Book) and Leger, may be ſaved the Trouble of writing the Waſte Book, where are others " that can conveniently do it; by which Means he " will have more Time and Leiſure for filling up the " Leger; and the Debtors and Creditors on the " Margin of the Waſte-Book." To this Reaſon of Mr. *Malcolm*'s we may add, that if the Waſte Book as firſt written be fair, then by marking the Debtors and Creditors in a Margin, (according to the Rules explained in the Article *Leger*,) it will anſwer the End of the beſt Journal without a Quarter of the Trouble.

6. The following Remark of Mr. *Huſcraft Stephens*, on the Uſe of the Word *To* in the Journal, is alſo a ſtrong Argument for the Uſe of the ſecond Method; *viz.* " In making Uſe of the Word *To*, in the Journal, it oftentimes ſeems to contradict our Senſe and " Reaſon; as for Example, Suppoſe A draws a Bill " for 100 *l.* upon B, payable to me; then, in the " Journal I ſhould ſay, B Debtor and A Creditor " 100 *l.* which is very plain and natural to be underſtood, but by joining the Debtor and Creditor " together by the Word *To*, though it ſignifies no " more than the pointing out the Debtor and Creditor Entries, *&c.* yet it would make the Tranſaction ſeem as if it was ſtated falſly; for Inſtance, " B Debtor to A 100 *l.* Now, properly ſpeaking, " if B was Debtor to A, then A muſt be Creditor to " B, the one owing, and the other giving him Credit for ſo much Money, but they neither of them " are indebted or give Credit to one another; for A " is Creditor to me, by Reaſon I am to account to " him for the Value of the Bill drawn; and B is " Debtor to me, as he hath accepted and promiſed

"Payment of the ſaid Bill." But in making Uſe of the ſecond Method, this ſeeming Contradiction is avoided. See Art. 5.

7. The * *Leger Book* is that in which each Subject has a diſtinct and ſeparate Space allowed it, containing an equal Portion of right and left Pages, with its Name written on the Head thereof; the Pages being firſt marked *Debtor*, and *Creditor*; the *left-hand* Pages being deſigned for *Debtor* Entries, and thoſe on the *Right-hand* for *Creditor* Entries. For the eaſier Inveſtigation of the principal Rules for making thoſe Entries, it is proper to conſider the ſeveral Kinds of Accounts, *perſonal*, *real*, and *imaginary*.

8. Perſonal Accounts are grounded on the mutual Credit any Perſon gives me, or that I give him. For I have no Account with any Perſon, till ſome Tranſaction happens, by which either that Perſon becomes under an Obligation to me, or I to him. It is evident that I may have Tranſactions with a Perſon and yet have no Account; for all Tranſactions wherein we exchange an equal Value of Goods, Money, *&c.* have no Relation to account betwixt us; for though I may be conſidered as Debtor to him for what I had of him, he muſt then be accounted Debtor to me for what he received from me, which being equal by the Suppoſition, they balance each other; and therefore, properly ſpeaking, we are neither of us indebted to the other, and conſequently there is no Neceſſity for an Account betwixt us. But when I receive from any Perſon more than he does from me; or he receives more from me than I from him, it is evident one of us will become under an Obligation to the other, and conſequently there will be an Account between us. And, to repreſent a

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* This Word is wrote by different Authors, *Leger*, *Ledger*, and *Leidger*. Mr. *Malcolm* thinks it to be derived from the *Latin*, *Legere, to gather*, becauſe in this Book all the Accounts are collected together. Mr. *Johnſon*, in his Dictionary, from the *Dutch* Word *Legger*, ſignifying ſomething kept in a Place, as this Book is in an Accounting Houſe. Others from an *Italian* Word.

juſt State of our Accounts, I may make him Debtor for all Obligations he is under to me; that is, put thoſe Articles on the Debtor Side of his Account, and make him Creditor for all thoſe Articles by which I am under an Obligation to him, that is, put thoſe Articles on the Creditor Side.

9. In *real Accounts*, *Debtor* and *Creditor* are uſed in an improper Senſe, taken from perſonal Accounts. The Reaſon of which is this: As all Things are valued by Money, ſo whatever Goods I have by me, or whatever Goods I buy, muſt be valued by Money; for if the Value be not expreſſed as well as the Quantity, there cannot be a juſt View of my Eſtate. Hence, as I may expect to make of my Goods as much as they coſt me, they are in Effect the ſame to me as if their Value was due to me from ſome Perſon; and as, in ſuch Caſe, that Perſon would be Debtor, ſo I may make the Goods in my Poſſeſſion Debtor for their firſt Coſt, and all ſubſequent Charges; and when they are ſold, as ſome Thing or Perſon muſt be accountable to me for them, the Goods ſold muſt be diſcharged, or made Creditor; that is, entered on the Creditor Side, to ſhew that they are ſold, as they were afore on the Debtor Side. And, if the Money received for the Goods ſold is more or leſs than they coſt me, (that is, more or leſs than the Sum for which they were made Debtor,) the Exceſs, or Defect, is certainly Gain or Loſs; and therefore, for this Exceſs, or Defect, I may erect an imaginary Account, entitled *Profit and Loſs*; (of which more hereafter.)

When Goods are given away or loſt, they muſt be entered on the Creditor Side, to ſhew that they are gone from me: And ſince, in ſuch Caſe, I may be ſaid to have loſt ſo much, (it being the ſame in Effect whether I give away or loſe any Thing,) and as there cannot be a Creditor without a Debtor, it will be neceſſary to have an Account entitled *Profit and Loſs*, to be made Debtor, or ſhew the reſpective Loſs.

As for a *Caſh Account*, I may charge and diſcharge it for what comes in and goes out reſpectively, in the ſame Manner as in other real Accounts.

10. *Imaginary Accounts* are found neceſſary for preſerving the Balance of the Leger; for by the foregoing Rules we know, that each Perſon in the Inventory muſt have a diſtinct Account, in which each reſpective Perſon is to be made Debtor for what he owes me, or Creditor for what I owe him; and every Thing, whatever it be, that I have in my Poſſeſſion muſt be made Debtor. But then, what muſt be the correſpondent Debtor or Creditor to preſerve the Balance? For all that I aim at here, is to ſhew, that ſuch Subjects belong to me, and that there are ſuch Articles of Debt and Credit betwixt ſuch Perſons and me, which are to be entered by proper Expreſſions. Hence, I muſt form an imaginary Account, called the *Stock Account*, which I muſt make Debtor for all I owe by the Inventory; and Creditor for all the Effects that are in my Hand, and Debts owing to me by the Inventory. Hence this Account may be conſidered as repreſenting myſelf, who am the Owner of the Books; the Articles on the Creditor Side ſhewing what I have in my Poſſeſſion, and what Debts are owing to me; and thoſe on the Debtor Side, what I owe.

11. The Uſe of the Profit and Loſs Account may be thus ſhewn. If I ſell Goods for more than they coſt me, the Exceſs being ſo much gained, I enter it on the Creditor Side of the *Profit and Loſs Account*; on the contrary, if, through their being damaged, or ſome other Misfortune, I am obliged to ſell them for leſs than they coſt me, it is evident, the Defect is, with Reſpect to me, ſo much loſt, and therefore, in Contradiſtinction to Gain, I enter it on the Debtor Side of this Account. Again, when I give away or loſe any Thing, as it is to me as a real Loſs, I enter it on the Debtor Side of this Account, to ſhew how much my real Eſtate is diminiſhed thereby. Conſequently, on the contrary, when I receive any Legacy or Preſent,

it being the ſame to me as if I had gained ſo much by Trade, I enter it on the Creditor Side, to ſhew how much my real Eſtate is augmented thereby. In a Word, every Thing given away, or received, or Debt contracted, for which there is Nothing exchanged in Return, or Obligation for it on any Perſon, is to be eſteemed as a Gain or Loſs, though in ſtrict Propriety of Speech it may not be ſuch. Thus, in particular, *Diſcounts* or *Rebates* allowed in Payments of Debts, *Intereſt of Money*, *Expences of Houſekeeping*, *&c.* belong to this Account; and the Method of entering theſe, or any other, muſt be plain, from what has been already ſaid.

12. From what has been ſaid it is evident,

1ſt, That the *Stock Account* is *Debtor* for all I *owe* by the Inventory; and *Creditor* for what I *have* in my Hands, or is owing to me, mentioned in the Inventory.

2dly, *Caſh* is *Debtor* for whatever is *received*, and *Creditor* for what Money is *paid* or *given away*.

3dly, *Profit and Loſs* is *Debtor* for all *Loſſes*, and *Creditor* for all *Gains*.

4thly, *A Perſon's Account* is *Debtor* for what I pay him, or he owes me; and *Creditor* for what he pays me, or I owe him.

5thly, An *Account of Goods* is *Debtor* for the Value of all received, with the Coſt and Charges thereto belonging; and *Creditor* for what any Ways goes out.

And in buying or ſelling, this Rule is general:

$$\left.\begin{array}{l}\text{What I receive}\\ \text{Or the Perſon receiving}\end{array}\right\}\text{is Debtor.}$$

$$\left.\begin{array}{l}\text{What I deliver}\\ \text{Or the Perſon delivering}\end{array}\right\}\text{is Creditor.}$$

13. From what has been obſerved, the following practical Rules naturally follow.

RULE 1. In the Leger every Subject muſt be entered double, *i. e.* for every *Debtor* entered I muſt enter its correſpondent *Creditor*, *&* *contra*.

14. Rule 2. When a Set of Books are to be opened, let the Account of *Stock* be made *Debtor* for all Sums owing by me; and *Creditor* for the Caſh, Goods, and Debts, belonging to me.

15. Rule 3. Let the Perſons to whom I owe Money, have Credit in their Accounts for the Sum due to them.

16. Rule 4. Let the Accounts of ſuch Perſons as owe Money to me, be made Debtor for the Sum due to me.

17. Rule 5. Let the Accounts of Caſh, Goods, *&c.* be made Debtor for their Value.

Hence, for every Article by which the Account of Stock is made Debtor, ſome other Account will be Creditor, *& contra*, agreeable to Rule 1.

18. Rule 6. When I receive Money.

In Payment of a Debt	Caſh is Debtor	to the Payer.
Borrowed		to the Lender.
As a Gift		to Profit and Loſs.

19. Rule 7. When I pay Money.

For a Debt	The Receiver	is Debtor to Caſh.
To a Perſon who borrows it	The Borrower	
As a Gift	Profit and Loſs	

20. Rule 8. When I buy Goods.

For Caſh	Goods bought Debtor	To Caſh.
On Truſt		To the Seller.
For Goods		To the Goods ſold.
Part Truſt		To the Seller, in Part.
Part Goods		To the Goods ſold, in Part.
Part Money		To Caſh in Part.

21. Rule 9. When I ſell Goods.

For Money	Caſh	Debtor to Goods ſold.
On Truſt	The Buyer	
For Goods	Goods received	
Part Money	Caſh in Part	
Part Truſt	Buyer in Part	
Part in Goods	Goods in Part	

22. Rule 10. Of Diſcounts and Abatements.

When I pay Money before due, and have Diſcount allowed	The Receiver Debtor	To Caſh for Money paid. To Profit and Loſs for Diſcount.
Money received by me before due, and I allow Diſcount thereon	Caſh D^r.	To Payer for Money received.
	Profit and Loſs D^r.	To Payer for the Diſcount.
Goods damaged	Profit and Loſs D^r.	To Goods for the Damage.

N. B. The Word *To*, in the above Rules, ſignifies, that the Subjects to which it is prefixed are Creditors.

23. Theſe Rules being ſufficient for a domeſtick Trade: I ſhall now proceed to illuſtrate them by a Variety of Examples, in a Waſte Book, *&c.* kept according to the new Method, hinted at in Art. 5. and make ſuch occaſional Remarks on ſome Tranſactions that may ſeem to require them.

N. B. The Figures in the Column of References in the Journal, refer to the Folio in the Leger where the reſpective Subject is entered. After the ſame Manner the References in the Leger refer back again to the Folio of the Journal.

The WASTE-BOOK and JOURNAL of me *A. B.* of *Bideford, Anno* 1757.

Beginning *December* the Firſt.

Number I.

	References to Leger.	*December* 1, 1757.	l. s. d.	l. s. d.
Sundry Accounts, *viz.* the ſeveral Articles that make up my Eſtate are *Debtors*. And Stock *Creditor*, 945*l.* 10*s.* Note, Stock is in Folio 1 of the Leger.		*The* INVENTORY *of the real and perſonal Eſtate of me* A. B. *of* Bideford, viz.		
	1	I have in Caſh —	500 : 0 : 0	
	2	In Sugar, 10 hhds. at 6*l*	60 : 0 : 0	
	2	Raiſins 10 Barrels at 1 *l.* 10*s.*	15 : 0 : 0	
	2	Broad Cloth 9 Pieces, *viz.*		
		N°. 1. 4 Pieces, each 20 Yds at 10*s. per* Yard	40 : 0 : 0	
		N°. 2. 5 Pieces, each 16 Yds. at 12*s.* —	48 : 0 : 0	
	2	A Piece of Linen, containing 20 Yds. at 2*s.* 6*d. per* Yard —	2 : 10 : 0	
	2	My Moveables and houſehold Furniture (as by particular Inventory thereof) are worth	150 : 0 : 0	
		The following Perſons owe me as follows. *Viz.*		
	2	*John Jackſon* owes me on Demand —	30 : 0 : 0	
	3	*William James* owes me by Bond, dated the 1ſt of *June* laſt, and payable the 1ſt of *January* next; (and to bear Intereſt at 5 *l. per Cent. per Annum*) The Principal is	100 : 0 : 0	
				945 10 —

	References	*l. s. d.*		*l.*	*s.*	*d.*
			I owe to ſeveral Perſons as under; *viz.*			
Stock (ſee *Leger* Folio 1) is *Debtor*, and ſundry Accounts, *viz.* the ſeveral Perſons to whom I am indebted, are *Creditors*	3 3	— — — — — — 90 : 0 : 0	*l.* To *John Williams* on Demand 40 To *Daniel Gregory*, payable the 10th of *January* next 50 *Here ends the* INVENTORY, *and now follow my daily Tranſactions in Trade.*	90	—	—
			December 4th, 1757.			
D^{r}. Caſh C^{r}. *J. Jackſon*	1 2	30 : 0 : 0	*John Jackſon* has paid me in full	30	—	—
			5th			
D^{r}. *J. Williams* C^{r}. Caſh	3 1	40 : 0 : 0	Paid *John Williams* in full —	40	—	—
			10th			
D^{r}. Caſh C^{r}. Sugar	1 2	8 : 0 : 0	Sold for ready Money 1 hhd. of Sugar — — — — — —	8	—	—
			12th			
D^{r}. Paper C^{r}. Caſh	3 1	6 : 0 : 0	Bought 10 Reams of Paper, at 12*s.* per Ream, for which I paid —	6	—	—
			14th			
D^{r}. Caſh D^{r}. *Stephens* C^{r}. BroadCloth	1 3 2	20 : 0 : 0 36 : 0 : 0 56 : 0 : 0	Sold Mr *John Stephens* 4 Pieces of Broad-Cloth, N^{o}. 1 each 20 Yards, at 14*s.* per Yard, is 56*l.* *l.* Of which he has paid me – 20 Reſt payable in 10 Days — 36	56	—	—
			17th			
D^{r}. Wine C^{r}. Caſh C^{r}. *W. James*	4 1 3	60 : 0 : 0 40 : 0 : 0 20 : 0 : 0 60 : 0 : 0	Bought of *William James* 10 Barrels of Wine, at 6*l.* per Barrel, is 60*l.* *l.* For which I paid ready Money 40 Reſt to pay on Demand — 20	60	—	—
			18th			
D^{r}. Serges C^{r}. Paper	4 3	8 : 0 : 0	Barter'd 10 Reams of Paper at 16*s.* per Ream is 8*l.* For 2 Pieces of Serges at 4*l.* is alſo — —	8	—	—

December 20th, 1757.

		l *s.* *d.*		*l.*	*s.*	*d.*
Dr. Caſh Cr. Profit & Loſs	1 1	21 : 0 : 0	*William Young* has made me a preſent of 20 Guineas —	21	—	—
			21			
Drs. Paper Holland Cr. *W. Emerſon*	3 4 4	8 : 8 : 0 60 : 0 : 0 68 : 8 : 0	Bought of *William Emerſon* 14 Reams of Paper at 12*s.* per Ream, is — — 8*l.* 8*s.* And 10 Pieces of *Holland* at 6 *l.* per Piece 60 0 I am to pay on Demand.	68	8	—
			22			
Drs. Caſh *W. James* Cr. Profit & Loſs	1 3 1	50 : 0 : 0 57 : 0 : 0 107 : 0 : 0	Took *John James* Apprentice for 107 *l.* Of which I have received *l.* 50 The remaining Part to be paid me at a Year's End, by *William James*, Father of the ſaid *John James* — — — 57	107	—	—
			23			
Drs. Caſh *Emerſon* Cr. Profit & Loſs	1 4 1	1000 : 0 : 0 300 : 0 : 0 1300 : 0 : 0	I am married to Miſs *Mary Johnſon*, whoſe Fortune is 1000*l.* in Caſh, which I have received *l.* 1000 *William Emerſon* owes her on Demand — — 300	1300	—	—
Dr. Profit & Loſs Cr. *W. James*	1 3	150 : 0 : 0	She owes *William James* on Demand — — — — —	150	—	—
			25			
Drs. Brd. Cloth Caſh Cr. *W. Emerſon*	2 1 4	200 : 0 : 0 31 : 12 : 0 231 : 12 : 0	Received of *William Emerſon* 10 Pieces of Broad Cloth, at 20*l* per Piece, is — 200*l.* 0*s.* And in Caſh — — 31 12 The whole being the Balance of his Account — — —	231	12	—
			26			
Dr Tobacco Cr. *J. Jackſon*	4 2	24 : 0 : 0	Bought of *John Jackſon* 2 hhds. of Tobacco at 12*l.* per hhd. payable in 6 Months — —	24	—	—

December 27th, 1757.

		l. : *s.* : *d.*		*l.*	*s.*	*d.*
Dr. *J. Jackſon*	2	24 : 0 : 0	*John Jackſon* being deſirous to have ready Money for the Tobacco that I bought of him Yeſterday, has offered to rebate me 2 *l.* which Propoſal I have accepted, and have paid him in full for the Tobacco *l.* 22 The Abatement 2	24	—	—
Crs. Caſh Profit&Loſs	1 1	22 : 0 : 0 2 : 0 : 0 22 : 0 : 0				
			28			
Dr. *J. Williams* Cr. *J. Stephens*	3 3	36 : 0 : 0	Received of *John Stephens* a Bill for 36 *l.* on *John Williams*, payable at 10 Days, which Do. *Williams* hath accepted —	36	—	—
			29			
Dr. *D. Gregory*	3	50 : 0 : 0	Paid *David Gregory* what I owed him, in Caſh and Goods, *viz.* *l.* In Caſh — — — — 2 6 Barrels of Wine at 8 *l.* each 48	50	—	—
Crs. Caſh Wine	1 4	2 : 0 : 0 48 : 0 : 0 50 : 0 : 0	Note, As he took moſt Part in Goods, I did not inſiſt on any Diſcount for paying my Money before due.			
			30			
Dr. Houſe-Expences Cr. Caſh	4 1	60 : 0 : 0	By my Book of Expences of Houſe-keeping, I find that I have paid for Rent, Victuals, idle Expences, &c. ſince the Beginning of this Book —	60	—	—
Dr. Charges of Merchandiſe Cr. Caſh	5 1	30 : 0 : 0	By my Book of Charges of Merchandiſe, I find, that ſince the Beginning of this Book, I have paid to my Clerk for his Wages, Poſtage of Letters, and other Charges on Goods not charged to particular Accounts	30	—	—

The LEGER ALPHABET. Number I.

A *Folio*	B *Folio*	C *Folio*
	Broad Cloth — 2 Balance — — 5	Caſh — — — 1 Charges of Merchandize 5
D	E	F
	William Emerſon — 4	
G	H	J
David Gregory — 3	Holland — — 4 Houſe Expences — 4	*John Jackſon* - — 2 *William James* — 3
K	L	M
	Linen — — — 2	Moveables — — 2
N	O	P
		Paper — — — 3 Profit and Loſs — 1
Q	R	S
	Raiſins — — 2	Stock — — — 1 Sugar — — — 2 *John Stephens* — 3 Serges — — — 4
T	V	W
Tobacco — — 4		*John Williams* — 5 Wine — — — 4
X	Y	Z

Folio 1.		Debtor.	References.	l.	s.	d.
		Stock.				
1757.						
December.	1	I owe to ſundry Perſons as by Inventory —	1	90	—	—
		My neat Eſtate — — — — — — —	—	2077	10	—
				2167	10	—
		Caſh.				
1757.						
December.	1	I have in ready Money, by the Inventory –	1	500	—	—
	4	Received of *John Jackſon* — — — — —	2	30	—	—
	10	Received for Sugar — — — — — —	2	8	—	—
	14	Received for Broad Cloth — — — — —	2	20	—	—
	20	Received a Preſent from *William Young* —	3	21	—	—
	22	Received with my Apprentice *John James* —	3	50	—	—
	23	Received as Part of my Wife's Fortune —	3	1000	—	—
	25	Received of *William Emerſon* — — —	3	31	12	—
				1660	12	—
		Profit and Loſs.				
1757.						
December.	23	My Wife owed *William James* when I was married to her — — — — — — —	3	150	—	—
		Brought from Houſe-Expences — — —	—	60	—	—
		Brought from Account of Charges of Merchandize — — — — — — — —	—	30	—	—
		Neat Gains ſince the 1ſt of *December*, carried t the Stock Account — — — — — —	—	1222	—	—
				1462	—	—

Creditor.

Stock.

1757.			R.	l.	s.	d.
December	1	My Money, Goods, and Debts owing to me, by the Inventory amount to	1	945	10	—
		Gains, brought from Profit and Loſs Account		1222	—	—
				2167	10	—

Caſh.

1757.			R.	l.	s.	d.
December	5	Paid *John Williams*	2	40	—	—
	12	Paid for Paper	2	6	—	—
	17	Paid for Wine	2	40	—	—
	27	Paid *John Jackſon*	4	22	—	—
	29	Paid *David Gregory*	4	2	—	—
	30	Paid Houſe Expences	4	60	—	—
	30	Paid Charges of Merchandize	4	30	—	—
		Balance remaining in my Hands	5	1460	12	—
				1660	12	—

Profit and Loſs.

1757.			R.	l.	s.	d.
December	20	Received as a Preſent from *William Young*	3	21	—	—
	22	*William James* has paid me 50*l.* and owes me 57*l.* more for taking his Son Apprentice	3	107	—	—
	23	My Wife's Fortune in Caſh is 1000*l.* and *William Emerſon* owes her 300*l.* In all	3	1300	—	—
	27	*John Jackſon* hath rebated me	4	2	—	—
		Gained by Sugar		2	—	—
		Gained by Broad Cloth		16	—	—
		Gained by Paper		2	—	—
		Gained by Wine		12	—	—
				1462	—	—

Folio 2.		*Debtor.*		R.	*l.*	*s.*	*d.*
1757.		*Sugar.*	hhds				
December	1	I have, at 6*l.* per hhd. as per Inventory	10	1	60	—	—
		Gained by this Account. — Carried to Profit and Loſs			2	—	—
					62	—	—
1757.		*Raiſins.*					
December	1	I have, at 1*l.* 10*s.* per Barrel, as per Inventory	Bar. 10	1	15	—	—
1757.		*Broad Cloth.*	Pieces				
December	1	I have, Numb. I. 4 Pieces, and Numb. II. 5 Pieces	9	1	88	—	—
	25	Bought of *William Emerſon* at 20l. per Piece	10	3	200	—	—
		Gained by this Account			16	—	—
					304	—	—
1757.		*Linen.*	Piece				
December	1	I have, as per Inventory	1	1	2	10	—
1757.		*Moveables.*					
December	1	I have Houſehold Goods, &c. as per Inventory, to the Value of		1	150	—	—
1757.		*John Jackſon*					
December	1	Owes me on Demand, as per Inventory		1	30	—	—
	27	I have paid for Tobacco 22*l.* and had 2*l.* Diſcount allowed. In all		4	24	—	—
					54	—	—

Folio 2. *Creditor.*

				R.	*l.*	*s.*	*d.*
		Sugar.					
1757.			hhds				
December	10	Sold for ready Money – – – – –	1	2	8	—	—
		Balance remaining, at 6*l.* per hhd. carried to the Balance Account – –	9	5	54	—	—
			10		62	—	—
		Raiſins.					
		Balance remaining, at 1*l.* 10*s.* per Barrel – – – – – – – – – – –	Bar. 10	5	15	—	—
1757.		***Broad Cloth.***	Pieces.				
December	14	Sold *John Stephens* of Numb. I. – –	4	2	56	—	—
		Balance remaining of Numb. II. 5 Pieces, alſo 10 Pieces. In all – –	15	5	248	—	—
					304		
		Linen.	Piece.				
		Balance remaining – – – – – –	1	5	2	10	—
		Moveables.					
		Balance remaining to the Value of – – –		5	150	—	—
1757.		***John Jackſon***					
December	4	Hath paid me in full – – – – – – –		2	30	—	—
	26	Bought of him Tobacco, payable in 6 Months		3	24	—	—
					54	—	—

Debtor.

William James

			R.	*l.*	*s.*	*d.*
1757.						
December	1	Owes me by Bond as per Inventory, the principal sum is — — — — — —	1	100	—	—
	22	Owes me at a Year's End for taking his Son Apprentice — — — — — — —	3	57	—	—
		Balance owing to him, carried to the Balance Account — — — — — — —		13	—	—
				170	—	—

John Williams.

			R.	*l.*	*s.*	*d.*
1757.						
December	5	Paid him in full — — — — — — —	2	40	—	—
	28	I have a Bill on him, payable in 10 Days, for		36	—	—
				76	—	—

David Gregory.

			R.	*l.*	*s.*	*d.*
1757.						
December	29	Paid him in Cash and Wine — — —	4	50	—	—

Paper,

			Reams.	R.	*l.*	*s.*	*d.*
1757.							
December	12	Bought at 12*s.* per Ream, with ready Money — — — — — —	10	3	6	—	—
	21	Bought of *William Emerson,* at 12*s.* per Ream — — — — — —	14		8	8	—
		Gained by this Account — — —			2	—	—
			24		16	8	—

John Stephens

			R.	*l.*	*s.*	*d.*
1757.						
December	14	Owes me for Broad Cloth sold him — —	2	36	—	—

Folio 3.		## *Creditor.*	Reams.	R.	*l.*	*s.*	*d.*
1757.		*William James.*					
December	17	I owe him on Demand for Wine — —		2	20	—	—
—	23	owe him, (being a Debt of my Wife's,) — —		3	150	—	—
					170	—	—
1757.		*John Williams.*					
December	1	I owe him, on Demand, as per Inventory		2	40	—	—
		Balance due to me — — — — — —			36	—	—
					76	—	—
1757.		*David Gregory.*					
December	1	I owe him, payable the 10th of *January* next, as per Inventory — — — — — —		2	50	—	—
1757.		*Paper.*					
December	18	Gave in Barter for Serges at 16*s.* per Ream — — — — — — —	10	2	8	—	—
		Balance remaining at 12*s.* per Ream	14		8	8	—
			24		16	8	—
1757.		*John Stephens.*					
December	28	Received of him a Bill on *John Williams* for		4	36	—	—

Folio 4.

Debtor.

				R.	*l.*	*s.*	*d.*
		Wine.					
1757.			Bar.				
December	17	Bought of *William James* at 6*l.* per Barrel — — — — —	10	2	60	—	—
		Gained by this Account — — —			12	—	—
					72	—	—
1757.		*Serges.*	Pieces.				
December	18	Received in Barter for Paper, at 4*l.* per Piece — — — — —	2	2	8	—	—
1757.		*William Emerson*					
December	23	Owes me, as Part of my Wife's Fortune —		3	300	—	—
					300	—	—
1757.		*Holland.*	Pieces.				
December	21	Bought of *William Emerson*, at 6*l.* —	10	4	60	—	—
1757.		*Tobacco.*	hhds				
December	26	Bought of *J. Jackson*, at 12*l.* per hhd. payable in 6 Months — — —	2	3	24	—	—
1757.		*House Expences.*					
December	30	Paid for House-Rent, Victuals, &c. — —		4	60	—	—

Folio 4.		*Creditor.*		R	£	s.	d.
		Wine.					
1757.			Bar.				
December	29	Sold *David Gregory*, at 8 *l.* — — —	6	4	48	—	—
		Balance remaining, at 6 *l.* per Barrel —	4		24	—	—
					72	—	—
		Serges.	Pieces.				
		Balance remaining at 4 *l.* per Piece —	2		8	—	—
		William Emerſon.					
1757.							
December	21	Bought of him Paper and Holland, to pay on Demand — — — — — — — — —		3	68	8	—
	25	Received of him Broad Cloth and Caſh —		3	231	12	—
					300	—	—
		Holland.	Pieces.				
		Balance remaining, at 6 *l.* per Piece —	10		60	—	—
		Tobacco.	hhds				
		Balance remaining, at 12 *l.* per hhd. —	2		24	—	—
		Houſe Expences.					
		Carried to the Debtor Side of the Profit and Loſs Account — — — — — — —			60	—	—

Debtor.

			R.	l.	s.	d.
1757.		*Charges of Merchandize.*				
December	30	Paid Book-keeper's Salary, *&c.* — — —	4	30	—	—
1757.		*Balance.*				
December	30	*Cash* remaining in my Hands — — —	1	1460	12	—
		Sugar, remaining 9 hhds, at 6*l.* per hhd. —	2	54	—	—
		Raisins, remaining 10 Barrels, at 1*l.* 10*s.* —	2	15	—	—
		Broad Cloth, remaining 15 Pieces — —	2	248	—	—
		Linen, remaining 1 Piece — — — —	2	2	10	—
		Moveables, *&c.* remaining — — — —	2	150	—	—
		John Williams owes me — — — —	3	36	—	—
		Paper, remaining 14 Reams at 12*s.* — —	3	8	8	—
		Wine, remaining 4 Barrels at 6*l.* — — —	4	24	—	—
		Serges, remaining 2 Pieces at 4*l.* — — —	4	8	—	—
		Holland, remaining 10 Pieces at 6*l.* — —	4	60	-	—
		Tobacco, remaining 2 hhds. at 12*l.* — —	4	24	—	—
				2090	10	—

Folio 5.		*Creditor.*	R.	*l.*	*s.*	*d.*
		Charges of Merchandize.				
		Carried to Debtor Side of Profit and Loſs Account — — — — — — —		30	—	—
1757.		***Balance.***				
December	30	***Owing to William James*** **(ſee his Account) —**		13	—	—
		My *neat Eſtate* brought from the Stock Account		2077	10	—
				2090	10	—

CHAP. II.

Some occaſional REMARKS *on the Set of Books Number* I.

24. THE firſt Thing worthy of Notice, concerning the making the Debtor and Creditor Entries, is, the Article relating to *William James* in the Inventory. In this Caſe ſome Authors would immediately make him Debtor for both Principal and Intereſt; but as Payment may be made either before or after the Day appointed, the Intereſt may be either more, or leſs, than the Intereſt for the Time mentioned. Nor can *William James*, juſtly ſpeaking, be chargeable for Intereſt at a Time when there is none due: Therefore, it is certainly the beſt Method at preſent to charge him only for the Principal, and omit taking any Notice of Intereſt till the Time of receiving the Debt.

25. *December* 10. Here, as I have ſold 1 Hogſhead of Sugar for 8*l.* which coſt me but 6*l.* it is evident I have gained by it 2*l.* and therefore 2*l.* may be carried immediately to the *Profit and Loſs* Account. But it is better, as it will ſave Trouble, and prevent the making that Account very long, to omit carrying the Gain or Loſs of each particular Account to the Profit and Loſs Account, till we come to balance the whole Account; and then to carry the Gain or Loſs upon the whole.

26. *December* 22. Here, as I have taken an Apprentice, it is evident that what I have received and am to receive, is, with Reſpect to me, as ſo much Profit; and therefore, *Profit and Loſs* muſt be Creditor 107*l.* And as I have received 50*l.* in Caſh, the *Caſh* Account muſt be Debtor for it, and *William James* for the Sum not paid.

27. *December* 28. As I have a Bill from *John Stephens* on *John Williams* which is accepted; *John Stephens* may be ſaid to have paid his Debt, and there-

fore muſt be *Creditor*, and *John Williams* having accepted the Bill becomes *Debtor* to me in the Place of *John Stephens*.

28. *December* 30. Here *Profit and Loſs* may be made immediately Debtor for the Houſe Expences, *&c.* they being, with Reſpect to me, as ſo much really loſt. But as it will be more ſatisfactory to ſee a diſtinct Account of the Expences of Houſekeeping, it will be proper to erect an Account, entitled *Houſe Expences*, or *Expences of Houſekeeping*, which I may make Debtor for all Payments relating to Houſekeeping: (And to prevent filling the grand Books with every trifling Expence of Houſekeeping, it will be convenient to keep a ſmall Book for this Purpoſe, and then, when I am cloſing my grand Books, the Total may be brought at once into the Waſte Book.) After the ſame Manner a ſmall Book may be kept for ſuch Charges of Merchandize as I may not think proper to charge to any particular Account; and the Total carried at once to the Account entitled *Charges of Merchandize*.

CHAP. III.

Of making a TRIAL BALANCE, *and the Method of diſcovering and correcting of Errors.*

29. HAVING poſted all the Journal Entries into the Leger, I muſt now proceed to balance the Accounts to make up the Leger, in Order to begin a new Set of Books; but firſt it will be convenient to make a Trial Balance on ſome waſte Paper, to examine the Truth of the Entries. In Order to which, I add up all the Debtor Sides of the Leger, and alſo all the Creditor Sides; then, if I have not committed ſome Error in poſting, the total Sum of the Debtor Sides muſt be equal to that of the Creditor Sides; becauſe, according to the Me-

thod of double Entry, every Debtor has an equal Creditor. Hence, if upon Trial the Sum of the Debtor and Creditor Sides are equal, we may reaſonably conclude the Leger Entries are truly made: For, the Sum of the Debtor and Creditor Sides cannot be equal, and yet ſome Error committed, unleſs there are two or more Errors committed on oppoſite Sides, which are equal, or balance each other; and let Reaſon judge if it is not improbable, that, when I am aiming at Truth, I ſhould commit two or more Errors ſo very extraordinarily circumſtanced; for though I may eaſily commit one or more Errors, yet it is very improbable, (though not impoſſible,) that I ſhould commit others exactly equal, and contrary thereto.

30. But when the Sum of the Debtor and Creditor Sides of the Leger are not equal, there muſt be an Error or Errors committed: And in Order to diſcover them I muſt firſt examine the *Journal* to ſee if the Debtors and Creditors are rightly formed, and then ſee whether they are properly carried into the *Leger*. If I find a wrong Entry in the Journal, it may be corrected by eraſing what is wrong, and writing the Correction in its Stead; or by tranſcribing the whole Tranſaction in a diſtinct Place choſen for that Purpoſe.

31. To prevent Eraſements, Errors in the Leger may be thus corrected: 1. When the *Debtor* is only entered, the *Creditor* being omitted; or the *Creditor* entered, and the *Debtor* omitted. In this Caſe, I have Nothing to do but to inſert that which was omitted.

32. *Second Caſe*. When the *Debtor* and *Creditor* are charged with different Sums of Money.

This may be explained by an Example.

Suppoſe in the Leger I had entered the Article of *December* the 4th thus:

Caſh

Caſh Debtor.	*Caſh Creditor.*
Received of *John Jackſon* 30 *l.*	
John Jackſon Debtor.	*John Jackſon Creditor.*
Error — — — 6 *l.*	Hath paid me in full 36 *l.*

Here I find there muſt be an Error, becauſe the Caſh Account ſays there was received of *John Jackſon* only 30 *l.* whereas his Account ſays he has paid 36 *l.* But looking back to the Journal I find he paid only 30 *l.* therefore I have given him too much Credit by 6 *l.* and conſequently muſt make him *Debtor* for the Error to bring the Account right; hence, when corrected, thoſe Accounts will be as above.

33. *Caſe 3d.* When ſome Tranſaction is entered in both Accounts on the *Debtor*, or both on the *Creditor* Side. For Example. Suppoſe the abovementioned Tranſaction was entered thus.

Caſh Debtor.	*Caſh Creditor.*
Received of *John Jackſon* 30 *l.*	
John Jackſon Debtor.	*John Jackſon Creditor.*
Hath paid me in full 30 *l.*	*Hath paid me in full* 30 *l.* *Error* — — 30 60

Here I have entered what *John Jackſon* hath paid on the Debtor Side of his Account, whereas it ought to have been on the Creditor Side. Therefore I muſt now enter it on the Creditor Side; and as it was at firſt entered on the Debtor Side, I muſt now enter 30 *l.* more on the Creditor Side to balance that Error; and then the Account will appear as above.

34. *Caſe 4th.* When an Article is written in an Account to which it does not belong. This may be corrected by placing the ſame on the other Side; and alſo entering it in the Account to which it belongs, if it is there omitted. An Example will make this plain. Suppoſe, in entering the Tranſaction of *December* the 4th, I had omitted making *Caſh Debtor* for the Money received of *John Jackſon*, and by Miſtake had entered it on the Debtor Side of Stock.

Thoſe Accounts, when corrected, will appear thus:

Caſh Debtor.	*Caſh Creditor.*
Received of *John Jackſon* 30*l.*	
Stock Debtor.	*Stock Creditor.*
Received of *John Jackſon* 30*l.*	*Error*, by inſerting in the Debtor Side of this Account the Money paid by *J. Jackſon*, 30*l.*

Here it is evident, that as I had made *Stock Debtor* 30*l.* by a Miſtake, and to correct it have made it *Creditor* 30*l.* it is the ſame as if no Miſtake had happened.

The Errors being corrected (if any,) the next Thing to be done is to *balance* the ſeveral Accounts.

CHAP.

CHAP. IV.

Of BALANCING *particular Accounts, or cloſing and balancing the* LEGER.

35. WHAT remains now to be done, is to draw up the Reſult of all the particular Accounts, in one particular Account opened for that Purpoſe, which, from its Uſe, is called the *Balance* Account. Which Account on the *Debtor* Side will ſhew what Money and Goods I have in my Hands, or belonging to me, with the Debts owing to me; and on the *Creditor* Side what Debts are owing by me.

36. To balance the *Caſh* Account.

In this Account the Exceſs of the Debtor Side above the Creditor is the Money or Balance remaining in my Hands, and therefore to be carried to the Debtor Side of the *Balance* Account. And it is plain that the Difference of Debtor and Creditor Sides of this Account being added to the Creditor Side, their Sums will be equal, and conſequently this Account be balanced.

37. Of a *Perſon*'s Account.

If the Debtor Side is greater than the Creditor, the Exceſs is the Balance owing by that Perſon to me, and therefore to be carried to the Debtor Side of the *Balance* Account. But if the Debtor Side is leſs than the Creditor, the Defect is the final Balance owing by me to that Perſon, and is therefore to be carried to the Creditor Side of the *Balance* Account. — The Balance being added to the leſſer Side will make it equal to the other Side.

Note, If at the Time of cloſing a Perſon's Account there is any Intereſt due by me to him, or from him to me, which is not already entered in his Account, it ought to be entered before I proceed to balance his Account.

38. Of Accounts of *Goods.*

In balancing Accounts of Goods, three Caſes naturally ariſe. 1. *When none are ſold.* Then the whole Quantity on the Debtor Side is the Balance due to me, and therefore to be carried to the Debtor Side of the *Balance* Account.

2. *When all are ſold*, or *any Way gone out.* Here it is evident, that the Difference of the Debtor and Creditor Sides is either Gain or Loſs; and muſt therefore, if Gain, be carried to the Creditor Side, but if Loſs, to the Debtor Side of the *Profit and Loſs* Account.

3. *When only* Part *of the Goods are ſold.* In ſuch Caſe, what remains at the prime Coſt is the Balance due to me, (to be put on the Creditor Side of that Account) and to be carried to the Debtor Side of the *Balance* Account. This done, the Difference betwixt the Sums of Money on the Debtor and Creditor Sides is either Gain or Loſs; — Gain, if the Creditor Side be greater, and therefore to be carried to the Creditor Side of the *Profit and Loſs* Account; but if the Debtor Side be greater, it is Loſs, and therefore to be put on the Debtor Side of the *Profit and Loſs* Account.

Note, In an Account of Goods, the Profit or Loſs (being the Difference of the Sums of Money on the Debtor and Creditor Sides) when added to the leſſer Side muſt make the Sides equal, or, in the Language of Accountants, the Account will be balanced. For an Example, ſee an Account of Goods in the Leger.

Perhaps it may be uſeful farther to remark, that if there happens to be an *Inlack*, that is, the Goods are ſhort of Meaſure or Weight; then, if all are ſold, the Quantity of Goods on the Creditor Side will be leſs than on the Debtor Side, by ſo much as is the Deficiency of Meaſure or Weight: Therefore that Deficiency being entered on the Creditor Side, by ſaying, *Deficient in Weight or Meaſure ſo much*, will make the Quantities of Goods on both Sides equal: But no Money muſt be placed againſt the Deficiency, becauſe no Money was received for it. But if, when all is ſold, it appears that I have made out more than I bought, it is manifeſt I had *over Meaſure*, or *Weight*, and the Exceſs of the Quantity on the Creditor Side over that on the Debtor, expreſſing the over Meaſure, or Weight, being entered on the Debtor Side, by theſe or the like Words, *Over Meaſure*, or *Over Weight*, ſo much, muſt make the Quantities on each Side equal; but then I muſt not put any Money againſt the Over-Meaſure, for that is already included in the Sum for which the Goods were ſold.

39. Of *Houſe-Expences*, *Charges of Merchandize*, *&c.* Such Accounts as theſe being really with Reſpect to me as ſo much loſt, they may be balanced by *Profit and Loſs*; and therefore may be carried to the *Debtor* Side of that Account.

40. All the Accounts being ſuppoſed to be now balanced, except the *Profit and Loſs*, *Balance*, and *Stock* Accounts, the next Thing to be done is to balance the *Profit and Loſs* Account, which may be thus done.

The Difference of the Debtor and Creditor Sides of this (the Profit and Loſs) Account, being added to the leſſer Side muſt balance this Account. And if the Debtor Side be greater, the Difference is what is gained on the whole Trade.

Then, if the Balance of the *Profit and Loſs* Account was *Gain*, being carried to the *Creditor Side* of the *Stock* Account; but if *Loſs*, to the *Debtor Side* of that Account; the Difference of the Debtor and Creditor Sides of the *Stock* Account muſt be my *neat Eſtate*, or real Worth after all my Debts are paid, if the Creditor Side be greater: But if I ſhould be ſo unfortunate as to have the Debtor Side the greater, then the Difference would be how much I am deficient; or how much I owe more than I have wherewith to pay. — This Difference being added to the leſſer Side, muſt balance the greater.

41. My *neat Eſtate* or real Worth (found by the laſt Article) being added to the *Creditor* Side of the *Balance* Account; or my *Deficiency*, if it ſhould ſo happen, being added to the Debtor Side of that Account, will (if there be no Miſtake) make that Side to which it is added equal to the other; and this is the laſt Proof of the Books being right.

42. That the *Balance* Account will be balanced by *my neat Worth* or *Deficiency*, as ſaid in the laſt Article, may be thus demonſtrated, *viz.* From what has been ſaid, it is manifeſt, that in all Accounts, whether perſonal, Accounts of Goods, or Expences, *&c.* wherein any Gain or Loſs has happened, the Gain or Loſs by each Account was carried to the *Profit and Loſs* Account; the Gains on the Creditor, and the Loſſes on the Debtor Side; and conſequently, the Difference of the Sides of that Account will be the Gain, or Loſs, gained or loſt upon the whole Trade; and therefore my Stock muſt be either increaſed, or decreaſed, ſo much as I have gained or loſt upon the whole; and if I carry this Increaſe or Diminution of the Stock to the *Stock* Account, *viz.* to the Creditor Side if Gain, to the Debtor if Loſs; then, as the Creditor Side of the Stock Account contains the preſent Value of what any Ways, or by whatever Means belongs to me, and the Debtor Side whatever is owing by me; it is evident the Difference of the two Sides of the Stock Account will be either my preſent Worth, or what I am deficient: Preſent Worth if the Creditor Side be greater, or Deficiency if the Debtor be greater. — Now, ſince whatever appears to be any Way belonging to me, by the balancing of the Accounts of Perſons, Goods, Houſe Expences, *&c.* has been entered on the Debtor Side of the *Balance* Account, and whatever is owing by me on the Creditor Side of that Account, it is clear, that the Difference of the Debtor and Creditor Sides of the Balance Account will be equal to my preſent Worth, or Deficiency alſo; preſent Worth, or my neat Eſtate, if the Debtor Side be the greater; or

Deficiency if the Creditor be greater: Conſequently, if my preſent Worth (already found by the Stock Account) be added to the Creditor Side; or the Deficiency (if it ſhould ſo happen) to the Debtor Side, the Sums of the Debtor and Creditor Sides of the *Balance Account* muſt then be equal, or balance each other; for it is an Axiom, that the Difference of any two Quantities being added to the leſſer, is equal to the greater Quantity.

From what has been ſaid in this Chapter, theſe practical Rules naturally follow.

43. To *balance* Books.

Make the *Balance* Account
Dr. To *Caſh*, remaining.
To *Goods*, remaining.
To *Perſons*, owing.
Cr. | By *Perſons* to whom I owe.

Make my Account of *Profit and Loſs* —
Dr. To *Goods*, for a Loſs.
To *Houſe Expences*, &c.
Cr. | By *Goods* for Gains.

Then, the Difference of
Profit and Loſs, carry to the oppoſite Side of *Stock*.
Stock, carry to the oppoſite Side of *Balance*.

And then, the Sum of the Debtor and Creditor Sides of *Balance* will be equal, if no Miſtake be made.

44. To draw a new *Inventory* in a new Set of Books.

The Dr. Side of the *Balance Account* Will be all I poſſeſs, or belongs to me.
The Cr. Side of the *Balance Account* Will be all I owe.

Thus much is ſufficient for a domeſtick, ſeparate Trade; therefore I now paſs on to Foreign Trade.

PART II.

OF THE

Eſſay on BOOK-KEEPING by Double Entry, in a Foreign Trade and Company Accounts.

CHAP. V.

Of Voyages, or ſending Goods abroad.

45. HAVING laid down the fundamental Rules of Book-keeping, ſo far at leaſt as relates to domeſtick Accounts, I now intend to attempt the Inveſtigation of Rules for Foreign and Company Accounts, as I propoſe very ſoon to put ſuch Rules in Practice.

The Method generally obſerved by Merchants in writing Journal Entries, is, 1ſt. To mention the Date. 2. The Debtor of the Article to be entered. 3. The reſpective Creditor. 4. The Sum of Money. 5. The Quantity and Quality of the Merchandize or Effects bought or ſold, or exchanged, *&c.* 6. The Conditions of Agreement, if any.

46. The firſt Thing that occurs to my Mind, is, that as *Bills of Exchange* frequently happen in foreign Trade, it will be proper to have two Accounts in the Leger, entitled *Bills of Exchange remitted me*, and *Bills of Exchange drawn upon me*, to ſhew what Bills I have by me, when payable, *&c.*

The Account of *Bills of Exchange remitted me* may be made Debtor for all Bills remitted me, and Creditor for thoſe I give out.

The Account of *Bills of Exchange drawn upon me* may be made Creditor for thoſe Bills which are drawn upon and accepted by me; and Debtor when I pay them.

Hence theſe practical Rules.

47. When I receive a Bill of Exchange.

For Goods immediately ſold — — —	Bills remitted me Dr.	To Goods ſold.
In Payment of a Debt		To the Remitter.
As a Gift — —		To Profit & Loſs.

48. When I give out a Bill of Exchange (which was afore remitted me.)

For Goods bought immediately — —	Goods bought	Debtor to Bills remitted me.
In Payment of a Debt	The Receiver	
As a Gift — —	Profit and Loſs	

49. When a Bill of Exchange is drawn on and accepted by me, payable by myſelf.

The *Drawer* is *Debtor* to *Bills drawn on me.*

And when I pay the Bill:

Bills drawn on me, is *Debtor* to *Caſh* for the Money paid.

50. To proceed to Foreign Trade.

Firſt then, when I ſhip *my own Goods* by *Order of my Correſpondent*, *for his Account*, it is manifeſt, ſo ſoon as they are on Ship-board *he* muſt become *Debtor* for them; the whole hazard of the Voyage being at his Riſque, therefore, I muſt make him *Debtor* to the *Goods ſhipped*, and charges on them till on Ship-board. And when I receive any Returns, *he* muſt be *Creditor*, that is, the *Effects* which I receive muſt be made *Debtor* to him.——If my Correſpondent lives in a Country where the Money in which Accounts are kept, is of a different Species from that in which my Accounts are kept, and I have at the ſame Time Dealings with him for my own Account, it will be proper for Diſtinction Sake, to entitle his Account thus, *A. B. (viz.* my Correſpondent) *his Account*; but otherwiſe this Diſtinction is not neceſſary.

51. Hence theſe practical Rules.

(*Of Voyages for my Correſpondent.*)

When I ſend Goods for Account of my Correſpondent — —	Correſpondent Debtor	To Goods ſent, & To Caſh for Charges paid by me on ſhipping them.
When I receive Returns in Goods in Bills	Goods received Bills remitted	Debtor to Correſpondent.

52. When I ſend Goods abroad on *my own Account*, conſigned to my Factor or Correſpondent beyond Sea. — As the Goods I ſend away muſt, if they are my own, (and ſtand already in my Leger,) be credited by the Rules aforegoing, there muſt be ſome Account Debtor, or accountable for them: And ſince the Perſon to whom I have conſigned them cannot be accountable for any Thing till the Goods are in his Poſſeſſion, I muſt for the preſent erect an imaginary Account, which may be entitled *Voyage to ſuch Place in ſuch Ship*, or rather *Merchandiſe conſigned to ſuch Perſon of ſuch Place for my Account*. Which I may make *Debtor* for all the Goods, and for all the Charges I have been at in ſhipping them; and for all other Expences that may happen upon their Account afterwards on the Voyage. But if the Goods are not already mentioned in my Books, being juſt bought, then this Account muſt be *Debtor* to the *Perſon* of whom I bought them if not paid for; or, to *Caſh* if I paid for them in Money; or, to other *Goods* if I bought them in Barter, *&c.* — This Account muſt alſo be *Debtor* to *Caſh* for the *Inſurance*, if I have paid the Inſurance in ready Money; but if not as yet paid, *Debtor* to the *Inſurers*. —— This Account muſt be *credited*, when the Perſon to whom they were conſigned has diſpoſed of them, and ſent me an Account of their *Sales; or in what Manner he has diſpoſed of them.

53.

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* **Some Authors, upon Advice that my Factor has received, but not ſold the Goods, would make,** *Goods per Ship — in the Hands*

53. When the *Goods* are *ſold*, as my Factor muſt be accountable to me for the neat Proceeds thereof, I muſt make him *Debtor* to the *Goods* conſigned to him, for ſo much as the *neat Proceeds* amount to: And if the Money or Coin in the Country in which my Factor lives, is of a different Species or Value from that in the Country where I reſide, it will be proper for Diſtinction Sake to entitle the Account in which he is to be made Debtor or Creditor for any Thing tranſacted for my Account, *(Such Perſon) my Account*. In this Account it will be proper to have double Money Columns, *viz.* one for the Species of the Place in which I live, the other for that of the Country in which my Correſpondent lives. And it may be proper here to remark that I am under a Neceſſity of leaving *Blanks*, in the Waſte Book, Journal, and Leger, for the *Sterling* Money, till I know what the Foreign thereto belonging produced in Sterling Money. And then the Blanks muſt be filled up at a Medium, if it has been remitted at different Rates of Exchange.

54. Hence theſe practical Rules.

Of Voyages.

When I ſhip Goods conſigned on my own Account	After they are entered in my Books	Merchandiſe conſigned to my Factor Debtor	To the Goods.
	When bought on Truſt & not entered in my Books		To the Seller.
	But if for Caſh		To Caſh.

***Hands of ſuch Factor*, *Debtor* for the Goods conſigned to him, *&c.* But as this is erecting more Accounts than are neceſſary; it is ſufficient, in ſuch Caſe, only to make a Memorandum under the Article of Goods ſent in the Waſte Book, that the Factor received thoſe Goods at ſuch a Time; and take no farther Account of them till I receive an Account of their being ſome Way or other diſpoſed of.**

Of Inſurances.

55. When I pay Money to be inſured	Merchandiſe inſured	Debtor to Caſh.
When I receive Money for a Loſs — —	Caſh Debtor.	To Merchandize loſt.
When I receive a Premium to inſure —		To Inſurance Account.
When I pay Money for Loſs as an Inſurer	Inſurance Account Debtor	To Caſh.

CHAP. VI.

Of FACTORAGE ACCOUNTS.

56. WHEN I *receive Goods* ſent me from abroad, to ſell for Account of the Perſon who ſent them, then I am his Factor. And as I have the Goods in my own Hands, they are my Security for whatever Charges I may be at on their Account; and therefore I may erect an Account, entitled, *Goods conſigned me by* A. B. *(my Factor) for his Account:* Which I may make *Debtor* for all Diſburſements that I lay out on their Account, and for my Commiſſion after the Goods are diſpoſed of, for my Trouble in ſelling them. — And *Creditor* when I ſell any of them, for the Value ſold. But ſince when this Account muſt be credited, ſome other Account muſt be Debtor in its Stead by the Nature of double Entry, it is evident, that *Caſh*, if I ſold the Goods for ready Money, but the *Perſon* to whom I ſold them if on Truſt, muſt become *Debtor* to this Account of Goods. Hence it appears that I have no Occaſion for an Account of *Time* (though ſome Authors uſe one) to ſhew what Debts of others to my Factor I have the Care of; becauſe I have made ſuch Perſons Debtor to his Account of Goods, and conſequently have made that Account Creditor by

them. Hence his Account of Goods will ſhew on the Creditor Side what Debts of others to him I have the Care of, and conſequently an Account of *Time* would be ſuperfluous. — When all the Goods are diſpoſed of, this Account ought to be balanced to ſhew the *neat Proceeds.* In Order to which, it will be proper to have an Account entitled, *A. B. (my Factor) his Account.* Which I make *Creditor* for the *neat Proceeds* of the abovementioned Account.

57. Hence theſe Rules. When I *receive Goods* to be ſold for Account of my Correſpondent *A. B.*

When I pay Charges on their Account	The Goods Debtor	To Caſh.
If I am to pay any Charges on them		To the Perſon to whom due.
When the Goods are all ſold, make		To Profit & Loſs for my Commiſſion.
To balance Account of his Goods, make for the *neat Proceeds* — —		To *A. B.* his Account.

C H A P. VII.

Of COMPANY ACCOUNTS.

58. HERE, as I have hitherto done, I ſhall only attempt to explain the general Principles, and a few particular Caſes; becauſe, to deſcend to all the particular Caſes and Suppoſitions that might be made concerning Companies, would be endleſs, if not uſeleſs. And I think, if the general Principles are rightly underſtood, there cannot be much Difficulty in applying them to particular Caſes, as Occaſion requires.

59. If I enter into Partnerſhip with one or more Perſons, to continue for a Time agreed on between us, and am jointly concerned in every Branch of the

Trade carried on by each of them, it will be proper to have the Tranſactions of the Company kept on ſeparate Books ſet apart for that Purpoſe: Becauſe, thoſe Books ought to be open to the Inſpection of every Partner; whereas, no Perſon would chuſe to let his private Books be at the Command of others.

As a Company who carry on any Trade are conſidered as one Perſon, the Method of making the Debtors and Creditors will be the ſame as that uſed by a Merchant who carries on Trade on his own Bottom and Stock, which has been already ſhewn. Only it may be hinted, that the Profit or Loſs which does ariſe on the Stock of the whole Company muſt be divided between us, in ſuch Proportion as was before agreed on.

60. But if I enter into a Partnerſhip for a particular Voyage, or for the Purchaſe of a Quantity of Goods to be ſold for Account of Company, it may not be thought worth the Labour of keeping ſeparate Books. And in ſuch Caſe, one of my Partners may have the chief Management of that Affair, being the Huſband, or chief Agent and Accountant. Or I ~~may myſelf have the principal Direction of that~~ Affair.

61. Firſt then, if one of my *Partners* is the *Huſband*, or *chief Agent*, he is obligated to keep a juſt Account of all Tranſactions; and therefore, when we ſhip Goods for a Voyage, *&c.* it will be ſufficient for my Purpoſe, to make the *Voyage*, or Account of the *Goods in Company*, *Debtor* for the Goods delivered into the Company for the intended Trade or Voyage, for my Part which I have therein.

62. Hence theſe practical Rules. When I deliver Goods into Company:

If they were my own	TheGoods in Company, or The Voyage (if ſhipped) make Dr.	To the Goods:
If juſt bought for ready Money —		To Caſh.
If bought on Truſt *&c.*		To the Seller. *&c.*

63. But when I *am* to have the *principal Direction* of the Company Affairs, being the *Huſband*, or *principal Agent*, I am obliged to keep a diſtinct Account of the Tranſactions relating to the Company.

When Goods are brought into Company, it is manifeſt that the Accounts from whence they come muſt be credited; and therefore, by the Nature of double Entry, ſome Accounts muſt be made Debtor to them. Hence, for Example Sake, if I *purchaſe Goods for Account of Company*, for a particular Voyage, the Account of the *Voyage*, or *Merchandize* bought, muſt be made *Debtor* for the whole Amount of the *Outſet* of the Voyage, or whole Purchaſe of the ſaid Merchandize. But if, at the Time of purchaſing the ſaid Merchandize, the Market to which the Goods are to be ſent is not reſolved on, it would be proper, for the preſent, to make the Account of *Goods*, ſo bought in Company, *Debtor* to the ſeveral Accounts from whence they came; which Account (of Goods) when the ſaid Goods are ſhipped off, muſt be credited; the Account of the *Voyage*, or *Merchandize ſent*, becoming *Debtor* in its Stead.

64. From the Method of Reaſoning in the laſt Article, theſe practical Rules naturally ariſe. When *I am Accountant* and *Manager*. ——

Firſt, the Goods brought into Company being for a particular Voyage agreed on:

If I bought the Goods for the Company, and paid for them — — —	Voyage Debtor	To Caſh.
If on Truſt — —		To the Seller.
And then as Partner has not paid his Part —	Partner Debtor	To Voyage.
But if each Partner paid his Part	Voyage Debtor	To Caſh for my Part. To each Partner for his Part.
But if I put in Goods of my own inſtead of Money —	Voyage Debtor	To the Account of Goods I put into Company.

If at the Time of the Purchaſe of the Goods for Company, the Market is not agreed on; in the above practical Rules, for *Voyage*, read *Goods bought in Company*. And then, when the Goods are ſhipped off, make *Voyage Debtor to Goods bought in Company.*

When I receive the neat Returns for the Company, I muſt ſee what each Perſon's Part is of the Gains; and then, to cloſe the Account relating to the Company, I muſt make

Voyage, or Merchandiſe ſold for Company	Debtor	To Profit & Loſs for my Share of Gain. To each Partner for his Gain.

But if, inſtead of Gain, there ſhould happen a Loſs, then

Profit and Loſs for my Loſs Each Partner for his Loſs	Debtor	To the Voyage, or Merchandize ſold.

65. *Corollary.* Hence it appears, that there is no Occaſion for ſuch a Number of particular Accounts as ſome Authors make Uſe of, and conſequently Company Accounts are not ſo difficult as is generally imagined: For the ſuperfluous Accounts, *&c.* of ſome Authors ſerve only to perplex, and not inſtruct the Learner. ——— A greater Variety of particular Caſes might have been added, but if the Learner underſtands the Reaſon of thoſe here given, he may be able of himſelf to form others as Occaſion may require.

THE

WASTE-BOOK, Number II.

Of me, A. B. Merchant in *Bideford.*

From the 1ſt of *January*, 1758.

Bideford, January 1, 1758.

The INVENTORY *of the whole Eſtate, both real and perſonal, of me* A. B. *collected from the Balance of the preceding* LEGER Number I. viz.

		l.	s.	d.
//	I have in Caſh — — — — —	1460	12	0
//	In Sugar, 9 hhds. at 6*l.* per hhd. — —	54	0	0
//	Raiſins, 10 Barrels, at 1*l.* 10*s.* per Barrel	15	0	0
//	Broad Cloth, 15 Pieces, *viz.*			
	Numb. 2. 5 Pieces, each 16 Yds. at 12*s.*	48	0	0
	6. 10 Pieces, at 20*l.* per Piece —	200	0	0
//	Linen, 1 Piece, containing 20 Yds. at 2*s.* 6*d.*	2	10	0
//	Moveables and Houſehold Furniture, as per particular Inventory thereof —	150	0	0
//	Paper, 14 Reams, at 12*s.* per Ream — —	8	8	0
//	Wine, 4 Barrels, at 6*l.* — — — —	24	0	0
//	Serges, 2 Pieces, at 4*l.* — — — —	8	0	0
//	Holland, 10 Pieces, at 6*l.* — — —	60	0	0
//	Tobacco, 2 hhds. at 12*l.* — — —	24	0	0
	The following Perſons owe me as under, *viz.*			
//	*John Williams*, by a Bill on him, payable the 8th of this Month for — — —	36	0	0
	Carried over	2090	10	0

	l. s. d.	l.	s.	d.
Brought over — — — — — —	2090 : 10 : 0			
″ *William James* owes me as under.				
Viz.				
By a Bond dated the 1ſt of *June* laſt, and this Day; the Principal is — — —	100 : 0 : 0			
N. B. I am to have Intereſt at 5 *l.* *per Cent. per Annum.*				
He owes me payable the 22d of *December* next, according to Agreement, made when I took his Son Apprentice — —	57 : 0 : 0			
		2247	10	—
I owe as under.				
″ To ſaid *William James* on Demand — — — — — —		170	—	—
January 3, 1758.				
″ *William James* has offered me to reckon his Debt of 57 *l.* which will not be payable till next *December*, as now due, in Order to balance Accounts, if I will give him the Intereſt which is due to me on his Bond. Which I have agreed to, and I have accordingly paid him the Balance of his Account, *viz* — — — — — —		13	—	—
9th				
″ *John Williams* having failed, I have compounded his Debt for 9 *l.* which I have received — — — — — —		9	—	—

January 10th, 1758.		l.	s.	d.
My Father J. B. in Order to encourage me to merchandiſe, has given me the Ship *Bideford-Merchant*, *Abraham Nauticus*, Maſter, now on the Sea homeward from *Virginia*, with all her Cargo of Tobacco; the Ship and Materials are valued at — — — — —	950 *l.*			
The Cargo of Tobacco — — — — —	1500			
		2450	—	—
30th				
Bought of *James Gregory* and *William Zeno*, ¼ of the Ship *True-love*, for which I paid them — — — —		200	—	—
February 5th				
My Uncle *William Bond* being dead, has left me a Houſe in *High-ſtreet*, now in the Poſſeſſion of *J. Rentwell*, valued in — — — — — — — —	500 *l.*			
The Ship *Molly*, now at Key, with all her Materials, valued in — — — — — —	680			
And in Caſh — — — — — — — — —	510			
All of which I have taken Poſſeſſion of		1690	—	—
Out of which I am to pay *William Tindel* — — —		80	—	—
20th				
The Ship *Bideford-Merchant*, *Abraham Nauticus*, Maſter, is arrived, and has brought Home 300 hhds. of Tobacco, Weight 2700 Cwt. which I value as before, at — — — — — — — — — —	1500 *l.*			
I eſteem the Freight of the Ship — — —	100			
I have paid Duty, Portage, Keyage, Sailor's Wages, &c. — — — — — — —	220			
		1820	—	—

March 6th, 1758.

Number			l.	s.	d.
	I have paid *William Tindel* the Debt of 80*l.* which I owed him, as under, *viz.*				
	With Sugar, 9 hhds. at 7*l.* per hhd. — —	63*l.*			
	In Linen, 1 Piece — — — — — —	3			
	In Serges, 2 Pieces, at 5*l.* — — — —	10			
	In Caſh — — — — — — — —	4			
			80	—	—
	25th				
	The Maſter of the Ship *Bideford Merchant* has brought in his Bill of Repairs; which I have paid, *viz.* — —		40	—	—
	April 10th				
	The *Bideford Merchant*, *Abraham Nauticus*, Maſter, is ſailed for *Bourdeaux*, with 300 hhds. of Tobacco, (lately brought Home in her from *Virginia*,) which I have conſigned to *Pierre Touzin*, to ſell for my Account.				
	The 300 hhds. of Tobacco is — — —	1720*l.*			
	Charges paid for Portage, *&c.* at Shipping —	30			
	Paid *John Anderſon* for inſuring the Ship and Tobacco — — — — — — — —	37			
			1787	—	—
	The Debenture, payable in 30 Days by the Commiſſioners of the Cuſtoms, is — — — — — —		100	—	—
	28th				
	Shipped on Board the *Jane* Gally, *John Sealove*, Maſter, for Account of *John Neſbit*, at *Cadiz*, the following Goods. Marked and Number as in the Margin, *viz.*				
1	10 Pieces of my own Broad Cloth, Numb. 6, at 24*l* per Piece — — — — — —	*l* s. 240 : 0			
2	30 Pieces of *Devonſhire* Kirſeys, which I now bought, and paid ready Money for, at 1*l.* 15*s.* per Piece — — — — — —	52 : 10			
	Paid Charges on the ſame — — — —	7 : 10			
	My Commiſſion at 2½ *per Cent.* — — —	7 : 10			
			307	10	—

May 9th, 1758.	*l.*	*s.*	*d.*
I have received the Debenture on the 300 hhds. of Tobacco that were lately ſent to *Bourdeaux*	100	—	—
30th			
Received Advice from *Pierre Touzin*, my Factor at *Bourdeaux*, that he has received and ſold my 300 hhds. of Tobacco: The neat Proceed, as per his Bill of Sales, (all Charges being deducted,) is 10500 Crowns, Exchange at 4*s.* per Crown	2100	—	—
30th			
Drew a Bill on *John Neſbit* at *Cadiz*, for 1537½ Pieces of Eight, at 4*s.* per Piece; payable to *John Traveller* for Value here received, *viz.*	307	10	—
June 1ſt			
The Maſter of the Ship *Truelove* has paid me my neat Share of the Freight	50	—	—
2d			
Received Advice from *Pierre Touzin*, my Factor at *Bourdeaux*, that he has ſent me per my Order, in my Ship *Bideford Merchant*, Salt; which, with Charges, &*c.* coſt 3375 Crowns, Exchange at 4*s.* per Crown	675	—	—

June 6th, 1758.		*l.*	*s.*	*d.*
My Ship *Bideford Merchant* is arrived with my Salt from *Bourdeaux*: The Charges, Wages, *&c.* which I have paid, amount to — — — — — —	30*l.*			
The Freight of the Veſſel I eſteem at — —	50	80	—	—
I have ſold my Salt on Board the Ship to Mr. *John Emerſon* (he being to pay all Charges in diſcharging) for —		830	—	—
10th				
Bought of *William Williams* 2000 Buſhels of Wheat, at 4*s.* per Buſhel; for which I have given a Bill drawn by me on *John Emerſon*; the Wheat I have ſhipped on Board my Ship *Molly*, and conſigned to *David Cornfactor* of *London*.				
The Corn 2000 Buſhels at 4*s.* is — — —	400*l.*			
Charges paid on the ſame — — — — — —	20			
John Emerſon has inſured 500*l.* on this Adventure, for which I am to give him Credit in his Account — — — — — — — — —	30	450	—	—

June 26th, 1758.	*l.*	*s.*	*d.*
Abraham Gonſalez, of *Amſterdam*, has ſent me, by the Ship *Sealove*, *Jacob Hadley*, Maſter, to ſell for his Account, as under : *Viz.* 60 lb. Weight of Bohea Tea. 20 Pieces of Holland, each 100 Ells Flemiſh. Cuſtom and Charges paid here by me on the above Goods amount to — — — — — — — — — —	50	—	—
July 10th			
David Cornfactor, of *London*, writes me that he has received and ſold my Corn. The neat Proceed of which, after his Commiſſion and Charges are deducted, is 570*l.* And that he has ſent me in Return, in the ſame Ship *Molly*, as follows, *viz.* 17 hhds. of Wine at 30*l.* per hhd. — — — 510*l.* Charges paid by him and his Commiſſion — — 60	570	—	—
11th			
Sold for ready Money *Abraham Gonſalez*'s (of *Amſterdam*) 60 lb. of Bohea Tea, at 4*s.* per lb. — — — —	12	—	—
20th			
The Ship *Molly* is arrived with my 17 hhds. of Wine ; which coſt me — — — — — — — 570*l.* I have now paid Charges, &c. here at Landing, and Seamens Wages — — — — — 25 The Freight of my Veſſel I value in — — — 25	620	—	—

July 21ſt, 1758.	*l.*	*s.*	*d.*
By Order of *Abraham Gonſalez*, I have ſhipped on Board the Sloop *Briſtol Trader*, *John Hopkins*, Maſter, his 26 Pieces of Holland, conſigned to *Abraham Andrew* my Factor at *Briſtol*, to ſell by my Order. — I have paid Charges at Shipping, *&c.* — — — — — —	3	—	—
31ſt			
Received Advice from *Abraham Andrew*, my Factor at *Briſtol*, that he has received and ſold for ready Money *Abraham Gonſalez*'s 20 Pieces of Holland; the neat Proceed, after his Commiſſion and Charges are deducted, is — — — — — — — — —	210	—	—
Auguſt 1ſt			
Received from *Abraham Andrew* (my Factor at *Briſtol*) a Bill drawn on *Peter Jackſon* payable to me in 10 Days — — — — — — — — —	210	—	—
N. B. Jackſon has accepted the Bill.			
14th			
Peter Jackſon has paid me *Abraham Andrews*'s Bill on him — — — — — — — — — —	210	—	—
After my Commiſſion of 28 *l.* is deducted on *Abraham Gonſalez*'s Goods, the neat Proceed is 141 *l.* — The neat Proceed and Commiſſion is — — — —	169	—	—
15th			
Abraham Gonſalez, of *Amſterdam*, has drawn a Bill on me for 100 *l.* Sterling, dated the 15th of *June* laſt, payable at double Uſance to *Peter Zeno*; which I have accepted — — — — — — — — — —	100	—	—

Paid *Abraham Gonſalez*'s Bill to *Peter Zeno* — — — 100 — —

20th

Abraham Gonſalez has ordered me to ſend to Don *Bleau*, his Factor at *Legborn*, the Balance of his Account in *Barnſtaple* Stuffs: In Compliance to which Order, I have bought of *Anthony Fox*, and ſhipped on Board the Ship *Sailwell*, *John Nimble*, Maſter, 20 Pieces of *Barnſtaple* Stuffs, at 2*l*. per Piece — — — 40*l*.
Paid Charges, *&c*. at Shipping — — — 1
— — — — — — — 41

N. B. Being deſirous of obliging *Abraham Gonſalez*, (for a particular Reaſon,) I have not charged him with any Commiſſion for purchaſing theſe Goods, as I ſhould otherwiſe have done by the Cuſtom of Merchants.

21ſt

John Tradewell having 200 Pieces of *Barnſtaple* Stuffs, which he is going to employ in a domeſtic Trade; I have agreed with him for a Half Share in the ſaid Trade, and am to allow him 2½ *per Cent*. for his Commiſſion. — I have accordingly paid him for my Half Share — — — — — — — — — 100 — —

30th

John Tradewell has ſold the 200 Pieces of *Barnſtaple* Stuffs, and brought in his Account, by which my Half Share, his Commiſſion being deducted, appears to be 150*l*. which he has paid me — — — — — 150 — —

Auguſt 31ſt, 1758.	*l.*	*s.*	*d.*
John Dawſon, *Peter Zenephon*, and Self, have bought in Company 20 Pieces of Linen at 2*l.* per Piece, (and each paid his Part) — — — — — — — —	40	—	—
Note, The Linen is in my Hands, and I am to have the chief Direction of the Trade.			
September 2d			
Sold to *Francis King*, for Account of *John Dawſon*, *Peter Zenephon*, and Self, the 20 Pieces of Linen, at 2*l.* 10*s.* per Piece. I have received in Part — — — — — 30*l.* Remains due on Demand — — — — — — 20	50	—	—
3d			
Francis King has paid me in full for the Linen in Company — — — — — — — — — — —	20	—	—
By inſpecting the Account of Linen in Company between *John Dawſon*, *Peter Zenephon*, and Self, I find the Company has gained 10*l* Whereof *John Dawſon*'s Third is — — 3 6 8 *Peter Zenephon*'s Third — — — — 3 6 8 My Third — — — — — — — 3 6 8	10	—	—
4th			
In Company with *John Dawſon* and *Peter Zenephon*, each a Third, I have bought of *Thomas Ormond* 3 hhds. of Brandy, at 12*l.* per hhd. to pay him in 4 Days, the Goods are in my Hands, and under my Diſpoſal —	36	—	—

September 8, 1758.	*l*	*s.*	*d.*
Paid *Thomas Ormond* in full for Brandy bought in Company with *John Dawſon* and *Peter Zenephon* — — —	36	—	—
9th			
Peter Zenephon and *John Dawſon* have each paid me 12 *l.* being what I paid *Thomas Ormond* for their Account —	24	—	—
10th			
Sold for ready Money 1 hhd. of Company's Brandy —	18	—	—
11th			
Peter Zenephon and Self have (with the Conſent of *John Dawſon)* parted the remaining 2 hhds. of Brandy betwixt us, he having taken one to himſelf at prime Coſt — — — — — — — — 12 *l.*			
And I the other — — — — — — — — 12			
	24	—	—
By inſpecting the Account of Brandy in Company, I find the Company has gained 6 *l.*			
Of which *John Dawſon*'s Third is — — — 2 *l.*			
Peter Zenephon's Third — — — — — — — 2			
My Third — — — — — — — — — 2			
	6	—	

September 14th, 1758.

		l.	s.	d.
John Dawſon, *Peter Zenephon*, and Self, have ſhipped on Board the *Cleveland*, *John Paſſoon*, Maſter, as follows, *viz.*				
John Dawſon put on Board 300 Buſhels of Wheat, at 5*s.* per Buſhel — — — — —	75 *l.*			
Peter Zenephon 80 Barrels of Spilchers, at 1 *l.* per Barrel — — — — — — — — —	80			
I have put on Board 10 Pieces of Holland, at 7 *l.* per Piece — — — — — — —	70			
I have paid Charges on ſhipping the above Goods	6			
	———	231	—	—

The above Goods being ſhipped in Company, we have agreed to make our Shares equal with Money; the Goods are conſigned to Don *Quixot* at *Cadiz*, to ſell for our Account.

15th

		l.	s.	d.
John Dawſon has paid me — — — — — —	2 *l.*			
And I have paid *Peter Zenephon* — — — — —	3			
	———	5	—	—

In Order to make each Perſon's Share equal in the above-mentioned Adventure to *Cadiz*.

November 20th

	l.	s.	d.
Received Advice from Don *Quixot*, our Factor at *Cadiz*, that he has received and ſold the Goods conſigned to him per *John Dawſon*, *Peter Zenephon*, and Self, each a Third. The neat Proceed of which, per his Bill of Sales, are 1334 Pieces of Eight, Exchange at 4*s.* 6*d.* per Piece — — — — — — — — — —	300	—	—

November 21ſt, 1758.

Number		l.	s.	d.
	Shipped on Board the *John* Galley, *Abraham Quick*, Maſter, conſigned to *John Snuffwell* at *Falmouth*, for our joint Account; (*viz. John Snuffwell* Two-thirds, and Self One-third.)			
	2 hhds. of Tobacco, Value — — — — — 24 *l.*			
	Paid Charges at Shipping — — — — 1			
	25			
	My Commiſſion on the 25 *l.* at ½ *per Cent.* is 0 : 2 : 6	25	2	6
	There is due from *John Snuffwell* for his Two-thirds of the above Tobacco — — — — — — —	16	15	—
	30th			
	This Day have received from Don *Quixot* of *Cadiz*, a Bill on *David Gregor*, dated *November* 20th, due *December* 30th, being the neat Proceed of the Adventure to *Cadiz*, in Company with *John Dawſon* and *Peter Zenephon*, *viz.* — — — — — — — —	300	—	—
	December 1ſt			
	Bought of *William Jacobs*, 3 Bales of Woollens, *viz.*			
1	60 Pieces of Stuffs, at 2 *l.* — — — — — 120 *l.*			
2	60 Pieces of Duroys, at 1 *l.* 10 *s.* — — — 90			
3	60 Pieces of Callimancoes, at 2 *l.* — — — 120			
3	Bales. 330			
	Which I paid for as under.			
	I gave him the Bill on *David Gregor* — — 300 *l.*			
	Paid him in Money — — — — — — — 30	330	—	—

December 1ſt, 1758.	*l.*	*s.*	*d.*
By inſpecting the Account of Voyage to *Cadiz* in Company with *John Dawſon* and *Peter Zenephon*, I find the Company has gained by that Adventure 69*l.*			
Whereof my Third is — — — — — 23*l*			
Peter Zenephon's — — — — — — — 23			
John Dawſon's — — — — — — — — 23			
	69	—	—
30th			
This Day received from *John Snuffwell* a Bill of Sale of the Tobacco ſent him for our joint Account; by which it appears that the neat Proceeds of my $\frac{1}{3}$ Share are 10*l.* 5*s.* — And with the Bill of Sale he has remitted me a Bill of Exchange for 27*l.* dated *December* 26th, payable at Sight, drawn on *John Paywell*; to balance Accounts — — — — — — — — —	27	—	—
31ſt			
By the Book of Houſe Expences there has been expended in Houſekeeping — — — — — — — —	70	—	—
Paid my Clerk his Salary, &c. — — — — —	60	—	—

End of the WASTE-BOOK.

THE JOURNAL-BOOK, Number II.

Of me, A. B. Merchant in *Bideford.*

From the 1ſt of *January*, 1758.

Bideford, January 1, 1758.

					l.	s.	d.
.1.	Sundry Accounts are Debtors to *Stock* 2247 *l.* 10 *s.* *viz.*						
		l.	s.	d.			
.3.	*Caſh* — — — — — — —	1460	12	0			
.4.	*Sugar*, 9 hhds. at 6 *l.* per hhd. — —	54	0	0			
.4.	*Raiſins*, 10 Barrels, at 1 *l.* 10 *s.* — —	15	0	0			
.4.	*Broad Cloth*, 15 Pieces, *viz.*						
	Numb. 2. 5 Pieces — — — —	48	0	0			
	6. 10 Pieces — — — —	200	0	0			
.4.	*Linen*, 1 Piece — — — — — —	2	10	0			
.4	*Moveables* and Houſehold Furniture —	150	0	0			
.4.	*Paper*, 14 Reams, at 12 *s* per Ream —	8	8	0			
.5.	*Wine*, for 4 Barrels, at 6 *l.* — — —	24	0	0			
.5.	*Serges*, for 2 Pieces, at 4 *l* — — — —	8	0	0			
.5.	*Holland*, 10 Pieces, at 6 *l.* — — —	60	0	0			
.5.	*Tobacco*, 2 hhds. at 12 *l.* — — —	24	0	0			
.5.	*John Williams*, by a Bill, due the 8th of this Month — — — — — —	36	0	0			
.6.	*William James*, by a Bond dated the 1ſt of *June* laſt, and payable this Day; the Principal is — — — — —	100	0	0			
.6.	*William James* alſo owes me, payable the 22d of *December* next, by Agreement, made when I took his Son Apprentice —	57	0	0			
					2247	10	—

	January 1ſt, 1758.	*l.*	*s.*	*d.*
1. 6.	*Stock* Debtor to ſaid *William James* on Demand — —	170	—	—
	3d			
6. 3.	*William James* Debtor to *Caſh* 13 *l.* for ſo much paid him in Balance of Accounts — — — — — — *Note*, I gave him the the Intereſt of his Bond 2 *l.* 10 *s.* due to me, on Account of his allowing his Debt of 57 *l.* as now due, in Order to balance Accounts.	13	—	—
	9th			
5. 3. 2.	Sundry Accounts Debtors to *John Williams* 36 *l.* *viz.* Debtor { *Caſh*, received in Compoſition for his Debt 9 *l.* { *Profit and Loſs*. Abated him — — — 27	36	—	—
	10th			
2. 6. 6.	Sundry Accounts Debtors to *Profit and Loſs* 2450 *l.* *viz.* *Ship Bideford Merchant*, *Abraham Nauticus*, Maſter; given me by my Father *J. B.* — — —, 950 *l.* *Voyage from Virginia homeward in Ditto Ship*, for the Cargo conſiſting of 300 hhds. of Tobacco, at 5 *l.* per hhd. — — — — — — 1500	2450	—	—
	30th			
6. 3.	Ship *Truelove* Debtor to Caſh 200 *l.* for my ¼ Part, bought of *James Gregory* and *William Zeno* — —	200	—	—

	February 5th, 1758.		*l.*	*s.*	*d.*
·2.	Sundry Accounts Debtors to Profit and Loſs 1690*l.* *viz.*				
·7.	Debtor { Houſe in *High-ſtreet* left me by my Uncle *William Bond*, valued in —	500*l.*			
·7.	Ship *Molly* given me by the ſame —	680			
·3.	Caſh given me per Ditto — — —	510			
			1690	—	—
·2. ·7.	Alſo Profit and Loſs Debtor to *William Tindel* 80*l.* which I am to pay him per my Uncle's Will — — —		80	—	—
	20th				
·5.	Tobacco Debtor to Sundry Accounts 1820*l.* *viz.*				
,6.	To Voyage from *Virginia* in the Ship *Bideford Merchant*, for her Cargo of 300 hhds. of Tobacco, Weight 2700 lb. Valued in —	1500*l.*			
,6.	To Ship *Bideford Merchant*, for Freight — —	100			
,3.	To Caſh, paid for Duty, Portage, Wages, &c.	220			
			1820	—	—
	March 6th				
7	*William Tindel* Debtor to Sundry Accounts 80*l.* *viz.*				
4	To Sugar, 9 hhds. at 7*l.* per hhd. — — —	63*l.*			
4	To Linen, 1 Piece — — — — — —	3			
5	To Serges, 2 Pieces, at 5*l.* — — — —	10			
3	To Caſh — — — — — — — —	4			
	Paid him in all		80	—	—

	March 25th, 1758.	*l.*	*s.*	*d.*
6 3	Ship *Bideford Merchant* Debtor to Caſh 40*l.* for ſo much paid for Repairs — — — — — — — —	40	—	—
	April 10th			
4	Merchandiſe conſigned *Pierre Touzin* of *Bourdeaux*, for my Account, Debtor to ſundry Accounts 1787*l.* *viz.*			
5	To 300 hhds. of Tobacco — — — — 1720*l.*			
3	To Caſh, paid for Portage, Inſurance, &c. 67	1787	—	—
8 5	And Commiſſioners of the Cuſtoms Debtor to the ſaid Tobacco 100*l.* being the Debenture payable in 30 Days — — — — — — — — — —	100	—	—
	28th			
8	*John Neſbit* of *Cadiz*, Debtor to ſundry Accounts 307*l.* 10*s.* *viz.*			
4	To Broad Cloth, for 10 Pieces of Numb. 6, at 24*l.* per Piece — — — — — — 240*l.*			
3	To Caſh, paid for 30 Pieces of Kirſeys, and Charges, &c — — — — — — 60			
2	To Profit and Loſs for my Commiſſion — — 7 : 10	307	10	—
	Note, The above Goods were ſhipped on Board the *Jane* Galley, *John Sealove*, Maſter.			

	May 9th, 1758.	l.	s.	d.
3/8	Caſh Debtor to the Commiſſioners of the Cuſtoms 100*l.* — Being the Debenture of the Tobacco ſent to *Bourdeaux*, which I have received — — — — — —	100	—	—
	30th			
8/7	*Pierre Touzin*, of *Bourdeaux*, my Account Debtor to Merchandiſe conſigned to him for my Account, 10500 Crowns: Being the neat Proceeds of my Adventure of Tobacco ſent him, which, at 4*s.* per Crown, is	2100	—	—
	30th			
3/8	Caſh Debtor to *John Neſbit* of *Cadiz* 307*l.* 10*s.* For a Bill for 1537½ Pieces of Eight, at 4*s.* per Piece, which I have drawn on him for Value received —	307	10	—
	June 1ſt			
3/6	Caſh Debtor to Ship *Truelove* 50*l.* being my Share of the Freight; which I have received — — — —	50	—	—
	2d			
9/8	Merchandiſe ſhipped on Board the *Bideford Merchant*, from *Bourdeaux*, Debtor to *Pierre Touzin* my Account 675*l.* For a Cargo of Salt which coſt 3375 Crowns, Exchange at 4*s.* per Crown — — — — — — — —	675	—	—
	6th			
9	Merchandiſe ſhipped on Board the *Bideford Merchant*, from *Bourdeaux*, Debtor to ſundry Accounts, 80*l.* *viz.*			
3	To Caſh paid on Arrival of the Ship for Wages, &c. 30*l.*			
6	To Ship *Bideford Merchant* for Freight — — 50			
		80	—	—

June 6th, 1758.

			l.	s.	d.
9/9	*John Emerſon* Debtor to Merchandiſe ſhipped on Board the *Bideford Merchant* from *Bourdeaux*, 830 *l.* —— For the Cargo of Salt ſold him on Ship-board — —		830	—	—
	10th				
9	Merchandiſe (ſhipped on Board my Ship *Molly*) conſigned to *David Cornfactor* of *London* for my Account, Debtor to ſundry Accounts 450 *l.* *viz.*				
9	To *John Emerſon* for a Bill drawn on him for 400 *l.* and for the Premium on inſuring 500 *l.* on this Voyage; 30 *l.* in all — — — —	430 *l.*			
3	To Caſh, for Charges paid at ſhipping 2000 Buſhels of Corn — — — — — —	20			
			450	—	—
	26th				
10/3	Merchandiſe conſigned me by *Abraham Gonſalez*, of *Amſterdam*, for his Account Debtor to Caſh 50 *l.* —— Paid for Cuſtom and Charges of 60 lb. Weight of Bohea Tea, and 20 Pieces of Holland, each 100 Ells *Flemiſh*; brought in the Ship *Sealove*, *Jacob Hadley*, Maſter —		50	—	—
	July 10th				
10/9	Voyage from *London* in Ship *Molly* Debtor to Merchandiſe conſigned to *David Cornfactor* of *London* 570 *l.* —— Being the neat Proceed of my Adventure of Corn conſigned him, which he has ſent me as under, *viz.*				
	17 hhds. of Wine at 30 *l.* per hhd. — — —	510 *l.*			
	Charges paid by him and his Commiſſion — —	60			
			570	—	—

	July 11th, 1758.	l.	s.	d.
3 10	Caſh Debtor to Merchandiſe conſigned me by *Abraham Gonſalez* of *Amſterdam* for his Account 12 *l.* Received for 60 lb. of Bohea Tea at 4 *s.* per lb. — — — —	12	—	—
	20th			
5	Wine Debtor to ſundry Accounts 620 *l.* *viz.*			
10	To Voyage from *London* for 17 hhds. of Wine – 570 *l.*			
7	To Ship *Molly* for Freight — — — — — — 25			
3	To Caſh, for Charges paid — — — — — — 25			
		620	—	—
	21ſt			
10 3	Merchandiſe conſigned me by *Abraham Gonſalez* of *Amſterdam* for his Account Debtor to Caſh 3 *l.* For Charges paid at ſhipping 20 Pieces of Holland, conſigned to my Factor *Abraham Andrew* at *Briſtol* —	3	—	—
	31ſt			
10 10	*Abraham Andrew*, my Factor at *Briſtol*, Debtor to Merchandiſe conſigned me by *Abraham Gonſalez* for his Account 210 *l.* For the neat Proceed of *Abraham Gonſalez*'s 20 Pieces of Holland — — — — —	210	—	—
	Auguſt 1ſt			
11 10	Bills remitted me Debtor to *Abraham Andrew*, my Factor at *Briſtol*, 210 *l.* for one on *Peter Jackſon* payable in 10 Days — — — — — — — — —	210	—	—

	Auguſt 14th, 1758.	*l.*	*s.*	*d.*
3 11	Caſh Debtor to Bills remitted 210 *l.* Received of *Peter Jackſon* for *Abraham Andrew*'s Bill on him — — — — — — — — — — —	210	—	—
10	Merchandiſe conſigned me by *Abraham Gonſalez* Debtor to ſundry Accounts 169 *l.* *viz.*			
2	To Profit and Loſs for my Commiſſion — — 28 *l.*			
11	To *Abraham Gonſalez* his Account for neat Proceeds 141	169	—	—
	15th			
11 11	*Abraham Gonſalez* Debtor to Bills of Exchange drawn on me 100 *l.* For his Bill to *Peter Zeno*, dated the 15th of *June* laſt, payable at double Uſance, due the 18th of *Auguſt* — — — — — — — — — —	100	—	—
	18th			
11 3	Bills of Exchange drawn on me Debtor to Caſh 100 *l.* Paid to *Peter Zeno*, *Abraham Gonſalez*'s Bill, dated *June* the 15th — — — — — — — — —	100	—	—
	20th			
11	*Abraham Gonſalez* of *Amſterdam* his Account Debtor to ſundry Accounts 41 *l.* *viz.*			
12	To *Anthony Fox*, for 20 Pieces of *Barnſtaple* Stuffs, bought of him at 2 *l.* per Piece; which I have ſhipped on Board the Ship *Sailwell*, *John Nimble*, Maſter, conſigned to Don *Bleau* of *Leghorn*, by Order of *Abraham Gonſalez* — — — — 40 *l.*			
3	To Caſh, for Charges, &c. paid at Shipping — 1	41	—	—
	21ſt			
12 3	Goods in Company with *John Tradewell* Debtor to Caſh 100 *l.* For my half Share of 200 Pieces of *Barnſtaple* Stuffs; which he is to employ in Trade for our joint Account. I have paid him — — — — —	100	—	—

Auguſt 30th, 1758.

			l.	s.	d.
3 12	Caſh Debtor to Goods in Company with *John Tradewell* 150*l.* Being my half Share of neat Proceeds of ſaid Goods, which he has paid me — — — — — —		150	—	—
	31ſt				
12	Linen, in Company with *John Dawſon* and *Peter Zenephon*, Debtor to ſundry Accounts 40*l.* *viz.*				
13	To *John Dawſon* his Account, for his Third Share of 20 Pieces of Linen, at 2*l.* per Piece —	13*l.* 6*s.* 8*d.*			
13	To *Peter Zenephon* his Account, for his Third Share of Ditto Linen — — — —	13 : 6 : 8			
3	To Caſh for my Third of Ditto Linen —	13 : 6 : 8	40	—	—
	September 2d				
12	Sundry Accounts Debtor to Linen, in Company with *John Dawſon* and *Peter Zenephon*, 50*l.* *viz.*				
14	*Francis King*, in Part for 20 Pieces of Company's Linen, ſold him at 2*l.* 10*s.* per Piece — —	20 *l.*			
3	Caſh, for the other Part, which I received — —	30	50	—	—
	3d				
3 14	Caſh Debtor to *Francis King* 20*l.* Received of him in full for Company's Linen ſold him — — — — — —		20	—	—
12	Linen in Company Debtor to ſundry Accounts 10*l.* *viz.*				
13	To *John Dawſon* his Account for his $\frac{1}{3}$ gained	3 : 6 : 8			
13	To *Peter Zeuephon* his Account for his $\frac{1}{3}$ gained	3 : 6 : 8			
2	To Profit and Loſs for my $\frac{1}{3}$ of Gains — —	3 : 6 : 8	10	—	—

	September 4th, 1758.	l.	s.	d.
14 14	Brandy in Company with *John Dawſon* and *Peter Zenephon* (each a Third) Debtor to *Thomas Ormond* 36*l.* For 3 hhds. of Brandy at 12*l.* per hhd. — — —	36	—	—
	8th			
14 3	*Thomas Ormond* Debtor to Caſh 36*l.* Paid him in full for the Brandy bought in Company with *John Dawſon* and *Peter Zenephon* — — — — — — — —	36	—	—
	9th			
3	Caſh Debtor to ſundry Accounts 24*l.* *viz.*			
13	To *Peter Zenephon*, received of him for his Third Share of Brandy in Company — — — — 12*l.*			
13	To *John Dawſon*, received for Ditto Share — 12	24	—	—
	10th			
3 14	Caſh Debtor to Brandy in Company with *John Dawſon* and *Peter Zenephon* 18*l.* Received for 1 hhd. ſold this Day — — — — — — — — — —	18	—	—
	11th			
14	Sundry Accounts Debtor to Brandy, in Company with *John Dawſon* and *Peter Zenephon*, 24*l.* *viz.*			
13	Deb- { *Peter Zenephou*, for 1 hhd. taken to himſelf 12*l.*			
15	tors { Brandy, for the other hhd. taken to myſelf 12	24	—	—
14	Brandy, in Company with *John Dawſon* and *Peter Zenephon*, Debtor to ſundry Accounts 6*l.* *viz.*			
13	To *John Dawſon* for his Third of Gains — — 2*l.*			
13	To *Peter Zenephon* for his Third of Gains — — 2			
2	To Profit and Loſs for my Third of Gains — — 2	6	—	—

	September 14th, 1758.	*l.*	*s.*	*d.*
15	Voyage to *Cadiz*, in Company with *John Dawſon* and *Peter Zenephon*, Debtor to ſundry Accompts 231 *l.* *viz.*			
13	To *John Dawſon*, for 300 Buſhels of Wheat, at 5 *s.* per Buſhel — — — — — — — 75 *l.*			
13	To *Peter Zenephon*, for 80 Barrels of Spilchers, at 1 *l.* per Barrel — — — — — — 80			
5	To Holland, for 10 Pieces at 7 *l.* — — — 70			
3	To Caſh, for Charges paid by me — — — 6	231	—	—
	Note, The above Goods are ſhipped on Board the *Cleveland*, *John Paſſoon*, Maſter; conſigned to Don *Quixot* at *Cadiz*.			
	15th			
3 13	Caſh Debtor to *John Dawſon* 2 *l.* Received of him, to make our Shares in the above Adventure equal —	2	—	—
13 3	*Peter Zenephon* Debtor to Caſh 3 *l.* Paid him to equal Shares in above Adventure — — — — —	3	—	—
	November 20th			
15 15	Don *Quixot* (of *Cadiz*) Debtor to Voyage to *Cadiz*, in Company with *John Dawſon* and *Peter Zenephon*, 300 *l.* The neat Proceed of Ditto Adventure being by the Bill of Sales received this Day, 1334 Pieces of Eight, at 4 *s.* 6 *d.* per Piece — — — — — — —	300	—	—

	November 21ſt, 1758.		l.	s.	d.
16	Merchandiſe conſigned to *John Snuffwell* at *Falmouth*, (for our joint Account, he Two Thirds and Self One Third) Debtor to ſundry Accounts 25*l.* 2*s.* 6*d.* for 2 hhds. of Tobacco ſhipped on Board the *John* Galley, *Abraham Quick*, Maſter, *viz.*				
5	To Tobacco, for 2 hhds. Value	24 *l.*			
1	To Caſh, for Charges paid at Shipping	1			
2	To Profit and Loſs for my Commiſſion on 25 *l.* at ½ *per Cent.*	0 : 2 : 6	25	2	6
16 16	*John Snuffwell* his Account Debtor to Merchandiſe conſigned to ſaid *John Snuffwell* for our joint Account, 16*l.* 15*s.* for his Two Thirds of ſaid Merchandiſe		16	15	—
	30th				
11 15	Bills of Exchange remitted me Debtor to Don *Quixot* of *Cadiz* 300*l.* for one on *David Gregory*, dated *November* 20th, payable *December* 30th		300	—	—
	December 1ſt				
16	Woollens Debtor to ſundry Accounts 330*l.* for 3 Bales bought of *William Jacobs*, *viz.*				
11	To Bills of Exchange remitted me, for the Bill on *David Gregor*, delivered *William Jacobs* in Payment for the 3 Bales	300 *l.*			
1	To Caſh paid him in Money	30	330	—	—
	Note, The 3 Bales are marked, &c. as under.				
	Numb. 1. 60 Pieces of Stuffs at 2 *l.*	120 *l.*			
	2. 60 Pieces of Duroys at 1 *l.* 10 *s.*	90			
	3. 60 Pieces of Callimancoes at 2 *l.*	120	330	—	—

December 1, 1758.

			l.	s.	d.
15	Voyage to *Cadiz*, in Company with *John Dawſon* and *Peter Zenephon*, Debtor to ſundry Accounts 69 *l.* *viz.*				
13	To *Peter Zenephon*, his Account for his $\frac{1}{3}$ of Gains	23 *l.*			
13	To *John Dawſon*, his Account for Ditto — —	23			
2	To Profit and Loſs for my $\frac{1}{3}$ of Gains — —	23	69	—	—
	30th				
11	Bills of Exchange remitted me Debtor to ſundry Accounts 27 *l.* for one on *John Paywell*, dated *December* 26th, payable at Sight, *viz.*				
16	To Merchandiſe conſigned *John Snuffwell* for our joint Account; for my Third of neat Proceeds, remitted by Bill on *John Paywell*	10 *l.* 5 *s.*			
16	To *John Snuffwell* his Account, remitted me by Bill on *John Paywell*, for his Two-thirds of Merchandiſe conſigned him — — —	16 : 15	27	—	—
	31ſt				
2 1	Profit and Loſs Debtor to Caſh 70 *l.* Paid at ſundry Times in Houſekeeping, as per particular Book does appear		70	—	—
2 1	Profit and Loſs Debtor to Caſh 60 *l.* Paid my Clerk for his Salary, &c. — — — — — — —		60	—	—

End of the JOURNAL.

Now follows the LEGER.

A.

Abraham Andrew — — — 10

B.

Broad Cloth — — — — 4
Bills remitted me — — 11
Bills drawn on me — — 11
Brandy in Company — — 14
Brandy — — — — 15
Balance — — — — 17

C.

Caſh — — — — — — 3
Commiſſioners of Cuſtoms — 8

D.

John Dawſon his Account — 13

E.

John Emerſon — — — — — 9

F.

Anthony Fox — — — 12

G.

Abraham Gonſalez his Account 11
Goods in Company with *J. Tradewell* — — — — 12

H.

Holland — — — — 5
Houſe in *Highſtreet* — — 7

J.

William James — — — 6

K.

Francis King — — — 14

L.

Linen — — — — — 4
Linen in Company with *J. Dawſon* and *P. Zenephon* - 12

M.

Moveables — — — — 4
Merchandiſe conſigned *Pierre Touzin* — — — — 7
Merchandiſe ſhipt from *Bourdeaux* in *Bideford Merchant* - 9
Merchandiſe conſigned *David Cornfactor* — — — 9
Merchandiſe conſigned me by *A. Gonſalez* — — — 10
Merchandiſe conſigned to *J. Snuffwell* — — — — 16

N.		O.	
John Neſbit	8	*Thomas Ormond*	14

P.		Q.	
Profit and Loſs	2	*Don Quixot*	15
Paper	4		

R.		S.	
Raiſins	4	Stock	1
		Sugar	4
		Serges	5
		Ship *Bideford Merchant*	6
		Ship *Truelove*	6
		Ship *Molly*	7
		John Snuffwell	16

T.		V.	
Tobacco	5	Voyage from *Virginia* in Ship *Bideford Merchant*	6
William Tindel	7	Voyage from *London* in Ship *Molly*	10
Pierre Touzin my Account	8	Voyage to *Cadiz* in Company	15

W.		X.	
Wine	5		
John Williams	5		
Woollens	16		

Y.		Z.	
		Peter Zenephon his Account	15

Debtor.

1757.		*Stock.*	R.	*l.*	*s.*	*d.*
January	1	To *William James* on Demand — — —	2	170	—	—
December	31	To *Balance*, my neat Stock, at this Time, appears to be — — — — — —	17	6850	16	8
				7020	16	8

1758.		*Stock.*	*l.*	*s.*	*d.*	F.	*l.*	*s.*	*d.*
January	1	By *Caſh* — — — — —	1460	12	0				
		By *Sugar*, 9 hhds. at 6*l.* —	54	0	0				
		By *Raiſins*, 10 Barrels at 1*l.* 10*s.*	15	0	0				
		By *Broad Cloth*, 15 Pieces —	248	0	0				
		By *Linen*, 1 Piece — — —	2	10	0				
		By *Moveables and Houſehold Furniture*	150	0	0				
		By *Paper*, 14 Reams at 12*s.* —	8	8	0				
		By *Wine*, 4 Barrels at 6*l.* —	24	0	0				
		By *Serges*, 2 Pieces at 4*l.* —	8	0	0				
		By *Holland*, 10 Pieces at 6*l.* —	60	0	0				
		By *Tobacco*, 2 hhds. at 12*l.* —	24	0	0				
		By *John Williams*, by a Bill on him	36	0	0				
		By *William James*, by a Bond	100	0	0				
		By *William James*, by an Agreement	57	0	0				
						1	2247	10	—
December	31	By *Profit and Loſs*, my neat Gains from *January* 1ſt to this Day, amount to — — —				2	4773	6	8
							7020	16	8

Debtor.

1758.		*Profit and Loſs.*	R.	*l.*	*s.*	*d.*
January	9	To *John Williams*, abated him — — —	2	27	—	—
February	5	To *William Tindel*, I am to pay him according to my Uncle *William Bond*'s Will — —	3	80	—	—
December	31	To *Caſh*, being Expences of Houſekeeping	1	70	—	—
		To *Caſh*, paid my Clerk his Salary, &c. —	1	60	—	—
		To *Stock*, my neat Gains — — — —	1	4773	6	8
				5010	6	8

1758.		*Profit and Loſs.*	R	*l.*	*s.*	*d.*
January	10	By *Ship Bideford Merchant*, given me	2	950	—	—
		By Cargo of ditto Ship, given me	2	1500	—	—
February	5	By ſundry Accounts (ſee Journal) given me by my Uncle *William Bond*'s Will	3	1690	—	—
April	28	By *John Neſbit*, for my Commiſſion	4	7	10	—
Auguſt	14	By *Merchandiſe conſigned me by Abraham Gonſalez* for my Commiſſion	8	28	—	—
September	3	By *Linen in Company* for my Third Part of Gains	9	3	6	8
	11	By *Brandy in Company* for my Third of Gains	10	2	—	—
November	21	By *Merchandiſe conſigned John Snuffwell our Account* for my Commiſſion	12	—	2	6
December	1	By *Voyage to Cadiz in Company* for my Third of Gains	13	23	—	—
	31	By *Sugar*, gained	4	9	—	—
		By *Broad Cloth*, gained	4	40	—	—
		By *Linen* gained	4	—	10	—
		By *Serges*, gained	5	2	—	—
		By *Holland*, gained	5	10	—	—
		By *Ship Bideford Merchant*, gained	6	110	—	—
		By *Ship Truelove*, gained	6	50	—	—
		By *Ship Molly*, gained	6	25	—	—
		By *Merchandiſe conſigned Pierre Touzin of Bourdeaux, my Account*, gained by that Adventure	7	313	—	—
		By *Merchandiſe ſhipped on Board the Bideford Merchant*, gained by that Adventure	9	75	—	—
		By *Merchandiſe conſigned to David Cornfactor of London for my Account*, gained by that Adventure	9	120	—	—
		By *Goods in Company with John Tradewell*, gained	12	50	—	—
		By *Merchandiſe conſigned to John Snuffwell for our joint Account*, gained	17	1	17	6
				5010	6	8

Caſh.

1758.			R.	*l.*	*s.*	*d.*
January	1	To *Stock* I have in my Hands — — —	1	1460	12	—
—	9	To *John Williams*, received of him — —	2	9	—	—
February	5	To *Profit and Loſs*, given by my Uncle *William Bond*, by his Will — — — — —	3	510	—	—
May	9	To *Commiſſioners of the Cuſtoms*, received a Debenture on Tobacco — — —	5	100	—	—
—	30	To *John Neſbit* of *Cadiz*, received for a Bill	5	307	10	—
June	1	To *Ship Truelowe*, received for Freight —	5	50	—	—
July	11	To *Merchandiſe conſigned me by A. Gonſalez*, received for his Tea — — — — —	6	12	—	—
Auguſt	14	To *Bills remitted me*, received for one on *Peter Jackſon* — — — — —	8	210	—	—
—	30	To *Goods in Company with J. Tradewell*, received for my Share of neat Proceeds —	8	150	—	—
September	2	To *Linen in Company with J. Dawſon and Peter Zinephon*, received in Part for 20 Pieces —	9	30	—	—
—	3	To *Francis King*, received of him for Linen in Company — — — — — —	9	20	—	—
—	9	To *Peter Zenephon*, received for his Third Share of Brandy in Company — — —	10	12	—	—
		To *John Dawſon*, received for his Third Share of Ditto — — — —	10	12	—	—
—	10	To *Brandy in Company*, received for 1 hhd. —	10	18	—	—
—	15	To *John Dawſon his Account* received of him	11	2	—	—
				2903	2	—

1758.		*Caſh.*	R.	*l.*	*s.*	*d.*
January	3	By *William James*, paid him — — —	2	13	—	—
	30	By *Ship Truelove*, paid for ¼ thereof — —	2	200	—	—
February	20	By *Tobacco*, paid for Duty, *&c.* — —	3	220	—	—
March	6	By *William Tindel*, paid him — — —	3	4	—	—
	25	By *Ship Bideford Merchant*, paid Repairs —	4	40	—	—
April	10	By *Merchandiſe conſigned Pierre Touzin, my Account*, paid for Portage, *&c* — — —	4	67	—	—
	28	By *John Neſbit*, paid for Kerſeys ſent him —	4	60	—	—
June	6	By *Merchandiſe ſhipped on Board the Bideford Merchant from Bourdeaux*, paid Wages, *&c.*	5	30	—	—
	10	By *Merchandiſe conſigned to David Cornfactor*, for Charges paid at ſhipping — — — — —	6	20	—	—
	26	By *Merchandiſe conſigned me by Abraham Gonſalez*, paid for Cuſtom and Charges — — —	6	50	—	—
July	20	By *Wine*, paid Charges on — — — —	7	25	—	—
	21	By *Merchandiſe conſigned me by A. Gonſalez* —	7	3	—	—
Auguſt	18	By *Bills drawn on me*, paid Bill to *Peter Zeno*, dated *June* the 15th — — — —	8	100	—	—
	20	By *Abraham Gonſalez his Account*, paid at ſhipping Goods for him — — — —	8	1	—	—
	21	By *Goods in Company with John Tradewell*, paid for my ½ Share of 200 Pieces of *Barnſtaple* Stuffs — — — — — — — —	8	100	—	—
	31	By *Linen in Company with J. Dawſon and Peter Zenephon*, paid for my Third Share —	9	13	6	8
September	8	By *Thomas Ormond*, paid him for Brandy —	10	36	—	—
	14	By *Voyage to Cadiz in Company*, paid Charges on ſaid Voyage — — — — — — —	11	6	—	—
	15	By *Peter Zenephon*, paid him to equal Shares of Voyage to *Cadiz* in Company — —	11	3	—	—
		Balance carried to Account of Caſh in Caſh-Book — — — — — — — —	1	1911	15	4
				2903	2	—

1758.		*Sugar.*	hhds	R.	*l.*	*s.*	*d.*
January	1	To *Stock* — I have, at 6 *l.* — —	9	1	54	—	—
December	31	To *Profit and Loſs*, gained — —	—	2	9	—	—
					63	—	—

1758.		*Raiſins.*	Bar.	R.	*l.*	*s.*	*d.*
January	1	To *Stock* — I have, at 1 *l.* 10 *s.* —	10	1	15	—	—

1758.		*Broad Cloth.*	Pces.	R.	*l.*	*s.*	*d.*
January	1	To *Stock* — I have, of Numb. 2 —	5	1	48	—	—
		And of Numb. 6	10	1	200	—	—
December	31	To *Profit and Loſs*, gained — — — —	—	2	40	—	—
			15		288	—	—

1758.		*Linen.*	Pce.	R.	*l.*	*s.*	*d.*
January	1	To *Stock* — I have — — — — —	1	1	2	10	—
December	31	To *Profit and Loſs*, gained — — —	—	2	—	10	—
					3	—	—

1758.		*Moveables, &c.*		R.	*l.*	*s.*	*d.*
January	1	To *Stock* — for Houſehold Furniture — —		1	150	—	—

1758.		*Paper.*	Rms.	R.	*l.*	*s.*	*d.*
January	1	To *Stock* — I have, at 12 *s.* — —	14	1	8	8	—

1758.		*Sugar.*	hhds.	R.	*l.*	*s.*	*d.*
March	6	By *William Tindel*, ſold him at 7 *l.* —	9	3	63	—	—

1758.		*Raiſins.*	Bar.				
December	31	By *Balance*, at 1 *l.* 10*s.* per Barrel —	10	17	15	—	—

1758.		*Broad Cloth.*	Pces.				
April	28	By *John Neſbit*, ſent him of N°. 6. at 24 *l.*	10	4	240	—	—
December	31	By Balance, remaining of N°. 2. —	5	17	48	—	—
			15		288	—	—

1758.		*Linen.*	Pce.				
March	6	By *William Tindel*, ſold him — —	1	3	3		—

1758.		*Moveables, &c.*					
December	31	By *Balance*, remaining Houſehold Furniture, *&c.* to Value of — — — — — —		17	150	—	—

1758.		*Paper.*	Rms.				
December	31	By *Balance*, remaining at 12*s.* — —	14	17	8	8	—

Wine.

1758.			hhds	Bar.	R.	l.	s.	d.
January	1	To *Stock* — I have at 6 *l.* — —	—	4	1	24	—	—
July	20	To ſundry Accounts, for — —	17	—	7	620	—	—
						644	—	—

Serges.

1758.			Pces.	R.	l.	s.	d.
January	1	To *Stock* — I have, at 4 *l.* — —	2	1	8	—	—
December	31	To *Profit and Loſs*, gained — —	—	2	2	—	—
					10	—	—

Holland.

1758.			Pces.	R.	l.	s.	d.
January	1	To *Stock* — I have, at 6 *l.* — —	10	1	60	—	—
December	31	To *Profit and Loſs*, gained — — —	—	2	10	—	—
					70	—	—

Tobacco.

1758.			hhds	R.	l.	s.	d.
January	1	To *Stock* — I have, at 12 *l.* — —	2	1	24	—	—
February	20	To *Sundry Accounts*, as per Journal, for	300	3	1820	—	—
			302		1844	—	—

John Williams.

1758.			R.	l.	s.	d.
January	1	To *Stock* — by Bill due 8th of this Month —	1	36	—	—

1758.		*Wine.*	hhds	Bar.	R.	*l.*	*s.*	*d.*
December	31	By *Balance*, remaining at 6 *l.* —	—	4		24	—	—
		And — — — — — — —	17	—	17	620	—	—
						644	—	—

1758.		*Serges.*	'ces	R.	*l.*	*s.*	*d.*
March	6	By *William Tindel,* ſold him at 5 *l* —	2	3	10	—	—

1758.		*Holland.*	Pces	R.	*l.*	*s.*	*d.*
September	14	By *Voyage to Cadiz in Company* for --	10	11	70	—	—

1758.		*Tobacco.*	hhds	R.	*l.*	*s.*	*d.*
April	10	By *Merchandiſe conſigned Pierre Touzin for my Account* — — — — — —	300	4	1720	—	—
		By *Commiſſioners of Cuſtoms* for Debenture on ſaid 300 hhds. — — — — —	—	4	100	—	—
November	21	By *Voyage conſigned to John Snuffwell for our Account* for — — — — —	2	12	24	—	—
			302		1844	—	—

1758.		*John Williams.*	R.	*l.*	*s.*	*d.*
January	9	By *Caſh*, received 9 *l.* and abated him 27 *l.* in all — — — — — — — —	2	36	—	—

1758.		William James.	R	l.	s.	d.
January	1	To Stock — by Principal of a Bond dated the 1ſt of June laſt, and payable this Day —	1	100	—	—
	1	To Stock — by Agreement, payable the 22d of December next — — — — — —	1	57	—	—
	3	To Caſh, paid him to balance this Account —	2	13	—	—
				170	—	—

1758.		Ship Bideford Merchant.	R	l.	s.	d.
January	10	To Profit and Loſs, given me by my Father —	2	950	—	—
March	25	To Caſh, paid for Repairs — — — —	4	40	—	—
December	31	To Profit and Loſs, gained per Ship — —	2	110	—	—
				1100	—	—

1758.		Voyage from Virginia in Ship Bideford Merchant.	R	l.	s.	d.
January	10	To Profit and Loſs, for Cargo of that Ship given me by my Father — — — — —	2	1500	—	—

1758.		Ship Truelove.	R	l.	s.	d.
January	30	To Caſh, for ¼ of ſaid Ship bought of James Gregory and William Zeno — — —	2	200	—	—
December	31	To Profit and Loſs, gained — — — —	2	50	—	—
				250	—	—

William James.

1758.			R.	*l.*	*s.*	*d.*
January	1	By *Stock* — I owe him on Demand — —	2	170	—	—

Ship Bideford Merchant.

1758.						
February	20	By *Tobacco,* for Freight — — — — —	3	100	—	—
June	6	By *Merchandiſe ſhipped from Bourdeaux,* for Freight	5	50	—	—
December	31	By *Balance,* remaining the Ship which I value in — — — — — — — — — —	17	950	—	—
				1100	—	—

Voyage from Virginia in Ship Bideford Merchant.

1758.						
February	20	By *Tobacco,* for her Cargo brought Home —	3	1500	—	—

Ship Truelove.

1758.						
June	1	By *Caſh,* received my Share of Freight —	5	50	—	—
December	31	By *Balance,* I value my ¼ of ſaid Ship in —	17	200	—	—
				250	—	—

Houſe in Highſtreet.

1758.			R.	*l.*	*s.*	*d.*
February	5	To *Profit and Loſs* — given me by my Uncle *William Bond*, by his Will — — —	3	500	—	—

Ship Molly.

1758.						
February	5	To *Profit and Loſs* — given me by my Uncle *William Bond*, by his Will — — —	3	680	—	—
December	31	To *Profit and Loſs* — gained per Ship —	2	25	—	—
				705	—	—

William Tindell.

1758.						
March	6	To *Sundry Accounts*, as per Journal — —	3	80	—	—

Merchandiſe conſigned Pierre Touzin of Bourdeaux, for my Account.

1758.						
April	10	To *Tobacco*, for 300 hhds. — — 1720 *l* To *Caſh*, paid for Portage, *&c.* — 67	4	1787	—	—
December	31	To *Profit and Loſs*, gained by this Adventure	2	313	—	—
				2100	—	—

Houſe in Highſtreet.

1758.			R.	*l.*	*s.*	*d.*
December	31	By *Balance*, I value the Houſe in — —	17	500	—	—

Ship Molly.

1758.			R.	*l.*	*s.*	*d.*
July	20	By *Wine*, for Freight of 17 hhds — —	7	25	—	—
December	31	By *Balance*, I value the Ship in — — —	17	680	—	—
				705	—	—

William Tindell.

1758.			R.	*l.*	*s.*	*d.*
February	5	By *Profit and Loſs* — I am to pay him by my Uncle *William Bond*'s Will — — —	3	80	—	—

Merchandiſe conſigned Pierre Touzin of Bourdeaux, for my Account.

1758.			*French Crowns.*	R.	*l.*	*s.*	*d.*
May	30	By *Pierre Touzin, my Account*, for the neat Proceeds of 300 hhds, *per Contra* — — — — —	10500	5	2100	—	—

				R	*l.*	*s.*	*d.*
1758.		*Commiſſioners of Cuſtoms.*					
April	10	To *Tobacco*, for Debenture on 300 hhds —		4	100	—	—
1758.		*John Neſbit of Cadiz.*					
April	28	To *Broad Cloth*, for 10 Pieces of Numb. 6, at 24 *l.* ſhipped per his Order on Board the *Jane* Galley, *Sealove* Maſter — — — — — —	240 *l.*				
		To *Caſh*, paid for 30 Pieces of Kerſeys, and Charges, ſhipped in ditto Ship	60				
		To *Profit and Loſs*, for my Commiſſion	7 : 10				
			———	4	307	10	—
1758.		*Pierre Touzin of Bourdeaux, my Account.*	*French Crowns*				
May	30	To *Merchandiſe conſigned him for my Account*, for neat Proceed of 300 hhds — — — — — —	10500	5	2100	—	—

Commiſſioners of Cuſtoms.

1758.			R.	*l.*	*s.*	*d.*
May	9	By *Caſh*, received the Debenture of 300 hhds.	9	100	—	—

John Neſbit of Cadiz.

1758.			*Spaniſh Pieces of Eight.*				
May	30	By *Caſh* — received in Exchange for a Bill which I have drawn on him, for —	1537½	5	307	—	—

Pierre Touzin of Bourdeaux, my Account.

1758.			*French Crowns.*				
June	2	By *Merchandiſe ſhipped on Board the Bideford Merchant from Bourdeaux,* for a Cargo of Salt, coſt — —	3375	5	675	—	—
December	31	By *Balance*, due to me — — —	7125	17	1425	—	—
			10500		2100	—	—

1758.		*Merchandiſe ſhipped on Board the Bideford Merchant from Bourdeaux.*	*French Crowns.*	R	*l.*	*s.*	*d.*
June	2	To *Pierre Touzin, my Account,* for a Cargo of Salt, coſt — — —	3375	5	675	—	—
	6	To *Caſh,* paid on Arrival of ſaid Merchandiſe, for Wages, &c. — —	—	5	30	—	—
		To *Ship Bideford Merchant* for Freight	—	5	50	—	—
December	31	To *Profit and Loſs,* gained by this Adventure — — — — — —	—	2	75	—	—
					830	—	—

1758.		*John Emerſon.*	R	*l.*	*s.*	*d.*
June	6	To *Merchandiſe ſhipped on Board the Bideford Merchant from Bourdeaux,* for the Cargo of Salt ſold him on Ship-board — — —	6	830	—	—

1758.		*Merchandiſe conſigned to David Corn-factor of London, for my Account.*	R	*l.*	*s.*	*d.*
June	10	To *John Emerſon* — — — — — — —	6	430	—	—
		To *Caſh,* paid at ſhipping 2000 Buſhels of Corn — — — — — — —	6	20	—	—
December	31	To *Profit and Loſs,* gained by this Adventure	2	120	—	—
				570	—	—

Merchandiſe ſhipped on Board the Bideford Merchant from Bourdeaux.

				l.	*s.*	*d.*
1758.						
June	6	By *John Emerſon*, for the Cargo ſold him on Ship-board — — — — — —	6	830	—	—

John Emerſon.

1758.						
June	10	By *Merchandiſe conſigned to David Cornfactor*, for a Bill on him for 400*l.* and for Inſurance 30*l.* — — — — — — — —	6	430	—	—
December	31	By *Balance*, due to me — — — —	17	400	—	—
				830	—	—

Merchandiſe conſigned to David Cornfactor of London for my Account.

1758.						
July	10	By *Voyage from London in Ship Molly*, for neat Proceeds of *per Contra* — — — —	6	570	—	—

Merchandiſe conſigned me by Abraham Gonſalez of Amſterdam, for his Account.

			R.	*l.*	*s.*	*d.*
1758.						
June	26	To *Caſh* — Paid for Cuſtom and Charges of 60 lb. Weight of Bohea Tea, and 20 Pieces of Holland; (brought by Ship *Sealove, Hadley* Maſter) — — — — — — —	6	50	—	—
July	2	To *Caſh*, paid at ſhipping his 20 Pieces of Holland — — — — — — —	7	3	—	—
Auguſt	14	To *Profit and Loſs*, for my Commiſſion —	8	28	—	—
		To *Abraham Gonſalez his Account* for neat Proceeds — — — — — — — —	8	141	—	—
				222	—	—

Voyage from London in Ship Molly.

			R.	*l.*	*s.*	*d.*
1758.						
July	10	To *Merchandiſe conſigned to David Cornfactor of London*, for neat Proceeds of my Adventure of Corn — — — — — — — —	6	570	—	—

Abraham Andrew of Briſtol.

			R.	*l.*	*s.*	*d.*
1758.						
July	31	To *Merchandiſe conſigned me by Abraham Gonſalez*, for neat Proceed of ditto *Gonſalez*'s 20 Pieces of Holland — — — — — — —	7	210	—	—

Merchandiſe conſigned me by Abraham Gonſalez of Amſterdam, for his Account.

1758.			R.	*l.*	*s.*	*d.*
July	11	By *Caſh* — received for 60 lb. of *per Contra* Tea — — — — — — — — —	6	12	—	—
	31	By *Abraham Andrew of Briſtol,* for neat Proceed of the 20 Pieces of *per Contra* Holland — — — — — — — —	7	210	—	—
				222	—	—

Voyage from London in Ship Molly.

1758.						
July	20	By *Wine,* for 17 hhds. — — — —	7	570	—	—

Abraham Andrew of Briſtol.

1758.						
Auguſt	1	By *Bills remitted me,* for one on *Peter Jackſon*	7	210	—	—

Bills of Exchange remitted me.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	1	To *Abraham Andrew*, for one on *Peter Jackſon*, payable in 10 Days — — — —	7	210	—	—
November	30	To *Don Quixot*, for one on *David Gregor*, dated *November* 20th, payable *December* 30th —	12	300	—	—
December	30	To *Sundry Accounts* as per Journal, for one on *John Paywell*, dated *December* 26th, payable 2d *January* next — — — — —	13	27	—	—
				537	—	—

Abraham Gonſalez of Amſterdam, his Account.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	15	To *Bills drawn on me*, for a Bill to *Peter Zeno* dated 15th *June* laſt — — — —	8	100	—	—
	20	To *Anthony Fox*, for 20 Pieces of *Barnſtaple* Stuffs, conſigned to Don *Bleau* at *Leghorn*, per Ship *Sailwell*, *Nimble*, Maſter — —	8	40	—	—
		To *Caſh*, paid Charges, &c. at ſhipping above Stuffs — — — — — — — —	8	1	—	—
				141	—	—

Bills of Exchange drawn on me.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	18	To *Caſh* — paid Bill to *Peter Zeno*, dated 15th *June* — — — — — — —	8	100	—	—

Bills of Exchange remitted me.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	14	By *Caſh*, received of *Peter Jackſon* for the Bill on him — — — — — — — —	8	210	—	—
December	1	By *Woollens*, per Bill on *D. Gregor*, dated *November* 20th, paid *William Jacobs* — —	12	300	—	—
	31	By *Balance*, remaining, a Bill on *John Paywell*, dated *December* 26th, payable 2d *January* next — — — — — — —	17	27	—	—
				537	—	—

Abraham Gonſalez of Amſterdam, his Account.

1758.						
Auguſt	14	By *Merchandiſe conſigned me by Abraham Gonſalez*, being the neat Proceed — — — —	8	141	—	—

Bills of Exchange drawn on me.

1758.						
Auguſt	15	By *Abraham Gonſalez*, for Bill to *Peter Zeno*, dated 15th *June* laſt, due 18th *Auguſt* —	8	100	—	—

Anthony Fox.

1758.			R.	*l.*	*s.*	*d.*
December	31	To *Balance*, due to him — — — —	17	40	—	—

Goods in Company with John Tradewell.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	21	To *Caſh*, paid for my half Share of 200 Pieces of *Barnſtaple* Stuffs in his Hand — —	8	100	—	—
December	31	To *Profit and Loſs*, gained — — — —	2	50	—	—
				150	—	—

Linen in Company with John Dawſon and Peter Zenephon.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	31	To *John Dawſon his Account*, for his Third Share of 20 Pieces of Linen — — — —		13	6	8
		To *Peter Zenephon his Account*, for a Third of Ditto — — — — — — —		13	6	8
		To *Caſh*, for my Third of Ditto — —	8	13	6	8
September	3	To *John Dawſon his Account*, for his Third of Gains — — — — — — —		3	6	8
		To *Peter Zenephon his Account*, for his Third of Gains — — — — — — —		3	6	8
		To *Profit and Loſs*, for my Third of Gains —	9	3	6	8
				50	—	—

			R.	*l.*	*s.*	*d*
1758.		*Anthony Fox.*				
August	20	By *Abraham Gonſalez his Account*, for 20 Pieces of *Barnſtaple* Stuffs — — — —	8	40	—	—

1758.		*Goods in Company with John Tradewell.*				
August	30	By *Caſh* — received for my neat Share of *per Contra* Stuffs — — — — — —	8	150	—	—

1758.		*Linen in Company with John Dawſon and Peter Zenephon.*					
September	2	By *Caſh*, received in Part for 20 Pieces ſold *Francis King* — — — —	30 *l.*				
		By *Francis King*, for remaining Part of ditto Linen — — — — —	20				
				9	50	—	—

1758.		*John Dawſon his Account.*	R.	*l.*	*s.*	*d.*
December	31	To *Balance*, due to him — — — —	17	130	—	—

1758.		*Peter Zenephon his Account.*	R.	*l.*	*s.*	*d.*
September	11	To *Brandy in Company*, for 1 hhd. taken to himſelf — — — — — — — —	10	12	—	—
		To *Caſh*, paid him to equal Shares of Voyage to *Cadiz* in Company — — — —	11	3	—	—
December	31	To *Balance*, due to him — — — —	17	118	13	4
				133	13	4

John Dawſon his Account.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	31	By *Linen in Company with him and Peter Zenephon*, for his Third Share of 20 Pieces — —	9	13	6	8
September	3	By *Linen in Company*, for his Third Share of Gains — — — — — — — —	9	3	6	8
	9	By *Caſh*, received for his Third Share of Brandy in Company — — — — — — —	10	12	—	—
	11	By *Brandy in Company*, for his Third of Gains	10	2	—	—
	14	By *Voyage to Cadiz in Company*, for 300 Buſhels of Wheat — — — — — —	11	75	—	—
	15	By *Caſh*, received of him, to equal Shares in Voyage to *Cadiz* in Company — — —	11	2	—	—
December	1	By *Voyage to Cadiz in Company*, for his Third of Gains — — — — — — — —	13	23	—	—
				130	13	4

Peter Zenephon his Account.

1758.			R.	*l.*	*s.*	*d.*
Auguſt	31	By *Linen in Company with him and John Dawſon*, for his Third Share of 20 Pieces — —	9	13	6	8
September	3	By *Linen in Company*, for his Third Part of Gains — — — — — — —	9	3	6	8
	9	By *Caſh*, received for his Third Share of Brandy in Company — — — — — —	10	12	—	—
	11	By *Brandy in Company*, for his Third of Gains — — — — — — — —	10	2	—	—
	14	By *Voyage to Cadiz in Company*, for 80 Barrels of Spilchers — — — — — —	11	80	—	—
December	1	By *Voyage to Cadiz in Company*, for his Third of Gains — — — — — — —	13	23	—	—
				133	13	4

Francis King.

1758.			R	*l.*	*s.*	*d.*
September	2	To *Linen in Company with John Dawſon* and *Peter Zenephon*, in Part for 20 Pieces ſold him	9	20	—	—

Brandy in Company with John Dawſon and Peter Zenephon, each a Third.

1758.			R	*l.*	*s.*	*d.*
September	4	To *Thomas Ormond*, for 3 hhds of Brandy at 12*l.* — — — — — — —	10	36	—	—
	11	To *John Dawſon*, for his Third of Gains —		2	—	—
		To *Peter Zenephon*, for his Third of Gains —		2	—	—
		To *Profit and Loſs*, for my Third of Gains —	10	2	—	—
				42	—	—

Thomas Ormond.

1758.			R	*l.*	*s.*	*d.*
September	8	To *Caſh* — paid for *per Contra* Brandy —	10	36	—	—

1758.		*Francis King.*	R.	l.	s.	d.
September	3	By *Caſh*, received of him, for Linen in Company — — — — — — — —	9	20	—	—

1758.		*Brandy in Company with John Dawſon and Peter Zenephon, each a Third.*	hhds	R.	l.	s.	d.
September	10	By *Caſh*, received for — — —	1		18	—	—
	11	By *Peter Zenephon*, taken to himſelf —	1	10	12	—	—
		By *Brandy*, taken to myſelf — —	1	10	12	—	—
					42	—	—

1758.		*Thomas Ormond.*	R.	l.	s.	d.
September	4	By *Brandy in Company with J. Dawſon and Peter Zenephon*, for 3 hhds. bought of him —	10	36	—	—

Brandy.

1758.			hhd.	R.	*l.*	*s.*	*d.*
September	11	To *Brandy in Company*, taken to myſelf	1	10	12	—	—

Voyage to Cadiz in Company with John Dawſon and Peter Zenephon, each a Third.

1758.			R.	*l.*	*s.*	*d.*
September	14	To *John Dawſon*, for 300 Buſhels of Wheat		75	—	—
		To *Peter Zenephon*, for 80 Barrels of Spilchers		80	—	—
		To *Holland*, for 10 Pieces — — — —		70	—	—
		To *Caſh*, for Charges paid by me — —	11	6	—	—
December	1	To *Peter Zenephon his Account*, for his Third of Gains — — — — — —		23	—	—
		To *John Dawſon his Account*, for his Third of Gains — — — — — —		23	—	—
		To *Profit and Loſs*, for my Third of Gains	12	23	—	—
				300	—	—

Don Quixot of Cadiz.

1758.			*Spaniſh Pieces.*	R.	*l.*	*s.*	*d.*
November	20	To *Voyage to Cadiz in Company* for neat Proceeds — —	1334	11	300	—	—

Brandy.

1758.			hhd.	R.	*l.*	*s.*	*d.*
December	31	By *Balance*, remaining — — —	1	17	12	—	—

Voyage to Cadiz in Company with John Dawſon and Peter Zenephon, each a Third.

1758.			R.	*l.*	*s.*	*d.*
November	20	By *Don Quixot*, for neat Proceeds 1334 Pieces of Eight — — — — — — —	11	300	—	—

Don Quixot of Cadiz.

1758.			R.	*l.*	*s.*	*d.*
November	30	By *Bills of Exchange remitted me*, for one on *David Gregor*, dated *November* 20th —	12	300	—	—

Merchandiſe conſigned to John Snuffwell at Falmouth, for our joint Account, viz. he Two-thirds, and Self One-third.

1758.			R.	*l.*	*s.*	*d.*
November	21	To *Tobacco*, for 2 hhds, ſhipped on Board the *John* Gally, *Quick*, Maſter — — — — — 24 *l.* To *Caſh*, paid Charges — — 1 To *Profit ana Loſs*, for my Commiſſion on 25 *l.* at ½ *per Cent.* — 0 : 2 : 6	12	25	2	6
December	31	To *Profit and Loſs*, I have gained by this Adventure — — — — — — — —	2	1	17	6
				27	—	—

John Snuffwell his Account.

1758.			R.	*l.*	*s.*	*d.*
November	21	To *Merchandiſe conſigned him for our joint Account*, for his Two-third Share of ſaid Goods	12	16	15	—

Woollens.

1758.			Pces.	R.	*l.*	*s.*	*d.*
December	1	To *Sundry Accounts*, (as per Journal,) for 3 Bales, *viz.* for					
		Numb. 1. Stuffs, at 2 *l.* per Piece —	60		120	—	—
		2. Duroys, at 1 *l.* 10 *s.* — —	60		90	—	—
		3. Callimancoes, at 2 *l.* —	60	12	120	—	—
			180	—	330	—	—

Merchandiſe conſigned to John Snuffwell at Falmouth, for our joint Account, viz. he Two-thirds, and Self One-third.

1758.			R.	*l.*	*s.*	*d.*
November	21	By *John Snuffwell his Account*, for his Two-thirds Share of *per Contra* Adventure —	12	16	15	—
December	30	By *Bills of Exchange remitted me*, for my Third of neat Proceeds of *per Contra* Tobacco —	13	10	5	—
				27	—	—

John Snuffwell his Account.

1758.			R.	*l.*	*s.*	*d.*
December	30	By *Bills of Exchange remitted me*, for one on *John Paywell*, dated 26th *December*, for his Two-thirds Part of Merchandiſe conſigned him	13	16	15	—

Woollens.

1758.			Pces	R.	*l.*	*s.*	*d.*
December	31	By *Balance*, remaining 3 Bales, *viz.* of					
		Numb. 1. Stuffs at 2 *l.* per Piece —	60		120	—	—
		2. Duroys, at 1 *l.* 10*s.* —	60		90	—	—
		3. Callimancoes, at 2 *l.* —	60	17	120	—	—
			180		330	—	—

1758.		*Balance.*	R.	*l.*	*s.*	*d.*
December	31	To *Caſh*, remaining, brought from the Caſh-Book — — — — — — —	1	1750	15	4
		To *Raiſins*, remaining 10 Barrels, at 1*l.* 10*s.* per Barrel — — — — — — —	4	15	—	—
		To *Broad Cloth*, remaining of Numb. 2, 5 Pieces — — — — — — —	4	48	—	—
		To *Moveables*, remaining, Houſehold Furniture, *&c.* to Value of — — — —	4	150	—	—
		To *Paper*, remaining 14 Reams, at 12*s.* per Ream — — — — — — —	4	8	8	—
		To *Wine*, 4 Barrels, at 6*l* — — —	5	24	—	—
		— — — and 17 hhds coſt — — —	5	620	—	—
		To *Ship Bideford Merchant*, valued in —	6	950	—	—
		To *Ship Truelove*, I value my ¼ in — —	6	200	—	—
		To *Houſe in Highſtreet*, valued in — —	7	500	—	—
		To *Ship Molly*, I value it in — — —	7	680	—	—
		To *Pierre Touzin, of Bourdeaux, my Account*, due to me 5125 French Crowns — —	8	1425	—	—
		To *John Emerſon*, due to me — — —	9	400	—	—
		To *Bills of Exchange remitted me*, for a Bill on *John Paywell*, dated *December* 26th, payable 2d *January* next — — — — —	11	27	—	—
		To *Brandy*, remaining, 1 Hogſhead —	15	12	—	—
		To *Woollens*, remaining 3 Bales, *viz.*				
		Numb. 1. 60 Pieces, Stuffs, at 2*l.* per Piece		120	—	—
		2. 60 Pieces, Duroys, at 1*l.* 10*s.*		90	—	—
		3. 60 Pieces, Callimancoes, at 2*l.*	16	120	—	—
		Proof		7140	3	4

1758.		*Balance.*	R.	*l.*	*s.*	*d.*
December	31	By *Anthony Fox*, due to him — — —	12	40		
		By *John Dawſon his Account*, due to him —	13	130	13	4
		By *Peter Zenephon*, his Account — — —	13	118	13	4
		By *Stock*, my neat Eſtate — — — —	1	6850	16	8
		Proof		7140	3	4

1758.		*Caſh Debtor.*	R.	*l.*	*s.*	*d.*
September	15	To *Account of Caſh in the Leger*, being the Balance of that Account — — — —	3	1911	15	4

1758.		*Per Contra Creditor.*	R.	*l.*	*s.*	*d.*
November	21	By *Merchandiſe conſigned J. Snuffwell, our Account*, paid Charges — — — —	12	1	—	—
December	1	By *Woollens*, paid *W. Jacobs* — — —	12	30	—	—
	31	By *Profit and Loſs*, for Houſe Expences —	13	70	—	—
		By *Profit and Loſs*, paid Clerk's Salary, &c. —	13	60	—	—
		By *Balance*, remaining in Hand, carried to the *Balance* Account in the Leger — —	17	1750	15	4
				1911	15	4

CHAP. VIII.

An EXPLANATION *of ſome particular Caſes in the Set of Books Number* II.

66. IN the *Inventory*, the only Thing worth Explanation is the Article concerning *William Jones*. — By the *Balance* Account in the Leger N° I. it is ſaid I owe him 13*l.* but if I turn to the Account of *William Jones* as is there directed, it appears, that he does not at preſent owe me any Thing, the Obligations not being at preſent in Force; whereas I now am abſolutely indebted to him 170*l.* ſo that he may demand 170*l.* immediately; and therefore, it is proper to enter in the Waſte-Book, that he owes me 157*l.* at ſuch Times hence, and that I owe him 170*l.* on Demand; ſo that I ſhall have a clear State of his Account, in the new Books, without having Recourſe to the old.

67. *January* 9. Here, as I have agreed to take 9*l.* in full for the whole Debt of 36*l.* *John Williams* muſt have Credit for the whole, and conſequently *Caſh* muſt be Debtor for the 9*l* received, and *Profit and Loſs* for the Money loſt, *viz.* 36 — 9 = 27*l.*

68. *January* 10. As the Ship *Bideford Merchant* is now my own, though on the Sea, I may make *it* Debtor, and *Profit and Loſs* Creditor 950*l.* but as to the *Tobacco*, it ſeems beſt not to make it Debtor till I have it in my Poſſeſſion, but to make *Voyage from Virginia in Ship Bideford Merchant* Debtor, and *Profit and Loſs* Creditor 1500*l.*

69. *February* 20. The *Voyage from Virginia* in the *Ship Bideford Merchant* was made Debtor for the Cargo; therefore, as the Voyage is now performed, that Account muſt have Credit, and conſequently the *Tobacco* muſt become Debtor, or accountable to me in its Stead. ——— Here it may be obſerved,

that though the Veſſel is my own, and ſo the Freight may be ſaid to coſt me Nothing; yet, to know what I get by my Veſſel, and what I ought to have for the Tobacco when ſold, it is proper to reckon the Freight of the Veſſel ſo much as I would have to lend it to another Merchant for the like Voyage: And then it follows, that the *Tobacco* muſt be made Debtor to the *Veſſel* for the Freight; and to *Caſh*, for the Duty, Portage, Keyage, Wages, *&c.* and conſequently, the *Veſſel*, and *Caſh*, Creditor by *Tobacco*.

70. *May* 30. As this is the ſame to me, as if *John Neſbit* had paid me himſelf, I muſt make Caſh Debtor to *John Neſbit*.

71. *June* 6. Here, as the Salt was not brought into my Cellars, nor entered in my Leger, I need not erect an Account of Salt, becauſe, if I did, it muſt be immediately diſcharged; therefore, it will be more compendious, only to make the *Perſon* that bought it, Debtor to *Merchandiſe ſhipped on Board the Bideford Merchant*.

72. *June* 10. Here, as the Corn was ſhipped off as ſoon as bought, it is not already mentioned in my Leger, and therefore I need not erect an Account for it; becauſe if I did, it muſt be immediately diſcharged, as the Goods are ſhipped off. Therefore, I make *Merchandiſe conſigned to David Cornfactor of London for my Account* Debtor to *John Emerſon* for the Bill drawn on him, and for the Premium due to him for Inſurance; and to *Caſh* for the Charges paid at ſhipping.

73. *July* 10. Here, as my Factor has immediately ſent me the Value of the neat Proceeds, there is no Occaſion to make him Debtor at all; but to make *Voyage from London in Ship Molly* Debtor to *Merchandiſe conſigned David Cornfactor*.

74. *November* 21. Here, as I ſend Goods conſigned for my Correſpondent, who is to have ⅔ Share

in that Adventure, the *Merchandiſe conſigned to him* muſt be *Debtor*, in Order to credit the ſeveral Accounts from whence it comes. And then as $\frac{2}{3}$ of the Riſque is my Correſpondent's, *he* muſt be *Debtor* for $\frac{2}{3}$ of the whole Value ſhipped, Charges, *&c.* included.

If the Rules afore laid down, and the Examples in the Sets of Books are compared together, we believe there will not be any Occaſion for farther Remarks.

Note, The two Dots in the Journal to the Leger References, ſignify, that the Article to which it is placed has been examined: That on the left Hand of the Figure is made when any Article of the Journal is examined by the Waſte-Book; and the Dot on the right Hand, when the Leger Entry has been examined by the Journal. ——— We have marked only a few Articles in this Manner, becauſe we would not give the Printer unneceſſary Trouble. We had almoſt forgot to hint, that the two Daſhes ″, in the Margin of the Waſte-Book, are thus to be underſtood, *viz.* one of them is when the Article to which it is placed has been entered in the Journal, the other when the Journal Entry has been examined by it.

CHAP.

CHAP. IX.

Of Comparative BOOK-KEEPING.

75. THE Books of any Perſon in Trade, more eſpecially of Merchants, ought to be kept in ſo regular a Manner, as to have a proper Connection with thoſe of any other Perſon with whom he tranſacts any Buſineſs: And the taking Notice of ſuch Connection is what I here mean by *Comparative Book-keeping.* One Example will be ſufficient.

76. Firſt then: By my Waſte-Book, Numb. II. it may be ſeen, that *April* 10th I conſigned *Pierre Touzin* of *Bourdeaux* 300 Hogſheads of Tobacco, *per* Capt. *Nauticus*, which coſt (all Charges till on Shipboard included) 1787*l.* but the Commiſſioners of Cuſtoms are Debtor for the Debenture 100*l.* payable in 30 Days; which Debenture I received the 9th of *May.* On the 30th of Ditto, received Advice from *Pierre Touzin*, that the neat Proceeds of my Tobacco amount to 8500 Crowns Exchange at 4*s. per* Crown. *June* 2d, Received Advice from *Pierre Touzin*, that he had ſhipped *per* my Order, on Board the *Bideford Merchant*, Capt. *Nauticus*, Salt which coſt, *&c.* amounts to 3375 Crowns, Exchange at 4*s. per* Crown. *June* 6th, My Ship arrived with the Salt, and I paid Charges, *&c.* 80*l.* which Cargo of Salt I ſold the 6th of *June* to *John Emerſon* for 830*l.* Now the whole of this Adventure, ſo far as *Pierre Touzin*, my Factor at *Bourdeaux*, is concerned in it, may be ſeen in my Leger under the Accounts entitled, *Merchandiſe conſigned Pierre Touzin, of Bourdeaux, for my Account*; *Pierre Touzin, of Bourdeaux, my Account.* And in *Pierre Touzin*, my Factor's Books, theſe Tranſactions would appear in a Form like the following. Which compare with the above in my Leger.

		Dibtor.	Crowns					Creditor.	Crowns		
1758.		*Merchandiſe conſigned me by A. B. of Bideford, for his Account.*				1758		*Merchandiſe conſigned me by A B. of Bideford, for his Account.*			
April	20	To Caſh, paid Charges on 300 hhds. of Tobacco, received per Capt. *Nauticus* — — — — — —	26	—	—	April	22	By Caſh, received for 300 hhds. of Tobacco ſold M. *Fourne* — — —	8700	—	—
		To Profit and Loſs, per my Commiſſion on 8700 Crowns — — —	174	—	—						
		To *A. B.* his Account, for neat Proceeds of ſaid Goods — — —	8500	—	—						
			8700	—	—						
1758.		*A. B. of Bideford, his Account.*				1758		*A B. of Bideford, his Account.*			
June	2	To ſundry Accounts, for a Cargo of Salt, ſhipped this Day per his Order, on Board Ship *Bideford Merchant*, Capt. *Nauticus* — — — —	3375	—	—	April	22	By Merchandiſe conſigned me by him, for the neat Proceeds of 300 hhds. of Tobacco, received per Capt. *Nauticus* — — — — —	8500	—	—
		To Balance, due to *A. B.* — —	5125	—	—						
			8500	—	—						

77. *Scholium.* Perhaps ſome may take Notice, that in this Eſſay we have not given the real Value of the Goods; to this it may be ſufficient to anſwer, that the Deſign of Treatiſes on Book-keeping is to ſhew, not the real Value of Goods, but how to keep Accounts in a regular Manner. A few other Objections may perhaps be ſtarted, but as they are not material, they are not worth anſwering; we ſhall now put an End to this Eſſay, not doubting, but if the Learner rightly underſtands what is contained herein, he will not find it a very difficult Matter, to keep a Set of Books in real Buſineſs.

The END.

BRITISH
12 OC76
MUSEUM

AN

ESSAY

ON THE

ELEMENTS

OF

PLANE GEOMETRY.

In which the moſt uſeful Propoſitions of the celebrated Geometricians, both ancient and modern, are clearly demonſtrated; and the Conſtructions and Uſes of the neceſſary Inſtruments explained.

By BENJAMIN DONN.

Ὁ Θεὸς ἀεὶ γεωμέτρει. PLATO.

PREFACE.

GEOMETRY and *Arithmetick*, as *Plato* juſtly called them, are " the two Wings by which the Minds of Men mount up to Heaven ;" that is, by which they contemplate and diſcover the Motions and Properties of the cœleſtial Bodies.

It is an Obſervation of *Plutarch*, that " GOD adorneth and layeth out all the Parts of the World according to Quantity, Proportion, and Similitude :" Or, in *Solomon*'s Words, " GOD diſpoſes all Things by Number, Weight, and Meaſure." Hence *Plato*, with Reaſon, ſaid, ὁ Θεὸς ἀεὶ γεωμέτρειν, GOD always acts according to Geometry.)

Geometry taken in its greateſt Extent, includes almoſt every Branch of the Mathematicks ; in which extenſive View, what we have ſaid of Mathematicks in general, in our Preface to the *Mathematical Eſſays*, is applicable to this. But even taken in the narrow Limits to which it is confined in this Eſſay, it is a neceſſary Introduction to almoſt all the Mathematicks, and the practical Sciences depending thereon ; for the Architect, Dialliſt, Gauger, Engineer, &c. could do nothing without Geometry, it being as it were the Spring which ſets them all at Work.

* " Beſides the Aids which every Branch " of the Mathematics derives from Geometry, " the Study of this Science is of infinite Ad- " vantage in the Uſes of Life. It is always " good to think and reaſon right; and it has " been juſtly ſaid, that there is no better " practical Logic than Geometry". It will " ſerve as the ſureſt Means to give our Rea- " ſon the firſt Habitude and Bent of Truth;" and will " teach us to operate upon Truths, " to trace the chain of them, ſubtle and al- " moſt imperceptible as it frequently is, and " to follow them to the utmoſt Extent of " which they are capable.

" The *geometrical Spirit* is not ſo much " confined to *Geometry*, that it cannot be " taken off from it, and transferred to other " Branches of Knowledge. Works of moral " Philoſophy, Politics, Criticiſm, and even " Eloquence, *cæteris paribus*, would have " additional Beauties if compoſed by *Geome-* " *tricians*. It ſo ſtrengthens and enlarges the " Mind, that, if we believe *Quintilian*, it " helps to make an Orator. The Order, Per- " ſpicuity, Diſtinction, and Exactneſs, which " have prevailed in good Books for ſome Time " paſt, may very probably derive themſelves " from this *geometrical* Spirit, which ſpreads " more than ever, and in ſome Sort commu- " nicates itſelf from Author to Author, even " to thoſe who know nothing of Geometry.

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* Supplement to Dr. *Harris's* Lexicon, under the Article *Geometry*, collected from *Lock* and others.

As to the Hiſtory of this ineſtimable Science, we ſhall for the moſt Part give it in the Words of Mr. *Thomas Lediard.*

This Science is ſaid to have been invented by the *Egyptians*, on Account of the Inundations of the *Nile*. This Reaſon might have induced them to cultivate Geometry with a greater Degree of Care and Attention, for diſtinguiſhing again each Man's Property of Land, after the Overflowings of the *Nile*; but Mr. *Rollin* thinks its Origin is undoubtedly of more ancient Date. If we may credit *Joſephus*, (of which Opinion were alſo *Pliny*, *Diodorus Siculus*, and *Cicero*,) the *Aſſyrians* and *Chaldeans* were the firſt after the Flood who applied themſelves to the Study and Practice of the Mathematics. From the *Aſſyrians* and *Chaldeans* it was transferred into *Egypt* by *Abraham*, who, by the Command of God, left his native Country, and coming thither, found the *Egyptians* to be of a remarkable Diſpoſition for learning mathematical Arts; he therefore communicated to them *Arithmetick*, *Geometry*, and *Aſtronomy*. The *Egyptians*, flouriſhing very much in theſe Sciences, gave Occaſion to *Ariſtotle* to ſay, that the *Egyptian Prieſts* were the firſt *Inventors* of the *Mathematical Arts*, who, by Reaſon of their Employments, had ſufficient Time upon their Hands for making Diſcoveries in thoſe Arts. From hence theſe Sciences were carried to *Greece* by *Thales* of *Miletus*, who travelled into *Egypt* for the Improvement of himſelf in Learning. He is ſaid to have found out the 5th, 15th, and 26th Propoſitions of the firſt

Book of *Euclid,* as alſo the 2d, 3d, 4th, and 5th of the fourth Book. He began to obſerve the Equinoxes and Solſtices, and was the firſt who foretold an Eclipſe of the Sun, and alſo one of the Moon to King *Cyrus* ; he is therefore juſtly accounted, and ought to be looked upon, as the firſt Founder of the *Mathematics* in *Greece.* He flouriſhed 584 Years before *Chriſt.* (*Vide Stanley*'s Lives of the Philoſophers.)

Thales was ſucceeded by *Pythagoras,* a Philoſopher of *Samos,* who greatly improved and adorned the Mathematics. He found out the 32d, 44th, 47th, and 48th Propoſitions of the firſt Book of *Euclid,* but is eſpecially celebrated for the Invention of the 32d and 47th, for which (as *Apollodorus* ſays) he ſacrificed a * *Hecatomb* to the Gods. He firſt laid open the Doctrine of Incommenſurables and the five regular Bodies. He was the firſt (that we read of) who opened a School wherein Youth might learn thoſe ſublime Arts and Sciences. *Pythagoras* was ſucceeded by *Anaxagoras* of *Clazomenæ* and *Oenopides* of *Chios* ; theſe were mentioned by *Plato :* and *Ariſtotle* affirms that *Anaxagoras* wrote a Treatiſe of *Geometry,* and *Proclus* aſcribes the 12th and 23d Propoſitions of the firſt Book of *Euclid* to *Oenopides.* After theſe came *Briſo, Antipho,* and *Hippocrates,*

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* *Hecatomb* (ἑκατόμβη. from ἑκατὸν a hundred, and βοῦς Oxen) was a Sacrifice of an hundred Oxen. *Strabo* ſays this Cuſtom came from the *Lacedemonians* ; who having 100 Cities in their Country, each City uſed to Sacrifice a Bullock every Year. Some derive the Word from ἑκατὸν and ποῦς a Foot, and hence conclude that a Hecatomb conſiſted only of 25 four-footed Beaſts. The Month in which this Sacrifice was made is called *Hecatombeon.*

Hippocrates, who are all celebrated by *Ariſtotle*, for attempting the Quadrature of the Circle; but of theſe *Hippocrates* was by far the moſt famous; from a Merchant he became a Philoſopher and Geometrician; he firſt attempted the doubling of the Cube by two mean Proportionals, and (as *Proclus* obſerves) was the firſt who wrote Elements, and digeſted into Order what Diſcoveries had yet been made both by himſelf and others. *Democritus* was admirable in Philoſophy and Mathematics, but his Writings were all deſtroyed, (as ſome Authors think) by *Ariſtotle*'s ambitious Deſire of having no other Writings but his own read. *Plato* (who is reported to have been a Scholar to *Theodorus Cyrenæus)* added great Luſtre to the mathematical Sciences: He vaſtly enlarged Geometry with great and noble Additions; he illuſtrated and ſet off his Books of Philoſophy in a mathematical Way, and encouraged whatſoever was admirable either in Mathematics or Philoſophy. Over the Door of his Academy was this Inſcription in Greek, "Let no one ignorant of Geometry enter here."

Out of *Plato*'s Academy proceeded a Number of Mathematicians; hence proceeded *Leodamus* the *Thracian*, who practiſing the Analyſis received from *Plato*, found out many uſeful Things by the Help of it; *Theætetus* the *Athenian*, who is rendered famous as well by *Plato*'s Encomiums (who inſcribed a Dialogue to his Name,) as by his own Inventions; 'twas he that wrote the *Elements* and the Inſcription of the regular Bodies: *Archytas* alſo wrote Elements, and his doubling the Cube is mentioned by *Eutocius*; he has alſo the Honour of being the firſt who applied Mathematics to Uſe in Life. *Gellius* reports, that by his Contrivance a wooden Pigeon was made to fly; he being followed therein by *Dædalus* and others, yielded Matter to the Poets for Fables. *Neoclides* is famous on Account of his Scholar *Leon*, who wrote Elements of the Mathematics, improved them, and made them more fit for the Uſes in Life. *Eudoxus* was not inferior to *Leon*, for to him we are indebted (as ſome Authors affirm,) for the whole 5th and the principal Propoſitions of the 12th Book of *Euclid*. He firſt framed the aſtronomical Hypotheſis, and following *Archytas*, derived down the Springs of Geometry to Mechanics. *Amyclas* the *Heracleot*, *Menechmus*, his Brother *Dinoſtratus*, and *Helicon* of *Cyzium*, all much improved and augmented Geometry. *Theudius* and *Hermotinus* rendered the Elements more perfect and full. *Xenocrates*, the Maſter of *Ariſtotle*, was a Man famous for the Mathematics: A Perſon ignorant of Geometry being deſirous to be his Auditor, he anſwered him thus, "Go thy Way, for thou wanteſt the Handles of Philoſophy." Theſe were all *Platoniſts*.

Ariſtotle's Books are all full of mathematical Arguments: Two of his Scholars are eſpecially celebrated; *Eudemus*, who compoſed a mathematical Hiſtory, and *Theophraſtus*, who wrote two Books of Numbers, four of Geometry, and one of Indiviſibles. To *Ariſteus*, *Iſidore*, and *Hypſicles* we are indebted for the Book of Solids. Laſtly, *Euclid* of *Alexandria*, gathering together the Inventions of others, diſpoſed them into Order, improved and demonſtrated them more accurately, and left us thoſe Elements which lay a ſolid Foundation for all the Branches of mathematical Knowledge. He taught in the Reign of *Ptolemy Lagus, A. R.* 454. The two laſt Books are attributed to *Hypſicles*, and not to him. He died *A. A. C.* 284. He was followed, about 100 Years after, by *Eratoſthenes* and *Archimedes*: The Name of the former was very famous, but his Writings are loſt. We have many Remains of *Archimedes*, but more are ſuppoſed to be loſt. *Polybius*, *Plutarch*, and others have delivered us his Inventions. From him proceeded that Saying, Δὸς πȣ̃ ϛῶ καὶ τὴν γῆν κινήσω, i. e. *Let me have where to ſtand, and I will move the Earth which Way I pleaſe.* *Conon* was cotemporary to him: He was a Geometrician and Aſtronomer, and his Death is lamented by *Archimedes* in his Book of the Quadrature of the Parabola. Theſe were followed by *Apollonius* of *Perga*, who was called the Great Geometrician on Account of his Conics, of which there are now extant ſeven Books. To him are alſo aſcribed the fourteenth and fifteenth Books of *Euclid*, which were contracted by *Hypſicles*. *Hipparchus* wrote twelve, and *Menelaus* ſix Books of Subtenſes in a Circle; there are alſo extant three Books of the latter concerning ſpherical Triangles. *Theodoſius* wrote three Books of ſpherical Triangles, which are pretty well known.

In the Year of *Chriſt* 70 flouriſhed *Claudius Ptolemy*: He was not only famous in *Aſtro-*

nomy, but in *Geometry* alſo; which (beſides other Things written by him) his Books of Subtenſes do eſpecially teſtify. He contracted the ſix Books of *Menelaus* and twelve of *Hipparchus* into five Theorems.

Plutarch's mathematical Problems are ſtill extant; and many are to this Day ſenſible of the learned Commentaries of *Eutocius* on *Archimedes*.

The Inventions of *Philo*, *Diocles*, *Nicomedes*, *Sporus*, and *Heron*, concerning doubling the Cube are recited by him. The Genius of the latter was excellent both for *Mechanics* and *Geometry*; his doubling the Cube is likewiſe recommended by *Pappus* before all others.

The Works of *Cteſibius*, to whom we owe the Invention of Pumps, are celebrated by *Proclus*, *Pliny*, and *Athenæus*. The Name of *Geminus* is not in the loweſt Place among *Mathematicians*: *Nicomachus* was famous for arithmetical, geometrical, and muſical Monuments; and *Serenus* well known for his two Books concerning the Section of a Cylinder.

Proclus, *Pappus*, and *Theon* are alſo celebrated among the Ancients: The learned Commentaries upon *Euclid* by *Proclus* ſhew what a Geometrician he was: The Praiſes of *Theon* alſo are deſervedly great; but I ſhall conclude with *Pappus*, the laſt in Time among the Ancients, who in Reputation ought to be reckoned the firſt: He was of *Alexandria*, the moſt fruitful City of great Men, and lived in the Year of *Chriſt* 400: The two firſt of the ſeven Books of his mathematical Collections are loſt; the other five abound with ſo many various and moſt noble Inventions in many Parts of the Mathematics, that they are eſteemed among the chief Monuments of the Ancients which have come down to us.

To particularize the Works of all who have written on Geometry would require a Volume; we ſhall therefore only give a Liſt of as many Authors as we can recollect; *viz.* *Contractus* in the Year 1000. *Cauſanus* 1440. *Regiomontanus* 1464. *Wernerus* 1480. *Barbarus* 1490. *Durerus* 1500. *Fineus* 1540. *Billingſly* 1570. *Candalla* 1578. *Keckermannus* 1603. *Clavius* 1604. *Severus* 1630. *Cavallerius* 1635. *Ramus* 1636. *Ghetaldus* 1640. *Torricellius* 1644. *P. Gregory* 1647. *Leotandus* 1654. *Fournier*, *Wallis* 1655. *S. Angelus* 1658. *Carragius* 1659. *Fabry* 1669. *Pardie* 1671. *Guarinus* 1671. *Malmſburienſi* 1673. *Bartholinus* 1674. *Dechales* 1674. *Ceva* 1678. *Barrow* 1678. *Henrion* 1683. *D. Gregory* 1684. *Scotus* 1688. *Sturmius* 1689. *Leybourn* 1690. *Oughtred* 1693. *Hugonus* 1698. *Gottignies* 1699. *Guidonus*, *Vivianus* 1701. *Tacquet* by *Whiſton* 1703. *Harris* 1705. *Ward* 1706. Meſſieurs de *Port-Royal* 1711. *Keil* 1728. *Stone* 1731. *Wolſius* 1732. *Martin* 1739. *Clareaut* 1741. *Le Clerc* 1742. *Simpſon* 1747. *Muller* 1748. *Halfpenny* 1753. *Cowley*, * *Simſon* 1756.

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* It is with Pleaſure we acknowledge our Obligations to this great Reſtorer of *Euclid*, (Profeſſor *Simſon* of *Glaſgow*,) for ſeveral valuable Remarks: And therefore the Theorems, &c. which are taken from *Euclid*, are frequently given in the Words of his correct Tranſlation.

What we have chiefly endeavoured in this Essay, has been to give clear Demonſtrations of the moſt valuable Propoſitions of *Euclid* and other celebrated Authors, both ancient and modern; that the Reader might acquire a competent Degree of Knowledge in Geometry, for an Introduction to the numerous Branches of Mathematics and Philoſophy, at a moderate Expence of Time and Application.

It is common indeed for Profeſſors to recommend *Euclid* to their Pupils; but though it muſt be allowed that the Demonſtrations of *Euclid* are in general both elegant and accurate, yet if it be conſidered, that at the Time *Euclid* wrote, it was thought ſufficient to underſtand his Elements to be a celebrated Mathematician; whereas they are now only an Introduction to other Arts: It would not, we think, be right to adviſe young Men to ſtudy *Euclid* thoroughly, as it would take up more Time than they can conveniently ſpare. Of this the learned and Reverend Mr. *Bedwell* was ſo ſenſible, that he ſays, [*] " In what Time, " think you, may a Man learn all *Euclid*, and " ſo by him be made ſkilful in this Art? By " himſelf I know not whether ever or never; " And with the Help of another, although " very expert, I will not promiſe him that he " ſhall attain to Perfection in many Years. — " *Ramus*, my Author, in reading him, con" feſſeth himſelf often to have been at a Stand; " often to have loſt himſelf; often to have hit

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[*] In his Preface to his Tranſlation of *Ramus*'s Geometry, printed in the Year 1636.

" on a Rock, when he thought he had " touched Land." —— He ſoon after adviſes his Reader by no Means to refuſe the Aſſiſtance of a Teacher, if he can get one, becauſe if he be not of a great Genius, he cannot otherwiſe attain unto any great Perfection, without much Labour and Loſs of precious Time.

The Method of Demonſtration we have choſen to make uſe of in this ESSAY, is the ſynthetick, that it might be underſtood independent of Algebra, and make the Reader acquainted with the ancient Method of Reaſoning, which hath been always juſtly admired for its Accuracy. As *Euclid's* Demonſtrations (except a few which were corrupted by *Theon* and ſome others,) have ſtood the Teſt of more than 2000 Years, we have endeavoured to follow him as nearly as might be conſiſtent with our Plan; and if we are not miſtaken, the Reader will here find all that is * neceſſary by Way of Elements, and may acquire a competent Degree of Knowledge in much leſs Time, and with greater Eaſe, than could be done by the Elements of that juſtly celebrated Author.

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* However, perhaps ſome Readers may find ſeveral Propoſitions in ſome modern Authors not taken Notice of in this Eſſay, which they may think too valuable to be paſſed by in Silence: If ſo, we would only hint, that being deſirous of having the Elements as ſhort as could be conveniently done, for the Eaſe of the young Student: We muſt beg Leave to refer ſuch Readers to the End of our intended Eſſay on Trigonometry, for a Number of other geometrical Propoſitions, as we intend in that Place to give a Collection of many, by Way of Exerciſe in Geometry, Trigonometry, and Algebra.

It is probable this Work will meet with Cenſures of oppoſite Kinds; ſome who are bigotted to Antiquity may perhaps condemn it, becauſe we have taken the Liberty to alter the Order and Demonſtrations of ſome of the Propoſitions; whilſt others who look upon the Ancients with too little Veneration, may blame us for not having taken greater Liberties: But as we have endeavoured to keep clear of each Extreme, the Cenſures of both theſe Sorts of Perſons will give us no Concern.

Bideford, Auguſt 15,
1759. B. DONN.

A
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BOOK I.

The INTRODUCTION.

1. GEOMETRY originally according to its Etimology ſignified the Art of meaſuring Land; but now is the Science which conſiders the Extenſion and Magnitude of Bodies in general.

2. Geometry is either Theoretical, or Practical. Theoretical Geometry is the Science of Quantity, Extenſion, or Magnitude, abſtractedly conſidered: And is what we ſhall principally treat of in this Eſſay.

3. Practical Geometry is the Application of Theoretical Geometry to the Conveniences of Life and Commerce, *&c.* which make up the principal Bulk of the Mathematicks. We ſhall therefore only juſt touch on this Part in this Eſſay; intending to treat of its ſeveral Branches in ſome future Eſſays.

4. Geometry may alſo be divided into *Plane* and *Solid*. Plane Geometry treats of the Properties of

Lines and Superficies; and is what we ſhall confine ourſelves to in this Eſſay.

5. Solid Geometry conſiders the Properties of Bodies, with Regard to their Length, Breadth and Thickneſs. Of which hereafter.

6. *Euclid*'s firſt Definition is,

A *Point* is that which hath no Parts, or which hath no Magnitude.

2.

7. A *Line* is Length without Breadth.

3.

8. The Ends, Bounds, or Extremes of a Line are called *Points.**

4.

9. A *right Line*, or *ſtraight Line*, is that which lies evenly between its Points; or which is the ſhorteſt Diſtance from its extreme Points. And a Line, which lies unevenly between its extreme Points, is called a curve Line.

5.

10. A *Superficies*, or *Surface*, is that which hath only Length and Breadth.

6.

11. A *Surface* is bounded or terminated by Lines, or the Extremes of a Superficies are Lines.

7.

12. A *plane Superficies* is that which lieth evenly between its Bounds; or that in which a right Line joining any two Points taken any where in it will be wholly in that Superficies.

8.

Is uſeleſs, and therefore omitted.

9.

13. A rectilineal Angle is the Inclination of two right Lines to one another, meeting in a Point;

---

* As ſome have objected to the firſt Definition, it may not be improper to remark, that hence it follows, a Geometrical Point is not a Quantity, but the End, Term, or Bound of a Quantity; and therefore indiviſible; and conſequently *Euclid*'s Definition that a Point hath no Parts, or Magnitude, is very juſt.

which Point is called the angular Point or Vertex. Or it is the Opening of two right Lines, which meet in a Point.

Note. When there are two or more Angles at the ſame Point, it is common to expreſs the Angle by three Letters, of which the middle one is at the Vertex, and the other two at the Lines forming the Angle. Thus DBC or CBD, *Plate* 1. *Fig.* 1. ſignifies the Angle at B, formed by the Inclination of the Lines DB and CB. And ABD, or DBA the Angle at B, formed by the Meeting of the Lines AB, DB. But if there is but one Angle at a Point, it is generally expreſſed by the ſingle Letter at that Point.

10.

14. When a right Line, ſtanding on another right Line, makes the Angles on either Side of it equal to each other; each of theſe Angles is called a *right Angle.* And the right Line which ſtands on the other is called a Perpendicular to that other. Plate 1. Fig. 2.

11.

15. An *obtuſe Angle* is that which is greater than a right Angle. Fig. 3.

12.

16. An *acute Angle* is that which is leſs than a right Angle. Fig. 4.

13.

Is uſeleſs and therefore omitted.

14.

17. A *Figure* is that which is incloſed by one or more Boundaries.

15.

18 A *Circle* is a plane Figure, contained by one Line, called the Circumference; to which all right Lines drawn from a certain Point within the Figure, are equal to each other. Which right Lines are called the *Radii*. Fig. 5.

16.

19. And that Point within the Figure from which the equal Lines are drawn, is called the *Center.*

17.

20. A *Diameter* of a Circle is a right Line drawn through the Center, and terminated on both Sides by the Circumference.

21. *Scholium.* At the End of this Definition moſt Copies add, "which divides the Circle into two "equal Parts." But this is not properly a Part of the Definition, but a Corollary following from it; the Truth of which plainly appears by ſuppoſing one of the Parts into which the Circle is divided to be laid on the other; for it is manifeſt in ſuch Caſe they will both coincide, otherwiſe the right Lines drawn from the Center to the Circumference could not be equal; and if they coincide, they muſt be equal, by Axiom 8.

18.

22. A *Semicircle* is a Figure contained by a Diameter, and that Part of the Circumference of a Circle cut off by the Diameter.

19.

23. A *Segment* of a *Circle* is a Figure contained by a right Line and the Circumference cut off by it. (This is not uſed in the Elements.)

20.

24. *Rectilineal Figures* are ſuch as are contained by right Lines.

21.

25. *Trilateral Figures*, or *Triangles*, by three right Lines.

22.

26. *Quadrilateral Figures* are contained by four right Lines.

23.

27. *Multilateral Figures*, or *Polygons*, by more than four right Lines.

24.

28. An *Equilateral Triangle* is that which has three equal Sides. Fig. 6.

25.

29. An *Iſoſceles Triangle* is that which has only two Sides equal. Fig. 7.

26.

30. A *Scalene Triangle* is that which has three unequal Sides. Fig. 8.

27.

31. A *right angled Triangle* is that which has one of its Angles a right Angle. Fig. 9.

28.

32. An *obtuſe angled Triangle* is that which hath an obtuſe Angle. Fig. 8.

29.

33. An *acute angled Triangle* is that which has three acute Angles. Fig. 7.

30.

34. A *Square* is a four ſided Figure, whoſe Sides are all equal, and Angles all right Angles. Fig. 10.

31.

35. An *oblong*, *rectangle*, or *right angled Parallelogram*, is that four ſided Figure which has all its Angles right Angles, but not all its Sides equal. fig 11

32.

36. A *Rhombus* is that which hath four equal Sides, but its Angles not right Angles. fig 12

33.

37. A *Rhomboides* is that which has its oppoſite Sides equal to one another, but all its Sides are not equal, nor its Angles right ones. fig 12

Note. For a Definition of a *Parallelogram*, ſee Art. 97.

38. *Scholium*. In moſt Copies of *Euclid*, there is another Condition added, *viz.* "that its oppoſite "Angles are equal." But this is not neceſſary, for we ſhall demonſtrate hereafter, that all quadrilateral Figures whoſe oppoſite Sides are equal, have alſo their oppoſite Angles equal. (See Art. 98.)

34.

34.

39. All other quadrilateral Figures beſides theſe are called *Trapeziums.*

35.

Fig. 14. 40. If two Lines AB, CD, meet any other Line BD, ſo as to make the Angles *a* and *b*, at the ſame Side equal, they are called *parallel Lines.*

N. B. We have here given a different Definition from *Euclid*, becauſe it will render ſome of our Demonſtrations more compendious, *&c.* However its Agreement with *Euclid*'s, that two parallel Lines if infinitely produced could never meet, is demonſtrated in Article 96.

POSTULATES.

1.

41. Let it be granted that a right Line may be drawn from any one Point to another.

2.

42. That a finite right Line may be produced to any Length in a right Line.

3.

43. And that a Circle may be deſcribed from any Center, at any Diſtance from that Center.

Scholium. And that Diſtance is called the *Radius.*

4.

44. That from a given Point and right Line may be drawn, either parallel to another right Line; or ſo as to make any given Angle with that other right Line.

N. B. In demonſtrating the following Theorems we ſhall not refer to theſe Poſtulates; becauſe the Learner will readily ſee where they are ſuppoſed.

AXIOMS.

1.

45. Things equal to one and the ſame Thing, or to equal Things, are equal to each other.

2.

46. If to equal Things, equal Things be added, the Whole will be equal.

3.

47. If from equal Things, equal Things be taken away, the Remainders will be equal.

4.

48. If equal Things be added to unequal Things, the Wholes will be unequal.

5.

49. If equal Things be taken from unequal Things, the Remainders will be unequal.

6.

50. Things which are double to one and the ſame Thing, are equal to one another.

7.

51. Things which are halves of the ſame or equal Things, are equal to one another.

8.

52. Magnitudes which coincide, that is, which exactly fill the ſame Space, are equal to one another.

9.

53. The Whole is greater than its Part.

10.

54. Two right Lines cannot incloſe a Space

11.

55. All right Angles are equal to one another.

12.

56. This ſeems to be improperly placed amongſt the Axioms, and has been the Cauſe of much Debate amongſt both the ancient and modern Geometricians; for which Reaſon it is here omitted, and a more uſeful one inſerted, *viz.*

All the Parts taken together are equal to the Whole. *Note.* This is not in *Euclid*, though that illuſtrious Author frequently makes Uſe of it.

Characters.

57. For Brevity the following Signs are made uſe of.

	Signification.
+	*Plus*, or *more*, or *and*, or to be *added.*
—	*Minus*, or *leſs*, or that the Quantity following is to be ſubtracted, or taken from another.
×	A *Rectangle*, thus AB×BD denotes the Rectangle contained under the Length and Breadth denoted by the Lines AB and BD. But we generally expreſs a Square by putting the Figure 2 over the Letter, or Letters, repreſenting its Side. Thus, AB^2 ſignifies the Square whoſe Side is AB, or which is the ſame the Rectangle AB×AB.
:	Is to, or to, } When four Quantities A, B, C, D, are proportional, that is, when as A is to B ſo is C to D; we generally write them thus, A : B :: C : D. But when three Quantities A, B, C, are proportional the ſecond is repeated thus, as A : B :: B : C.
::	So is.
=	*Equal* to.
Γ	The Greek Letter Gamma denotes *greater than.*
Λ	The Greek Letter Lambda in a vertical Poſition ſignifies *leſs than*; but,
∠	In an horizontal Poſition ſignifies *Angle*; and with the Addition of the Letter *s*, as ∠'s denotes *Angles*.
‖	In a vertical Poſition ſignifies *Parallel.*
△	Signifies *Triangle*, and with the Addition of the Letter *s*, as △'s, is for Triangles.
R	*Radius.*

Rt.	Right. Thus, Rt. ∠. is for Right Angle.
⊥	Denotes a Perpendicular.

When a Number is prefixed to any Quantity, it ſhews how many Times that Quantity is to be taken or repeated: Thus, 6 A, denotes ſix Times the Quantity ſignified by A.

In the following Propoſitions, for *Right Line*, we ſhall only write, *Line*.

Though we ſhall make Uſe of the algebraic Signs as above explained, yet we ſhall give Demonſtrations purely geometrical; for, to uſe the Words of * Mr. *Simpſon*, (who is juſtly celebrated for his great Skill in mathematical Learning,) “ It is not the Uſe of
“ Symbols, (which ſome more ſcrupulous than diſcerning
“ have condemned,) but the Ideas annexed
“ to them, that render the Conſiderations *geometrical*
“ or *ungeometrical*. In pure Geometry Regard
“ is always had to the abſolute Quantity of ſome
“ one of the three Kinds of Extenſion, abſtractedly
“ conſidered; and whatever Symbols are uſed here,
“ are to be conſidered as expreſſive of the Quantities
“ themſelves, and not as any Meaſures, or numerical
“ Values of them. Thus, by A × B, taken
“ in a geometrical Senſe, we have an Idea, not
“ of the Product of two Numbers, (as in the algebraic
“ Notation,) but of a real, rectangular
“ Space, contained under two Right Lines, repreſented
“ by A and B, and two others equal to them.
“ So likewiſe, $\frac{B \times C}{A}$ is not to be underſtood here
“ in the Light of an algebraic Fraction, but as a
“ Right Line, which is a fourth Proportional to
“ three Right Lines repreſented by A, B, and C.—
“ Theſe Diſtinctions are abſolutely neceſſary to thoſe
“ who would have an accurate Idea of the Subject.”

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* In his ſelect Exerciſes.

Pl. I. Fig. 15. 58. Theorem 1. *If two Triangles,* ABC, DEF, *have two Sides of the one,* AB, AC, *equal to two Sides of the other,* DE, DF, *each to each,* viz. AB *equal to* DE, *and* AC *equal to* DF, *and the Angles contained between thoſe equal Sides equal,* viz. *the Angle* A *equal to the Angle* D; *they ſhall have the other Side* BC *equal to the other Side* EF, *the Triangle* ABC *equal to the Triangle* DEF, *and the other Angles in one Triangle ſhall be equal to their correſponding ones in the other Triangle, each to each,* viz. *thoſe to which the equal Sides are oppoſite; that is, the Angle* B *equal to the Angle* E, *and the Angle* C *equal to the Angle* F.

For if the △ ABC be applied to DEF, ſo that the Point A may coincide with D, and the Line AB with DE, the Point B will coincide with E, becauſe AB = DE; and the Side AC will coincide with DF, becauſe ∠ A = ∠ D; and AC being = DF, the Point C will fall upon the Point F. Hence the Points, B and C, coinciding reſpectively with the Points E and F, the Baſe BC muſt be = the Baſe EF, otherwiſe two Lines would contain a Space, which is [a] abſurd. ∵ the Side BC coincides and is = the Side EF; ∵ the whole △ ABC coincides and is = △ DEF; ∵ ∠ B coincides and is = ∠ E; and ∠ C with and = ∠ F. Q. E. D.

[a] 10 Ax.

Pl. I. F. 16. 59. Theorem 2. *The Angles* B, C, *at the Baſe of an Iſoſceles Triangle* ABC, *and oppoſite to the equal Sides* AC, AB, *are equal to each other; and a Line* AD *biſecting the Angle* BAC, *divides the Baſe into two equal Parts, and is perpendicular thereto.*

For becauſe the Line AD biſects the ∠ A, that is, makes the ∠ BAD = ∠ CAD, and Side AB is = AC, and AD common to both, the △ ABD = [a] △ ACD; ∠ B = ∠ C, and ∠ ADB = [b] ∠ ADC; ∵ AD ⊥ to BC. Q. E. D.

[a] 58.
[b] 14.

60. Corollary. *Hence every equilateral Triangle is alſo equiangular.*

For the Sides AB, AC, being equal, from above, $\angle B = \angle C$; and if the $\triangle$ be equilateral, the Side $CA = CB$, and $\therefore$ by this Theorem, the oppoſite $\angle$s are equal, *viz.* $\angle A = \angle B$ $\because$ $\angle A = \angle B =$[b]$\angle C$. Q. E. D. [b] 1 Ax.

61. Scholium. This Theorem is the fifth Propoſition of the firſt Book of *Euclid*, and is by many called the *Aſſes Bridge*, becauſe thoſe who are not capable of learning that Book, generally ſtop at this Place; but as *Euclid*'s Demonſtration is certainly ſomething too difficult to be ſo ſoon entered on, we have taken the Liberty to give a different, and it is ſuppoſed, a much eaſier Demonſtration.

62. Theorem 3. *If two Angles* B, C, *of a Triangle* ABC, *be equal, then the Sides* AB, AC, *ſubtending the equal Angles, will be equal between themſelves.* Pl.1.F17.

For, if AB be not $=$ AC, one of them is Γ the other. Let AB be the greater, and let $DB =$ AC. Then, becauſe in the $\triangle$s DBC, ACB, we have $DB = AC$, BC common to both, and the contained $\angle$s equal, *viz.* $\angle B = \angle C$, the $\triangle$ DBC will in every Reſpect be $=$[a] $\triangle$ ACB, the leſs $=$ the greater, which is abſurd. $\because$ AB is not unequal to AC, $\because$ AB muſt be $=$ AC. Q. E. D. [a] 58.

63. Corollary. *Hence every equiangular* $\triangle$ *is alſo equilateral.*

For, by this Theorem, the $\angle B$ being $= \angle C$, the Side AB will be $=$ AC; and in an equiangular $\triangle$, the $\angle A = \angle C$, $\because$ the oppoſite

Sides, BC, AB, are equal. ∵ AC=BC=AB, by Axiom 1ſt. Q. E. D.

Pl.1.F18. 64. THEOREM 4. *A Line* AB *ſtanding on another Line* DC, *making with it Angles*, ABD, ABC, *thoſe Angles taken together are equal to two Right Angles.*

For, let EB be a ⊥ to DC, then the ∠s, EBD,
[a] 14. EBC, are [a] Right Angles. But ∠ABD=[b]∠EBD
[b] 56. +∠EBA; and ∠EBA+∠ABC=∠EBC; that is,
the ∠ABD= a Rt. ∠+ ∠EBA, and ∠EBA + ∠
ABC= a Rt. ∠.

∵ by adding thoſe equal Things together, *viz.*
the Antecedents to the Antecedents, and the Con-
ſequents to the Conſequents, we have ∠ABD+∠EBA
[c] 46. +∠ABC [c]= 2 Rt. ∠s+∠EBA; and by taking
away the ∠EBA, common to both Sides of the
[d] 47. Equation, we find ∠ABD + ∠ABC [d]= 2 Rt. ∠s.
Q. E. D.

65. COROLLARY 1. *Hence if one of the ∠s be a Rt. one, the other is alſo a Rt. ∠; and if one of them*, ABC, *is acute, the other*, ABD, *is obtuſe*, et contrà.

66. COROLL. 2. *If two or more Lines ſtand on the ſame Line at the ſame Point, the Angles which they make, taken together, will be = 2 Rt. ∠s*; for all the Parts are equal to the Whole.

67. COROLL. 3. *Two Rt. Lines croſſing each other, make 4 ∠s, which taken together are = 4 Rt. ∠s.*

68. COROLL. 4. *All the ∠s made about a Point are = 4 Rt. ∠s, by Coroll.* 2.

Pl.1.F.19. 69. *Theorem* 5. *If two Right Lines*, AB, CD, *interſect each other, the oppoſite Angles are equal*, viz. ∠AEC = ∠BED, and ∠CEB = ∠AED.

For ∠AEC + ∠CEB = [a] 2 Rt. ∠s, and ∠CEB [a] 64.
+ ∠ BED = 2 Rt. ∠s; ∵ ∠ AEC + ∠ CEB =
[b] ∠CEB + ∠ BED; from which taking away the [b] 45.
∠CEB, common to both, there remains ∠ AEC =
[c] ∠BED. After the ſame Way it may be ſhewn [c] 47.
that ∠CEB = ∠AED. Hence the oppoſite ∠s are
equal. Q. E. D.

70. THEOREM 6. *Any two Sides of a Triangle are together greater than ~~third~~ the third Side.*

For the neareſt Diſtance to any two Points being Pl.1.F.15.
[a] a Rt. Line, BA + AC muſt be ⊏ BC. Q. E. D. [a] 9.

71. *Scholium.* The *Epicureans*, as *Proclus* obſerves, derided this Theorem, it being manifeſt to the weakeſt Capacity, and ∵ required no Demonſtration: However, as the Number of Axioms ought not to increaſed without a Kind of Neceſſity, *Euclid*, who is ſcrupulouſly exact, thought proper to give a formal Demonſtration of it. —— What we have ſaid above is ſufficient.

72. THEOREM 7. *If a Line* EF, *croſſes two parallel Lines*, AB, CD, *the alternate Angles*, AGH, GHD, *are equal to each other; and the two interior Angles*, BGH, GHD, *upon the ſame Side, taken together are equal to two Right Angles.* Pl.1.F.20.

For, the Lines AB, CD, being ||, the ∠ AGH
[a] = CHF; and ∠ GHD = [b] ∠CHF, ∴ ∠ AGH [a] 40. [b] 69.
[c] = ∠GHD. (After the ſame Manner it might be [c] 45.
ſhewn that ∠ CHG = ∠ HGB.) Again, it has
been juſt ſhewn that ∠ AGH = ∠ GHD, ∵
∠ BGH + ∠ AGH [d] = BGH + ∠ GHD. But [d] 46.
∠ BGH + ∠ AGH [e] = 2 Rt. ∠s. ∵ ∠ BGH + [e] 64.
∠GHD [c] = 2 Rt. ∠s. Q. E. D.

Pl.1.F.20. 73. THEOREM 8. *If a Line* EF, *falling upon two other Lines,* AB, CD, *makes the alternate Angles* AGH, GHD, *equal to each other; the Lines* AB, CD, *will be parallel.*

For the $\angle AGH$ being $= \angle GHD$ by the Suppoſition,
[a] 69. and $\angle AGH$[a] $= EGB$, the $\angle EGB$[b] $=$
[b] 45. $\angle GHD$; ∵ AB[c] ∥ CD. Q. E. D.
[c] 40.

Pl.1.F.21. 74. THEOREM 9. *If two Lines* AB, CD, *be each of them parallel to another Line* EF, *thoſe Lines* AB, CD, *are parallel to each other.*

For AB being ∥ to EF, the $\angle AGH$[a] $= \angle EHI$;
[a] 40. and the Line CD being ∥ to EF, the $\angle EHI$[a] $=$
[b] 45. $\angle CIK$; ∵ $\angle AGH$[b] $= \angle CIK$; ∴ AB is[a] ∥ CD. Q. E. D.

75. *Scholium.* From the bare Definition of parallel Lines, plainly follows this COROLLARY, *that if a Line is* ⊥ *to another Line, it is alſo* ⊥ *to all the Parallels of that other Line.* — See Art. 96.

Pl.1.F.22. 76. THEOREM 10. *If any Side* BC *of a* △ ABC *be produced, the external Angle* ACD *will be equal to the two internal and oppoſite Angles* A *and* B.

For let the Line CE be ∥ to BA ; then the
[a] 40. $\angle ECD$[a] $= \angle B$; and, as the Line AC croſſes two
[b] 72. ∥ Lines BA, CE, the alternate ∠s are [b] equal, *viz.*
[c] 46. $\angle ACE = \angle A$. ∵ $\angle ECD + \angle ACE$ [c] $=$
[d] 56. $\angle B + \angle A$. But $\angle ECD + \angle ACE$ [d] $= \angle ACD$,
[e] 45. ∵ $\angle ACD$[e] $= \angle B + \angle A$. Q. E. D.

77. COROLLARY. *Hence the external Angle* ACD *is greater than either of the internal and oppoſite Angles* A, *or* B ; becauſe it is, by this, equal to both.

78. **Theorem 11.** *The three (interior) Angles of every Triangle are equal to two Right Angles.* Pl. 1. F. 22.

For, the $\angle A + \angle B$[a] $= \angle ACD$, and to theſe equals adding $\angle ACB$, we have $\angle A + \angle B + \angle ACB$[b] $= \angle ACD + \angle ACB$: But $\angle ACD + \angle ACB$[c] $= 2$ Rt. ∠s; ∵ $\angle A + \angle B + \angle ACB$[d] $= 2$ Rt. ∠s. Q. E. D.

[a] 76.
[b] 46.
[c] 64.
[d] 45.

79. **Corollary 1.** *The three Angles of any Triangle, taken together, are equal to the three Angles of any other Triangle taken together.*

80. **Coroll. 2.** *If in one Triangle, two of its Angles, taken together, are equal to two Angles of another Triangle taken together, in ſuch Caſe the remaining Angle in one is equal to the remaining Angle in the other Triangle.*

81. **Coroll. 3.** *If two Triangles have one Angle of the one equal to one Angle of the other, then the Sum of the two remaining Angles of one Triangle is equal to the Sum of the two remaining Angles of the other.*

82. **Coroll. 4.** *If in any Triangle one of its Angles is a Right Angle, the Sum of the other two is a Right Angle.*

83. **Coroll. 5.** *Hence, if in a right-angled Triangle one of the acute Angles is given, the other is alſo known, it being the Difference between the given Angle and a Right Angle.*

84. **Coroll. 6.** *When the Angle contained between the Sides of an Iſoſceles Triangle is a Right Angle, the other two Angles are each of them half a Right Angle.*

For one of the Angles being a Rt. ∠ by the
[a] 82. Suppoſition, the Sum of the other two is [a] = Rt. ∠.
[b] 59. But the ∠s at the Baſe of an Iſoſceles Δ are [b] equal;
∴ each of them muſt be = $\frac{1}{2}$ a Rt. ∠, both being
equal to a Rt. ∠, as hath been juſt ſhewn.

[a] 60. 85. Coroll. 7. *An equilateral Triangle being [a] equiangular, each Angle muſt be equal to one Third of two Rt. Angles; or, which is equivalent, two Thirds of one Right Angle.*

86. Coroll. 8. *If in any Triangle, one of its Angles is either a right or an obtuſe Angle, the other two will be each of them acute.*

87. Coroll. 9. *All the interior Angles of any rectilineal Figure, together with four Right Angles, are equal to twice as many Right Angles as the Figure has Sides.*

Pl.1.F.23. For any rectilineal Figure, ABCDE, can be divided into as many Triangles as the Figure has Sides,
by drawing Lines from any Point F within the Figure, to each of its angular Points. And all the
[a] 78. Angles of theſe Triangles are [a] = twice as many Rt.
∠s as there are Triangles, that is, as the Figure has Sides. And the ſame ∠s are = the ∠s of the Figure, together with the ∠s at the Point F, which is the common Vertex of the Triangles; that is, [b] together with four Rt. ∠s. Therefore this Corollary is true.

[b] 68. 88. Coroll. 10. *All the exterior Angles of any rectilineal Figure are together equal to four Right Angles.*

Pl.1.F.24. Becauſe every interior ∠ABC, with its adjacent
[a] 64. exterior ∠ABD, is [a] = 2 Rt. ∠s; ∴ all the interior
together, with all the exterior ∠s of any Figure,

are = twice as many Rt. ∠s as the Figure has Sides, that is, by the laſt Corollary, they are = all the interior ∠s of the Figure, together with four Rt. ∠s. ∵ all the exterior ∠s are = 4 Rt. ∠s.

89. THEOREM 12. *The greater Side of every Triangle is oppoſite to the greater Angle:* That is, in △ABC, let the Side AC be greater than AB, then, I ſay, the Angle B is greater than the Angle C. Pl.1.F.25.

For AC being Γ AB by the Suppoſition, let D be a Point in the Side AC, ſo that AD be = AB; and ſuppoſe B, D, joined. Then, ∠ADB being the exterior ∠ of △ BDC, the ∠ADB[a] Γ the interior and oppoſite ∠C: But AD being = AB, the ∠ADB[b] = ∠ABD; ∵ as it has been juſt ſhewn that ∠ADB is Γ ∠C, its equal ∠ABD muſt alſo be Γ ∠C, and much more muſt the whole ∠B (which is Γ ∠ABD) be Γ ∠C. Q. E. D.

[a] 77.

[b] 59.

90. COROLLARY. *Hence, in every Triangle, the greater Angle is ſubtended by (that is, is oppoſite to,) the greater Side.*

For by the laſt Article, the greater Side is oppoſite to the greater Angle; conſequently, the greater ∠ is oppoſite to the greater Side. For if any Thing, A, is oppoſite to another Thing, B, it is manifeſt that the Thing B muſt be oppoſite to the Thing A. —— Here again we have taken the Liberty to deviate from *Euclid*'s formal Demonſtration.

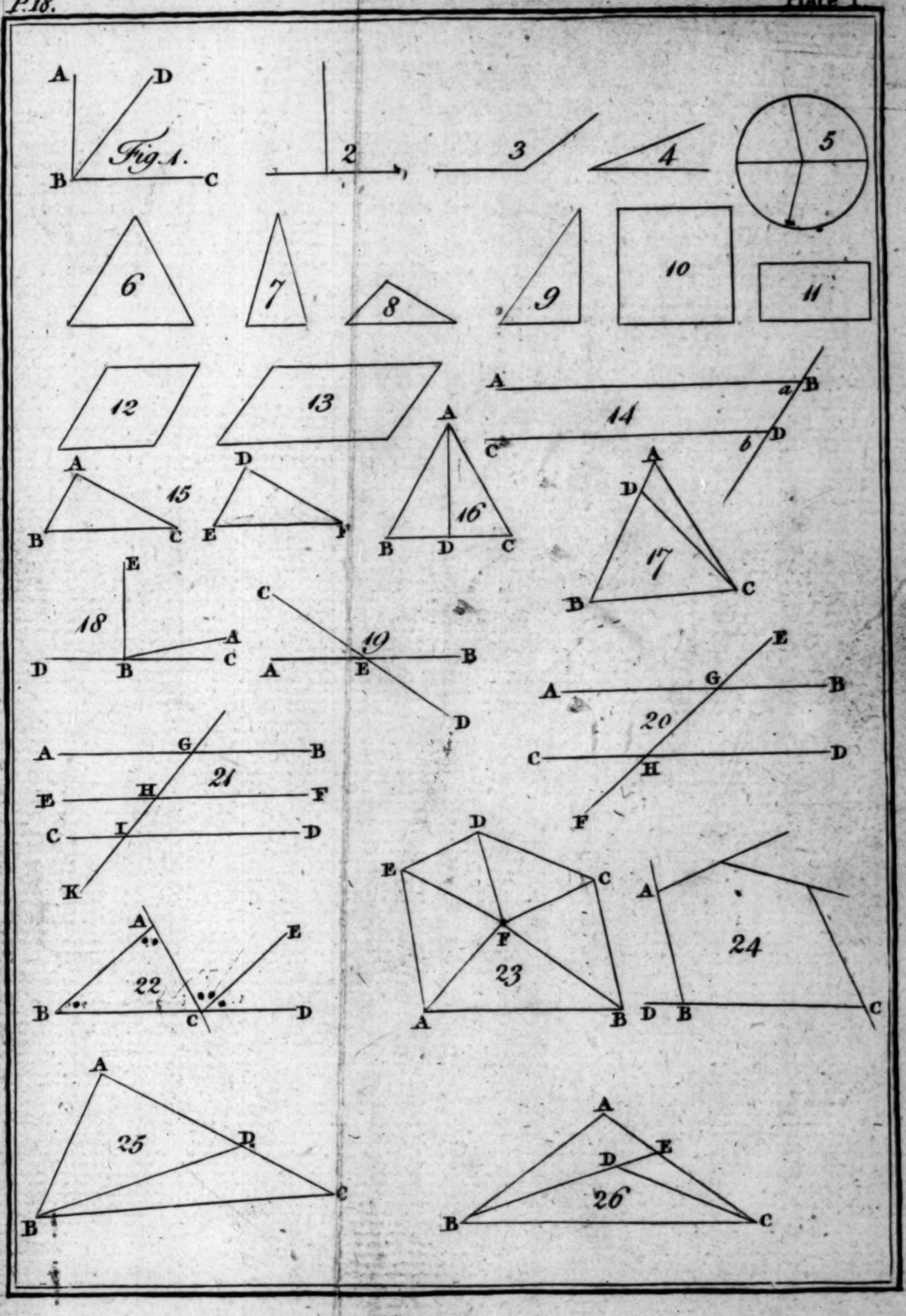
91. THEOREM 13. *If two Triangles,* ABC, DBC, *have the ſame common Baſe, the two Sides* BD, DC, *of the inſcribed Triangle, taken together, will be leſs than the two Sides* BA, AC, *of the circumſcribing Triangle: But the vertical Angle* BDC *of the inſcribed Triangle will be greater than the vertical Angle* A *of the circumſcribing Triangle.* Pl.1.F.26.

As the Sides BD, DC, are contained within the Sides BA, AC, it is manifeſt the Sum of the circumſcribing Sides $BA + AC$ muſt be Γ thoſe which they circumſcribe, *viz.* $\Gamma\ BD + DC$: For whatever Thing includes another, muſt itſelf be Γ that other. This is ſo plain, that Monſieur *Clairault*, a celebrated *Frenck* Geometrician, cenſures *Euclid* for taking the Pains to demonſtrate it. But if a Demonſtration be required, as the Number of Axioms ought not to be increaſed without a Kind of Neceſſity, it may be done thus. Suppoſe BD produced to meet the Side AC in E. Then in Δ ABE we have $BA + AE$ [a] $\Gamma\ BE$, and to each of theſe adding EC, we find that $BA + AE + EC$ is $\Gamma\ BE + EC$, but $AE + EC = AC$, ∵ $BA + AC\ \Gamma\ BE + EC$. Again, [a] 70. in Δ EDC we know that $CE + ED$ [a] $\Gamma\ DC$, and to each of theſe adding DB, we find that $CE + ED + DB\ \Gamma\ DC + DB$; but $ED + DB = BE$ ∵ $CE + BE\ \Gamma\ DC + DB$. But it has been proved that $BA + AC\ \Gamma\ CE + BE$, much more then is it $\Gamma\ DC + DB$. Q. E. D.

[b] 77. 2. That the ∠BDC is Γ ∠A may be thus ſhewn. The exterior ∠BDC of the ΔCDE is [b] Γ the interior and oppoſite ∠CED; and the exterior ∠CEB of of the Δ ABE is Γ the interior and oppoſite ∠A. Hence we have proved that the ∠BDC is Γ ∠CED, (or, which is the ſame, CEB,) which is Γ ∠A; much more then muſt ∠BDC be Γ ∠A. Q. E. D.

Pl.2.F.1. 92. THEOREM 14. *If two Triangles* ABC, DEF, *have two Sides of the one equal to two Sides of the other, each to each,* viz. $AB = DE$, *and* $AC = DF$; *but the contained Angle of one greater than the contained Angle of the other,* viz. *the Angle* A *greater than the Angle* EDF; *that which has the greateſt Angle between the equal Sides will have the greateſt Baſe,* viz. BC *will be greater than* EF.

P.18. Plate I.


A D B C Fig. 1.
2
3
4
5
6
7
8
9
10
11
12
13
A B a C b D 14
A B D C 16
A B C 15
D E F
A D B C 17
E 18 A D B C
C 19 A E B D
E G A B 20 C H D F
A G B 21 E H F C I D K
A E 22 B C D
D E C F 23 A B
A 24 D B C
A 25 D B C
A E D 26 B C

Of the two Sides DE, DF, let DE be that which is not Γ the other; and let DG be a Line = DF, making an ∠ EDG = ∠ A; and let the Points F and G be joined. Then the two Sides AB, AC, of the △ ABC are reſpectively = the two Sides of the △ DEG, and ∠ BAC = ∠ EDG, by the Suppoſition; ∴ the Baſe BC [a] = Baſe EG. And, becauſe DG = DF, the ∠ DFG [b] = DGF; but the ∠ DGF Γ ∠ EGF, and ∴ ∠ DFG is Γ ∠ EGF, and much more ſhall the ∠ EFG be Γ ∠ EGF. And the ∠ EFG of the △ EFG, being Γ ∠ EGF, the Side EG is [c] Γ EF: But EG = BC, and ∴ BC muſt be Γ EF. Q. E. D. [a] 58. [b] 59. [c] 89.

93. Theorem 15. *If two Triangles* ABC, DEF, *have two Sides of one*, AB, AC, *equal to two Sides of the other*, DE, DF, *each to each*, viz. AB = DE, AC = DF; *but the Baſe of one*, BC, *greater than the Baſe of the other*; *the Angle contained by the equal Sides of that which has the greater Baſe, ſhall be greater than the Angle contained by the equal Sides of the other*; viz. *the Angle* A *greater than the Angle* D. Pl. 2. F. 2.

This is manifeſt from the preceding Article, being only the Reverſe of that; for if that which has the greateſt ∠ will have the greateſt Baſe, conſequently the greateſt Baſe muſt have the greateſt ∠. However, *Euclid* has thought proper to demonſtrate it in the following Manner, *viz.* If the ∠ A be not Γ ∠ D, it muſt be either = or Λ ∠ D: But it is not equal, for then the Baſe BC would be [a] = EF, but it is not; ∴ ∠ A is not = ∠ D. Nor can the ∠ A be Λ ∠ D; for then the Baſe BC would be Λ [b] EF, but it is not; ∴ ∠ A is not Λ ∠ D, and it has been ſhewn not to be = it; ∴ ∠ A muſt be Γ ∠ D. Q. E. D. [a] 58. [b] 92.

94. Theorem 16. *If two Triangles have two Angles of one equal to two Angles of the other, each to*

each, and one Side of one equal to one Side of the other, the Triangles are equal in all Reſpects.

The two ∠s of one being = two ∠s of the other, each to each, the remaining ∠ of one [a] = the remaining ∠ of the other; and ∵ the three Angles of one = the three ∠s of the other, each to each. Let ABC, DEF, be the two △s, in which Side BC = Side EF, ∠B = ∠E, ∠C = ∠F, and ∠A = ∠D. Then if the △DEF be ſuppoſed to be laid on △ABC, ſo that Side EF agrees with its equal BC; then, ſince ∠B = ∠E, the Side ED will coincide with the Side BA, and ∵ the Point D will be ſomewhere in Line BA; and ∠F being = ∠C, Side DF will coincide with CA, and ∵ Point D muſt be ſomewhere in Line CA; and conſequently, as the Point D is both in the Line BA and CA, it muſt be in the Point where they meet, *viz.* in the Point A, and ſo DE = BA, FD = CA, and the △s in every Reſpect = each other. Q. E. D.

[a] 80.

Pl. 2. Fig. 3.

Note. Euclid having demonſtrated this Propoſition, by proving the contrary to be abſurd, we imagined this would be more acceptable, as it is both direct and eaſy to be underſtood.

95. THEOREM 17. *If two Triangles have the three Sides of one equal to the three Sides of the other, each to each, the Angles of one will be equal to the Angles of the other, each to each: Or, the Triangles are equal in all Reſpects.*

Pl.2.F.3. For AB being = DE, and AC = DF, if the ∠s A, and D, be one r the other, the Baſes BC, EF, will be one [a] r the other; but they are equal by the Suppoſition: ∵ ∠A muſt be equal to ∠D, and ∵ the △s ABC, DEF, are [b] equal in all Reſpects; and conſequently the ∠A = ∠D, ∠B = ∠E, and ∠C = ∠F. Q. E. D. —— This Demonſtration is alſo different from *Euclid*'s, as are ſome others not taken Notice of.

[a] 92.

[b] 58.

96. THEOREM 18. *Two parallel Lines* AB, CD, *though indefinitely continued, would never meet; or, in other Words, they continue to be at the ſame Diſtance from each other.*

Let GF be a Perpendicular to the Line AB, and HE ⊥ to CD, and ſuppoſe the Points F, E, joined; then the △s EGF, FHE, have FE common, ∠ GEF =[a] ∠ EFH, and ∠s G, and H, each[b] Rt. ∠s; and ∵ the △s have 2 ∠s and one Side of one = 2 ∠s and one Side of the other, each to each; and ∵ the ſaid △s are[c] equal in all Reſpects, and conſequently the Side GF = Side EH. Q. E. D. Pl.2.F.4. [a] 72. [b] 14. [c] 94.

97. THEOREM 19. *The oppoſite Sides and Angles of* * *Parallelograms are equal to each other, and the Diameter (or Diagonal) biſects them, that is, divides them into two equal Parts.*

Let the Parallelogram ABCD be divided into two Parts by the Diagonal BD; then, becauſe AB is ∥ DC, the ∠ ABD[a] = BDC; and becauſe AD is ∥ BC, the alternate ∠s are equal, *viz.* ∠ ADB = DBC; ∵ the two △s ABD, CBD, have two ∠s ABD, ADB, in one, = two ∠s BDC, DBC, of the other, each to each, and the Side BD common to both △s; ∵ △s ABD, CDB are[b] equal in all Reſpects, (and conſequently the Parallelogram is divided into two equal Parts;) ∵ AB = DC, and BC = AD; ∠ A = ∠ C, and by adding equals, ∠ ABD + ∠ DBC = ∠ ADB + ∠ BDC, that is, ∠ ABC = ∠ ADC; but it has been juſt ſhewn that ∠ A = ∠ C, ∵ the oppoſite Sides, &c. as in the Theorem. Q. E. D. Pl.2.F.5. [a] 72. [b] 94.

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* A *Parallelogram* is a four-ſided Figure, whoſe oppoſite Sides are parallel. And the Diameter, or Diagonal, is the Line joining two of its oppoſite Angles. And the Altitude is the perpendicular Diſtance betwixt the two oppoſite Sides.

98. THEOREM 20. *All quadrilateral Figures, whoſe oppoſite Sides are equal, have their oppoſite Angles equal.*

Pl.2.F.5. Let the four-ſided Figure ABCD be divided into
two △s by the Diagonal BD; then AB being =
DC, BC = AD, and BD common to both △s,
the △s ABD, CDB, have the three Sides of
one = the three Sides of the other, each to each;
and ∵ the △s are in all Reſpects [a] = each other:
[a] 95. ∵ ∠A = ∠C; ∠ABD = CDB, ∠ADB = ∠
CBD, and conſequently, by adding Equals, ∠ABD
+ ∠CBD = ∠ADB + ∠CDB, that is, ∠ABC =
∠ADC, but ∠A has been juſt ſhewn to be = ∠C;
∵ the oppoſite ∠s are equal. Q. E. D.

99. THEOREM 21. *Parallelograms, or any other Figures between the ſame Parallels, have equal Altitudes.*

This plainly follows from Article 96; for by that Article the Altitude or ⊥ Diſtance between two || Lines, wherever taken, are equal.

100. THEOREM 22. *Rectangles contained under equal Sides are equal.*

Pl.2. F6. For ſuppoſe the Diagonals BD, FH, to be drawn:
Then, becauſe AD = EH, and AB = EF, by the
[a] 35. Suppoſition, and the Angles A and E both [a] right
∠s; in the △s ABD, EFH, are two Sides, AD,
AB, and ∠A of one = two Sides EH, EF, and ∠
E of the other, each to each, and ∵ the △s ABD,
[b] 58. EFH, are [b] equal in all Reſpects. In the ſame
Manner it may be ſhewn that the △s BDC, FGH,
are equal; and conſequently the whole Rectangle
ABCD = the whole Rectangle EFGH. Q. E. D.

101. COROLLARY. *Hence, if two Squares have the Side of one equal to the Side of the other, the Squares are equal to each other.*

102. THEOREM 23. *Parallelograms upon the ſame Baſe and between the ſame Parallels are equal.*

If the Sides AD, DF, of the Parallelograms ABCD, DBCF, be terminated in one and the ſame Point D, it is plain that each of the Parallelograms is [a] double of the △BDC, and conſequently they are [b] equal to each other. Q. E. D.

Pl. 2. F. 7.
[a] 97.
[b] 50.

But, if the Sides AD, EF, oppoſite to the Baſe BC of the Parallelograms ABCD, EBCF, be not terminated in the ſame Point, then, becauſe ABCD is a Parallelogram, AD is [c] = BC; by the ſame Reaſon EF = BC; ∴ AD [d] = EF, and ∴ by adding for Fig. 8. and ſubtracting for Fig. 9. DE, which is common, we have (AD + DE [e] = EF + DE for Fig. 8. and AD — ED [f] = EF — ED for Fig. 9. that is,) AE = DF; AB is alſo [g] = DC, and ∠A = ∠FDC, from the Nature of [h] parallel Lines; ∴ △s ABE, DCF, have two Sides of one AB, AE, and contained ∠A = two Sides DC, DF, and contained ∠FDC of the other, and ∴ the ſaid △s are [i] equal in all Reſpects; ∴ if each △ be taken from the whole Figure ABCF, there will remain ABCD [k] = EBCF. Q. E. D.

[c] 97.
[d] 45.
[e] 46.
[f] 47.
[g] 97.
[h] 40.
[i] 58.
[k] 47.

103. COROLLARY 1. *Hence Parallelograms having the ſame Baſe and equal Altitudes, are equal.* See Art. 99.

104. COROLLARY 2. *Every Parallelogram is equal to a Rectangle having the ſame Baſe and equal Altitudes.*

105. Coroll. 3. *Hence Parallelograms having equal Baſes and equal Altitudes, are equal.*

106. Theorem 24. *If a Triangle and Parallelogram ſtand upon the ſame Baſe, and are between the ſame Parallels, the Triangle is half of the Parallelogram.*

Pl.2.F.5. For let BDC be the $\triangle$, and let AB be $\parallel$ and $=$
[a] 97. DC, and AD $\parallel$ and $=$ BC, then is ABCD a [a] Pa-
rallelogram, on the ſame Baſe and betwixt the ſame
Parallels. Now, one Side BD of the $\triangle$BDC is a
Diagonal to the Parallelogram ABCD, and ∵ the
$\triangle$ BDC $= \frac{1}{2}$ the Parallelogram ABCD: But the
[b] 102. Parallelogram ABCD is [b] $=$ any other Parallelogram
upon the ſame Baſe and between the ſame Parallels,
∵ the $\triangle$BDC [c] $= \frac{1}{2}$ of any Parallelogram on the
[c] 45. ſame Baſe and between the ſame Parallels. Q. E. D.

107. Corollary 1. *Hence, if a Parallelogram and Triangle have equal Baſes, and are between the ſame Parallels, or, which is the ſame* [a] *Thing, have*
[a] 99. *equal Altitudes, the Triangle is Half of the Parallelogram.*

108. Coroll. 2. *Hence, if a Triangle and Rectangle have equal Baſes and Altitudes, the Triangle is Half the Rectangle.*

109. Coroll. 3. *Hence, if two Triangles have equal Baſes and Altitudes, they are equal to each other.*

For the Triangles being the Halves of Parallelograms of equal Baſes and Altitudes, which Parallelograms are equal, it follows, that if the Wholes (*viz.* the Parallelograms) are equal, their Halves (*viz.* the $\triangle$s) muſt be equal alſo.

110. Scholium 1. On theſe two laſt Theorems and their Corollaries are grounded the Methods of meaſuring, or finding the Areas, or ſuperficial Contents, of Parallelograms and Triangles. But in Order to this it muſt be firſt ſhewn how to find the Area of a Rectangle; which may be thus illuſtrated. Let AB Pl.2,F.10. be a Line which we ſuppoſe to be moved towards DC, ſo as to continue always || to its firſt Poſition, *viz.* let the Point A be moved in the ⊥ Line AD, and the Point B in the ⊥ Line BC; then it is manifeſt by ſuch Motion it will deſcribe a Rectangle. And if we imagine the Line AB to be divided into any Number of equal Parts, when the Line is gone in Length one ſuch Part towards D, *viz.* got into the Poſition *ab*, it will have deſcribed a Rectangle A*ab*B, containing as many little Squares as AB was divided into Parts; and when it gets into the Poſition *dc*, two ſuch Parts diſtant from its firſt Poſition, it will have deſcribed a Rectangle containing twice as many Squares as AB contains Parts: And ſo when it arrives at DC, it will have deſcribed the whole Rectangle ADCB = as many little Squares (each Side = one Part of AB) as there are Units in the Product of the Number of Parts in AB multiplied by the Number of Parts in BC: And this is called *Meaſuring*, or finding the *Area* of ſuch Figure. Hence appears the Reaſon why the *Product* of any two Numbers is by *Geometricians* called the *Rectangle.*

Scholium 2. Hence may be ſeen the Reaſon why $3 \times 7 = 7 \times 3$, or generally $A \times B = B \times A$; which is demonſtrated algebraically in our *Mathematical Eſſays.*

111. Hence, ſince any Parallelogram is equal to a Rectangle of equal Baſe and Altitude, *to meaſure any Parallelogram we have this Rule. Multiply the Number of Parts into which the Baſe is ſuppoſed to be*

divided, by the Number of like Parts contained in its Altitude, and the Product will be the Area required: Thus, for Example. If the Baſe of a Parallelogram be 6 Feet, and Altitude 2 Feet, then its Area is = 6 × 2 = 12 ſquare Feet.

112. As to *Triangles*, every Triangle being half of a Parallelogram of equal Baſe and Altitude, Nothing further need be ſaid.

113. THEOREM 25. *The Complements of Parallelograms which are about the Diagonal of any Parallelogram, are equal to each other.*

Pl.2 F.11. Let ABCD be a Parallelogram whoſe Diagonal is AC; and EH, FG, the Parallelograms about AC, that is, thrô which AC paſſes; and BK, KD, the other Parallelograms, which make up the whole Figure, and are therefore called *Complements.* Then will the Complement BK be = the Complement KD. For, becauſe ABCD is a Parallelogram, and AC
[a] 97. its Diagonal, the △ABC [a] = △ADC; and becauſe EKHA is a Parallelogram whoſe Diagonal is AK, the △AEK = △AHK; and for the ſame Reaſon △KGC = △KFC; ∵ if from the △ABC we take away the △s AEK and KGC, and from the △ADC take away the △s AHK and KFC, the
[b] 47. remaining Quantities muſt be [b] equal, *viz.* the Parallelogram BK = the Parallelogram KD. Q. E. D.

Pl.2.F.12. 114. THEOREM 26. *The Diagonals* AC, DB, *of any Parallelogram biſect each other.*

For let the Point of their Interſection be denoted
[a] 97. by E. Then, becauſe AD is [a] =, and ‖ BC, and
[b] 72. the alternate ∠s [b] equal, *viz.* ∠ACB = ∠DAC, and ∠CBD = ADB, the △s AED, BEC, have two ∠s DAE, ADE, and one Side AD of one = two ∠s ECB, EBC, and one Side BC of the other,
[c] 94. each to each; ∵ the △s AED, BEC, are [c] equal in

in all Reſpects, and conſequently the Side AE = Side EC, and Side DE = Side BE, that is, the Diagonals divide each other into two equal Parts. Q. E. D.

115. THEOREM 27. *In any right angled Triangle the Square upon the Side, ſubtending the right Angle, is equal to both the Squares upon the Sides containing the right Angle.*

Let ABC be the △. If we ſuppoſe the Line CA Pl. 2. F. 13.
produced to G, ſince ∠ CAB is a Rt. ∠, the ∠
BAG will be ſo [a] too, and ∵ one Side of the Square [a] 65.
upon BA will fall on the Line [b] AG, and ſo with the
Line CA will make one Line CG. For the ſame
Reaſon one Side of the Square on CA will with BA
make one Line BH. Now, ſuppoſe the Squares de-
ſcribed, and the Points F, C; B, K; A, D; A, E; [b] 34.
joined by Lines; alſo AL ∥ BD. Then becauſe the
∠ DBC = ∠ FBA, each being a right [b] one, ∵
adding the common ∠ ABC, we have the whole
∠ DBA [c] = the whole ∠ FBC; and the Side FB [b] = [c] 46.
BA, and BD = DC: Hence the △s FBC, DBA,
have two Sides of one, FB, BC, and contained ∠
FBC = two Sides BA, BD, and contained ∠ DBA
of the other, each to each, and ∵ △s FBC, DBA
are [d] = each other in all Reſpects: But FB, GC, [d] 58.
are ∥ by the Nature of Squares (for the ∠ FBA =
GAB, being each a [b] Rt. ∠, and ∵ FB [e] ∥ GC;) [e] 40.
∵ the Square BG and △ FBC are on the ſame Baſe,
and between the ſame Parallels, and ∵ the Square
BG [f] = twice the △ FBC; and BD, AL, being [f] 106.
Parallels, the Parallelogram BL = twice the △ DBA;
but the △s FBC, DBA, have been juſt ſhewn to be
= each other, conſequently the Square BG [g] = Pa- [g] 45.
rallelogram BL. After the ſame Manner it may be
ſhewn that the Square CH = the Parallelogram CL;
∵ the Square BG + the Square CH [c] = (the Pa- [c] 46.
rallelogram BL + the Parallelogram CL, that is =)
the Square BE. Q. E. D.

116. Scholium. This is one of the moſt extenſive and valuable Theorems in Geometry. —— For one Example of its Uſe ſee 1ſt. of the *Mathematical Eſſays*, Article 462.——We ſhall give another, and eaſier Demonſtration after we have treated of *ſimilar Triangles*; when it will alſo be ſhewn to be only a particular Caſe of a more general Theorem.

BOOK II.

Definitions.

117. *Def.* 1. EVery right-angled Parallelogram is ſaid to be contained by any two of the Lines which contain one of the right Angles.

118. *Def.* 2. In every Parallelogram either of the Parallelograms about the Diameter, together with
Pl.2.F.11. the two Complements, is called the *Gnomon*. Thus the Parallelogram EH, together with the Complements BK, KD, is the Gnomon, which is more compendiouſly expreſſed by the Letters BHF, or GED, which are at the oppoſite Angles of the Parallelograms which make the Gnomon.

Pl.2.F.14. 119. Theorem 1. *If a Line* AB *be divided into any two Parts in* C, *the Square of the whole Line* AB, viz. ABED, *is equal to the Squares of the two Parts* AC, CB, *together with twice the Rectangle contained by the Parts* AC, CB.[*]

Let

[*] It may be demonſtrated algebraically thus. Let $r =$ AC, $x =$ CB, then $r + x =$ AB $=$ AD, and ∵ the Area of the
* 111. Square ABED* $= \overline{r + x} \times \overline{r + x} =$ (by 1ſt. of the *Mathemat. Eſſays*, Art. 455.) $r^2 + 2rx + x^2$, which is the ſame as the Theorem. Q. E. D. —— Hence, if $x = r$, by writing r for $x = rr + 2rr + rr = 4rr$, the Corollary.

Let CF be ∥ AD or BE, and let AH be = CB;
and let HK be a ∥ to AB or DE. Then, becauſe
AB [a] = AD, by taking away equals CB, AH, [a] 34.
there remains AC [b] = DH. Becauſe HG, DF are [b] 47.
Parallels, alſo HD ∥ GF, the oppoſite ∠s of the
Figure HGFD are [c] equal to each other; but the [c] 98.
∠D is a Rt. ∠ ∵ the other 3 are Rt. ∠s, and ſo
HGFD is a [a] Square each Side = AC. By the ſame [a] 34.
Reaſon CBKG is a Square each Side = CB; and by
the like Method of Reaſoning ACGH and KEFG are
Rectangles whoſe Sides are = AC and CB. But the
Squares DG, GB, and two Rectangles AG, GE,
make up the whole Square ABED. Q. E. D.

120. COROLLARY 1. *Hence the Square of any Line is equal to four Times the Square of half that Line.*

121. THEOREM 2. *A Rectangle made by the Sum and Difference of two right Lines is equal to the Difference of the Squares of the ſaid Lines.**

Let AB, AC, repreſent the two Lines, and AK,
AG, their Squares. Let HG be produced to HF, Pl. 2. Fig. 15.
till GF = AB, or, which is the ſame, till HF =
AC + AB. Let FE be ∥ to AH, and let IK be
produced to meet FE in E. Then, becauſe HF =
AC + AB, the Sum of the Lines, and IK = AH
— AI [a] = AC — AB = the Difference of the Lines, [a] 47.
'tis plain the Rectangle HE is contained under the
Sum HF, and Difference IK, of the Lines AC,
AB,

---

* This Theorem may be eaſily demonſtrated algebraically. Let the greater Line be denoted by a, the leſſer by m, then $a+m$ = their Sum, and $a-m$ = their Difference; then their Rectangle is per Art. 111 = $\overline{a+m}\times\overline{a-m}$ = by the Operation in the Margin, a^2-m^2. the ſame as affirmed in the Theorem. Q. E. D.

$$\begin{array}{l} a+m \\ a-m \\ \hline a^2+am \\ \quad -am+m^2 \\ \hline a^2 \quad - \quad m^2 \end{array}$$

AB, which we are now to ſhew to be $=$ the Gnomon HDB, the Difference of the Squares AK, AG; which may be eaſily done, for GF being $=$ AB by
[b] 34. the Suppoſition, $=$ [b] BK, and DG ($=$ CG $-$ CD
[a] 47. $=$ [a] AC $-$ AB) $=$ BC, the Rectangles BD, DF,
[c] 100. are contained under equal Sides, and are ∴ [c] equal to each other; ∴ to each adding the Rectangle HD
[d] 46. common, we have BD $+$ DH $=$ [d] DF $+$ HD, that is, the Gnomon HD $=$ the Rectangle HE. Q. E. D.

122. Theſe two Theorems are ſufficient for our preſent Purpoſe; we ſhall therefore paſs on to the third Book. —— We intend hereafter to give algebraic Demonſtrations to all the Theorems of this Book.

BOOK III.

123. *Def.* 1. EQUAL Circles are thoſe whoſe Diameters are equal, or from whoſe Centers the Lines to the Circumferences are equal: Or, in other Words, they are thoſe that have equal Radii.

124. *Scholium.* This is not, ſtrictly ſpeaking, a Definition, but a Theorem, whoſe Truth is manifeſt; for if the Circles are laid on one another ſo that their Centers may coincide, the Circumferences will alſo coincide, as the Radii are equal, and ∴ the Circles will exactly cover, and be ∴ $=$ each other.

125. *Definition* 2. A Line is ſaid to touch a Circle when it meets the Circle in one Point, and being produced does not touch it in any other Point: And ſuch Line is called a Tangent to the Circle.

126. *Definition* 3. Circles are ſaid to touch one another, which meet, but do not cut each other.

127. *Definition* 4. Lines are ſaid to be equally diſtant from the Center of a Circle, when the Perpendiculars drawn to them from the Center are equal.

128. *Def.* 5. But the Line on which the greater Perpendicular falls, is ſaid to be further from the Center.

129. *Def.* 6. A Segment of a Circle is the Figure contained by a Line and the Circumference it cuts off, (ſee Pl. 2. F. 16.) and the Line is called the Chord.

130. *Def.* 7. The Angle in a Segment is that which is contained by two Lines drawn from any Point in the Circumference (or Arch) of the Segment to the Extremities of the Chord Line, or Baſe of the Segment. See Pl. 2. F. 17.

131. *Def.* 8. And an Angle is ſaid to ſtand upon the Circumference which the Lines that contain the Angle intercept between them.

132. *Def.* 9. The Sector of a Circle is the Figure contained by two Lines drawn from the Center, and the Circumference between them. See Pl. 2. Fig. 18.

133. *Def.* 10. Similar Segments are thoſe in which the Angles are equal, or which contain equal Angles.

134. THEOREM 1. *If a Line* CD, *drawn through the Center* E *of a Circle, biſect any Chord* AB, *it will cut it at right Angles. And if the Diameter* CD *is at right Angles to the Chord* AB, *it will biſect it.* Pl. 2. F. 19.

For firſt, Let EA, EB, be joined; then, becauſe
a 18. AF = FB by the Suppoſition, and AE [a] = EB being
Radii, and FE common, the △s AEF, BEF, have
three Sides of one = the three Sides of the other,
b 95. each to each, and ∵ they are [b] equal in all Reſpects;
conſequently the ∠ AFE = ∠ BFE, and ∵ each =
c 14. [c] a Rt. ∠. Q. E. D.

Secondly. But if CD is at Rt. ∠s to AB, then,
d 59. ſince AE = EB, the ∠ EAF [d] = ∠ EBF; but ∠
AFE = ∠ BFE, each being a Rt. ∠ per Art. 14.
Hence, in the △s AFE, BFE, are two ∠s and
one Side of one = two ∠s and one Side of the
e 94. other, each to each, and ∵ the △s are [e] equal in all
Reſpects: Conſequently, their Baſes FA, FB, are
equal to each other. Q. E. D.

Pl. 2. F. 20. * 134. THEOREM 2. *If any Point* F, *which is not the Center, be taken in the Diameter* AD *of the Circle* ABCD, *whoſe Center is* E; *then, of all the Lines* FB, FC, FG, &c. *that can be drawn from* F *to the Circumference, that* FA, *in which the Center is, is the longeſt, and the other Part* FD *of that Diameter* AD *is the leaſt; and of the others, that which is nearer to the Line* FA *is longer than one more diſtant; thus,* FB *greater than* FC, *and* FC *greater than* FG. *And from the ſame Point* F *there can be drawn to the Circumference only two Lines equal to each other,* viz. *one on one Side and the other on the other Side of the Diameter* AD, *making equal ∠s with it.*

For 1ſt. Let BE, CE, GE, be joined; then be-
a 70. cauſe two Sides of any △ are [a] Γ the third, BE +
b 18. EF Γ BF; but BE = [b] AE, ∵ AE + EF (=
BE + EF, and ∵) Γ BF. Again, BE being [b] =
CE, EF, common, and ∠ BEF Γ ∠ CEF, the Baſe
c 92. BF of the △ BEF is [c] Γ the Baſe CF of the △ CEF;
and for the ſame Reaſon the Baſe CF Γ Baſe GF.
Again, becauſe GF + FE is [a] Γ GE, and GE =
ED,

ED, GF + FE muſt be Γ ED, and taking away the Part FE common, GF muſt be Γ FD. Hence we have proved that AF Γ BF, BF Γ CF, CF Γ GF, and GF Γ FD: Which were to be demonſtrated.
Secondly. Let DFH be an ∠ = GED, then we affirm the Line FH is = FG, and that no other can be equal to it; for that FH = FG may be thus
ſhewn. Becauſe GE = [b] EH, EF common, and [b] 18.
∠ GED = DEF by the Suppoſition, the Baſe
GF [d] = Baſe FH. Q. E. D. And that no other [d] 58.
Line can be = FG, we thus prove. For if poſſible, let FK be a Line = FG; and FH has been juſt
ſhewn to be = FG, and ∴ FK would be [e] = FH, [e] 45.
that is, the two Lines FK, FH, = each other, though one is more remote from the Line which paſſes through the Center than the other; which is contrary to what has been proved above, and ∴ FK is not = FH, and ∴ cannot be = to FG. Q. E. D.

135. Theorem 3. *Let* ABC *be a Circle, and* D *any Point without it, from which Lines are drawn to the Circumference. Then we affirm, that of theſe which fall upon the concave Part of the Circumference* AEFC, *the greateſt ſhall be* DA, *which paſſes thro' the Center* M; *and the nearer to it greater than the more remote*, viz. DE Γ DF, *and* DF Γ DC. *But of thoſe which fall on the convex Part of the Circumference, the leaſt is* DG, *which coincides with the Diameter* AG *produced; and the nearer to it is always leſſer than the more remote*, viz. DK Λ DL, *and* DL Λ DH. *And only two equal Lines can be drawn from the Point* D *to the Circumference, one upon each Side of the leaſt.* Pl. III. F. 1.

For, 1ſt. Let the Points ME, MF, MC, MK,
ML, MH, be joined; then AM [a] = EM, ∴ ad- [a] 18.
ding MD common, EM + MD ([b] = AM + [b] 46.
MD) = AD; but EM + MD [c] Γ DE, ∴ alſo [c] 70.
DA Γ DE. Again, becauſe ∠ EMD Γ ∠ FMD,

a 18. EM [a] = FM, and MD common to both Δs, d 58. EMD, FMD, the Baſe DE [d] Γ Baſe DF. By the ſame Method of Reaſoning DF Γ DC. Hence we have ſhewn that DA Γ DE, DE Γ DF, and DF Γ DC. Q. E. D.

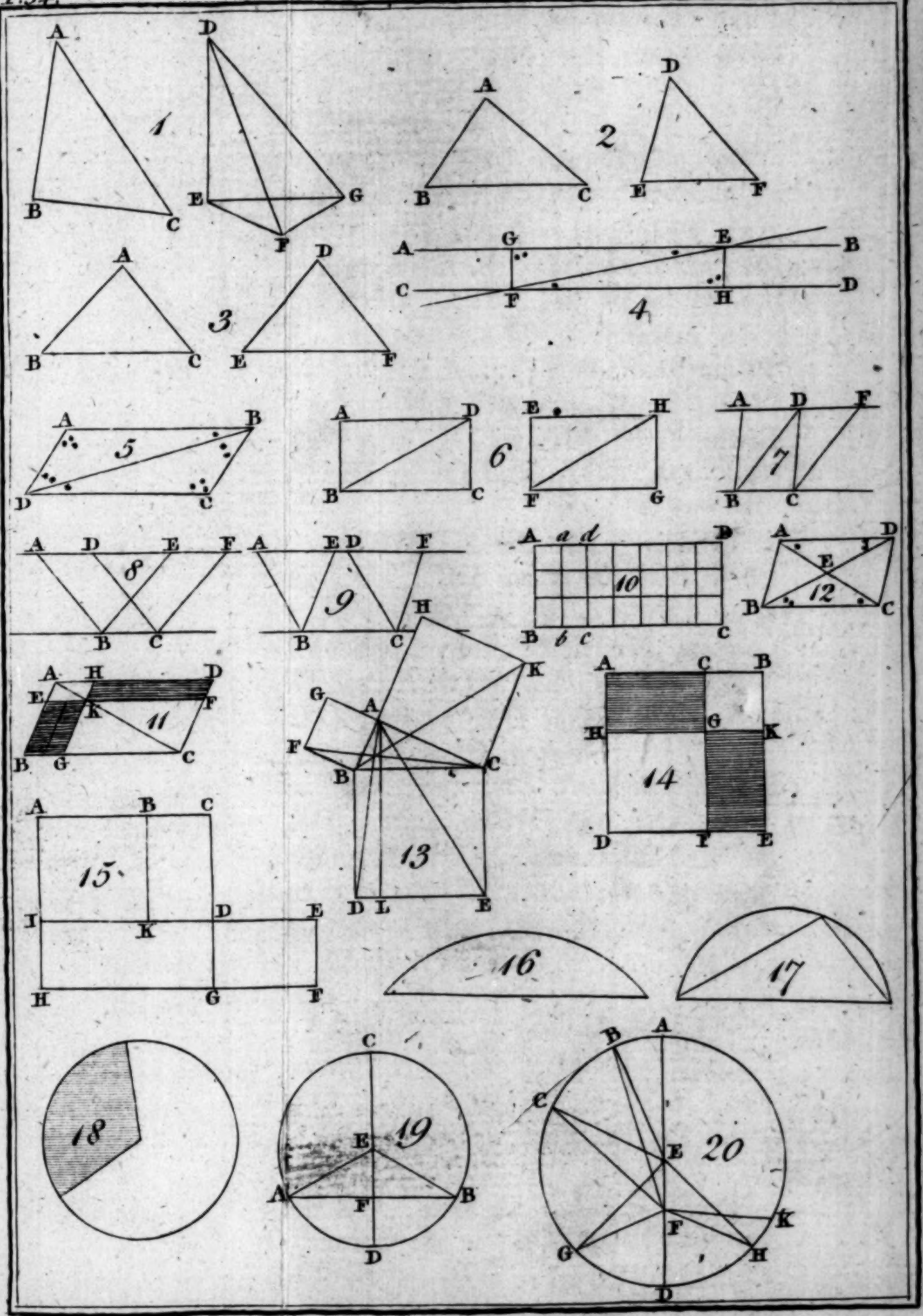
c 70. Second. Becauſe MK + KD [c] Γ MD, taking away a 18. equals MK, [a] MG, there will remain KD Γ DG. And becauſe the Δ MKD is within the Δ MLD, the two Sides MK, KD, taken together muſt be Λ the two Sides ML, LD, which include them, (for it is manifeſt a Thing containing muſt be Γ the Thing contained ;) and taking away equals MK, [a] ML, the Remainder KD Λ the Remainder LD. In the ſame Manner DL is Λ DH ; ∵ DG is the leaſt, DK Λ DL, and DL Λ DH. Q. E. D.

Thirdly. Let the ∠DMB = ∠DMK ; then we affirm that the Line DB = DK, and that no other Line DN can be = DK : For, that DB = DK, a 18. may be thus demonſtrated. Becauſe MK [a] = MB, MD common, and ∠DMB = ∠DMK by the Sup- d 58. poſition, the Baſe DB [d] = Baſe DK. Q. E. D.

And that no other Line DN can be = DK, may be thus ſhewn : DK is = DB, as has been juſt proved ; ∵ if DN be alſo = DK, then will DN e 45. be [e] = DB, that is, the nearer to the leaſt = the more remote, which is contrary to what has been above demonſtrated, and conſequently impoſſible : ∵ DN is not = DB. Q. E. D.

Pl.3.F.2. 136. THEOREM 4. *If a Point* D *be taken within a Circle, from which there fall more than two equal Lines to the Circumference*, viz. DA, DB, DC, *that Point* D *is the Center of the Circle.*

For if it is poſſible that D is not the Center, let any other Point E be the Center, in the Diameter b 134. FDEG. Then, if D be not the Center, DG is [a] the greateſt Line from it to the Circumference, and DC Γ DB, P.34. Plate II.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Γ DB, and DB Γ DA; but they are alſo equal by the Suppoſition, which is impoſſible: ∵ E is not the Center of the Circle ABC, and in the ſame Manner it may be demonſtrated that no other Point but D is the Center. Q. E. D.

137. THEOREM 5. *One Circle cannot cut another in more than two Points.*

For if it be poſſible, let the Circle ABC cut the Circle DEF in more than two Points, *viz.* in B, G, F; and let K be the Center of the Circle ABC, and BF, GK, FK, be joined; then, becauſe within the Circle DEF there is taken the Point K, from which to the Circumference DEF fall more than two equal Lines, KB, KG, KF, the Point K muſt be the [a] Center of the Circle DEF; but it is alſo the Center of the Circle ABC, and ſo the two Circles ABC, DEF, would have the ſame common Center and Radii, and conſequently cannot cut each other, but would coincide, and ∵ properly ſpeaking, would be but one Circle: ∵ one Circle cannot cut another in three Points. Q. E. D. Pl. III. F. 3. [a] 136.

138. A LEMMA, or THEOREM 6. *If two right-angled Triangles have two Sides of one,* AC, CB, *equal to two Sides of the other,* DF, FE, *each to each* (viz. AC = DF, CB = FE;) *the remaining Side of one* AB, *will be* = *the remaining Side of the other* DE. Pl. III. F. 4.

For $AB^2 + BC^2$ [a] $= AC^2$, and $DE^2 + EF^2 = DF^2$: But AC = DF by the Suppoſition, ∵ $AC^2 = DF^2$, ∵ $AB^2 + BC^2$ [b] $= DE^2 + EF^2$; but BC = EF by the Suppoſition, and ſo $BC^2 = EF^2$, ∵ taking away theſe equals, there remains AB^2 [c] $= DE^2$, and conſequently AB = DE. Q. E. D.—— This Demonſtration proves the Theorem to be true, when the Side oppoſite to the Rt. ∠ and one of the other Sides of one, are = the Side oppoſite to the Rt. [a] 115. [b] 45. [c] 47.

∠ and one of the other Sides of the other △. But it is equally true when the two Sides containing the Rt. ∠ in one = the two Sides containing the Rt. ∠ in the other: for it then becomes the ſame as Article 58.

Pl. III. F. 5. *138. Theorem 7. *Equal Lines*, AB, CD, *in the Circle* ABDC, *are equally diſtant from the Center* E; *and thoſe* AB, CD, *equally diſtant from the Center, are equal to each other.*

First, that if AB = CD, they are equidiſtant
from the Center, may be thus demonſtrated. Let
EF be ⊥ to AB, EG ⊥ to CD, and join AE and
CE. Then the ∠s AFE and CGE being Rt. ∠s,
a 31. the △s AFE, CGE, are [a] Rt. ∠ △s; and AB,
b 134. CD, are [b] biſected by the Lines EF, EG; and ∵
as their Wholes are equal, their Halves muſt be
equal alſo, that is, AF = CG. Hence, in the two
Rt. ∠ △s AFE, CGE, are two Sides of one AE,
AF, reſpectively = two Sides of the other CE, CG,
(for AE, CE, are Radii of the ſame Circle and ∵
c 138. equal,) the remaining Side FE of one is [c] = the remaining
Side GE of the other; which is all that is
meant by ſaying, the Lines AB, CD, are equidiſtant
from the Center. Q. E. D.

Secondly. If AB, CD, are equidiſtant from the
Center, it may be eaſily demonſtrated that they are
= each other. For AE being = CE, and EF =
EG, by the Suppoſition, the Rt. ∠ △s AFE,
CGE, have two Sides of one AE, EF, = two Sides
c 138. of the other, CE, EG, and ∵ AF [c] = CG; but
b 134. AF, CG, are the [b] Halves of their reſpective
Wholes AB, CD; and as the Halves AF, CG, are
proved to be equal to each other, their Wholes muſt
be ſo too; that is, AB = CD. Q. E. D.

Pl. 3. F. 6. 139. Theorem 8. *The Diameter* AD *is the greateſt Line in a Circle* ABCD; *and of any others* BC, FG, *that*

that BC *which is nearer to the Center* E, *is always greater than that more remote* FG.

Let EI, EH, be the Diſtances of BC, FG, from the Center E; and let BE, CE, FE, GE, be joined; then, it is manifeſt that AE being[a] = BE, ED = EC, AD is = BE + CE; but BE + EC[b] Γ BC, ∵ AD Γ BC. Again, the △s BEC, FEG, have two Sides BE, EC, of one = two Sides FE, GE, of the other; but the contained ∠ of one BEC Γ the contained ∠ of the other FEC, ∵ BC[c] Γ FG. Hence we have proved that AB Γ BC, and BC Γ FG; conſequently Γ any other Line at the ſame Diſtance from the Center (becauſe Lines at the ſame Diſtance from the Center are[d] = each other.) Q. E D.

[a] 18. [b] 70. [c] 92. [d] 138.

140. *Hence a Line greater than the Diameter, drawn from any Point within the Circle, muſt cut the Circumference; and ∵ any Line within a Circle, being produced both Ways indefinitely, muſt cut the Circumference.*

141. THEOREM 9. *A Line* ED *meeting the Circle* ACG *in the Point* A, *at right Angles to the Radius* BA, *is a Tangent to the Circle at the Point* A. *And no Line* AK *can be drawn from the ſaid Point* A *between the Line* ED *and the Circumference, which does not cut the Circle; or, which is the ſame Thing, no Line* AK *can make ſo great an acute Angle* BAK *with the Diameter* CA, *or ſo ſmall an Angle* FAK *with the Line* AD, *but that it ſhall cut the Circle.* Pl.3.F.7.

Firſt. That ED is a Tangent, may be thus demonſtrated. Let any Line BF be drawn; then the △BAF being a Rt. ∠ △, BF^2 [a] = $BA^2 + AF^2$, ∵ BF^2 Γ BA^2 ∵ BF Γ BA the Radius, wherever the Point F is taken in the Line ED, and ∵ the Point F will fall without the Circle, as BF has been juſt ſhewn to be Γ the Radius BA, and conſequently Γ its[b] equal the Radius BL. ∵ ED touches

[a] 115. [b] 18.

touches the Circle in no Point but A, and ∴ it is
[c] 125. a [c] Tangent to the Circle at the Point A. Q. E. D.

2dly. If poſſible, let the Line AK be drawn from A, between the Line ED and the Circumference; and let H be a Point taken in AK, ſo that BH may be at Rt. ∠s to AK, and let G be the Point where the Line GH meets the Circumference.
Then, becauſe BHA is a Rt. ∠ △, Rt. ∠d at H,
[a] 82. the ∠BHA [a] = ∠BAH + ∠ABH, the ∠ BHA
[b] 89. [c] 18. Γ BAH, and ∴ Side BA [b] Γ Side BH; but BA [c] =
BG, ∴ BG Γ BH, the leſs Γ the greater, which is abſurd; ∴ no Line AK can be drawn from the Point A, between ED and the Circumference, which does not cut the Circle; or, in other Words, "no Line AK can make ſo great an acute ∠BAK with the Diameter CA, or ſo ſmall an Angle FAK with the Line AD, but that the Circumference ſhall paſs between the Tangent AD and Line AK." Q. E. D. And this is all that *Euclid* muſt be underſtood to mean when he ſays, (in the *Greek* Text, and alſo in moſt Tranſlations, that) the ∠ of the Semicircle is Γ any rectilineal ∠, and the remaining ∠ Λ any rectilineal ∠.

142. COROLLARY 1. *Hence if a Line touches a Circle in any Point, a right Line drawn from the Center to that Point will be perpendicular to the touching Line.*

For if that Line be not perpendicular to the Radius at the Point of Contact, it will cut the Circle by this Propoſition.

143. COROLL. 2. *If a Line touches a Circle, and from the Point of Contact a Line be drawn perpendicular to the touching Line, the Center of the Circle ſhall be in that Line.*

144.

144. *Scholium.* From the Manner in which this Theorem is worded in ſeveral Copies of *Euclid*, many ſtrange Paradoxes and Abſurdities have been deduced by ſome modern Mathematicians, which would not have been had they rightly conſidered what *Euclid* muſt mean, and not what the Words ſeem to import.

145. THEOREM 10. *The Angle* BEC *at the Center of a Circle, is double of the Angle* BAC *at the Circumference, upon the ſame Baſe; that is, upon the ſame Part of the Circumference* BC. Pl. III. Fig. 8 & 9.

Becauſe EA[a] $= EB$, the Angle EAB[b] $= \angle$ [a] 18.
EBA, $\because \angle EAB + \angle EBA = 2\angle FAB$; but $\angle$ [b] 59.
FEB[c] $= \angle EAB + \angle EBA$, $\because \angle FEB$[d] $= 2\angle$ [c] 76.
FAB. After the ſame Manner may be ſhewn that [d] 45.
the $\angle FEC = 2\angle FAC$; hence, adding in Figure
8th, we have $\angle FEB + \angle FEC$[e] $= 2\angle FAB + 2\angle$ [e] 46.
FAC, that is, $\angle BEC = 2\angle BAC$. Q. E. D. Or
ſubtracting in Figure 9th, we find $\angle FEC -$
$\angle FEB$[f] $= 2\angle FAC - 2\angle FAB$, that is, $=$ [f] 47.
$\overline{\angle FAC - \angle FAB} \times 2 = 2\angle BAC$. Q. E. D.

146. THEOREM 11. *The Angles in the ſame Segment of a Circle are equal to one another.*

Firſt then, let ABCDE be a Circle, and F the Pl. III.
Center; then, if the Segment BAED be Γ a Semi- F. 10.
circle, as in Figure 10th. the $\angle$s BAD, BED,
BFD, ſtand upon the ſame Baſe BD, $\because$ the $\angle$
BAD[a] $= \frac{1}{2}\angle BFD$, and $\angle BED = \frac{1}{2}\angle BFD$; $\because$ [a] 145.
$\angle BAD$[b] $= BED$, wherever the Points A, E, are [b] 45.
taken in the Circumference BAED. Q. E. D.

Secondly. But if the Segment BAED be Λ a Semicircle, let F be the Center, and ſuppoſe Lines drawn as repreſented in the Figure. Then the Segment Fig. 11.

ment BAEC being Γ a Semicircle, the ∠s in it upon
the ſame Baſe BC are by the firſt Caſe equal to each
other, that is, ∠ BAC = ∠BEC ; and by the
ſame Reaſon the ∠s in the Segment CAED on the
ſame Baſe CD are = each other, *viz.* ∠CAD =
∠CED ; ∵ by adding equals, ∠BAC + ∠CAD
[c] 46. [c] = ∠BEC + ∠CED, that is, ∠BAD = ∠BED.
Q. E. D.

Pl. III. F. 12. 147. THEOREM 12. *The oppoſite Angles of any quadrilateral Figure,* ABCD, *deſcribed in a Circle, are together equal to two right Angles.*

[a] 146. Join AC, BD ; then the ∠ CAB [a] = ∠CDB,
being in the ſame Segment BADC, and upon the
ſame Baſe BC ; and the ∠ACB = ∠ADB, being
in the ſame Segment ADCB, and upon the ſame
Baſe AB. ∵ adding equals, the ∠ADB + ∠
[b] 46. CDB [b] = ∠ ACB + ∠CAB, that is, ∠ADC =
∠ACB + ∠CAB ; to each of theſe equals adding
the ∠ABC, we have ∠ADC + ∠ABC = ∠ACB
+∠CAB + ∠ABC ; but the ∠ACB + ∠CAB +
[c] 78. ∠ABC [c] = 2 Rt. ∠s, ∵ ∠ADC + ∠ABC [d] =
[d] 45. 2 Rt. ∠s. In the ſame Manner it may be ſhewn
that the ∠BAD + ∠BCD = 2 Rt. ∠s. Q. E. D.

Pl. III. F. 13. 148. THEOREM 13. *The Angle in a Semicircle* BAC *is a right Angle ; but the Angle in a Segment* ABC, *greater than a Semicircle, is leſs than a right Angle ; and the Angle in a Segment* ADC, *leſs than a Semicircle, is greater than a right Angle.*

Let E be the Center, and Lines be drawn as in
[a] 18. the Figure. Then, AE being [a] = EB, the ∠EAB
[b] 59. [b] = ∠EBA ; and EA being = EC, the ∠EAC =
∠ECA. ∵ adding equals, ∠ EAB + ∠ EAC
[c] 46. [c] = ∠ EBA + ∠ ECA, that is, ∠ BAC = ∠
[d] 76. CBA + ∠BCA ; but, ∠FAC [d] = ∠CBA + ∠
[e] 45. BCA, ∵ ∠BAC [e] = ∠FAC, ∵ each of them
is

is a [f] right ∠, ∵ the ∠BAC in a Semicircle is a Rt. f 14.
∠. Q. E. D.

2dly. The △BAC having been juſt ſhewn to be
Rt. ∠d. *viz.* the ∠BAC a Right ∠, the ∠ABC
(in the Segment ABC) muſt be [g] Λ a Rt. ∠. g 86.
Q. E. D.

3dly. The Figure BADC being quadrilateral, the
∠ABC + ∠ADC [h] = 2 Rt. ∠s; but it has been h 147.
juſt ſhewn that ∠ABC Λ a Rt. ∠, conſequently
∠ADC muſt be Γ a Rt. ∠. Q. E. D.

149. THEOREM 14. *In an oblique Triangle* ACD, Pl. III.
inſcribed in a Circle, the vertical Angle ADC *exceeds,* F.14.&15.
or is leſs than a right Angle, by the Angle CAB, *made
by the Baſe* AC *and the Diameter* AB, *drawn from
either Extremity of the Baſe.*

Join DB; then, AB being a Diameter, ADCB a 22.
is [a] a Semicircle, and ∵ ∠ADB [b] = a Rt. ∠, and b 148.
the ∠CDB [c] = ∠CAB, (being in the ſame Seg- c 146.
ment CDAB, and ſtanding on the ſame Baſe BC;) ∵
adding equals in Fig. 14. the ∠ADB + ∠CDB [d] d 46.
= a Rt. ∠ + ∠CAB; but ∠ADB + CDB =
the whole ∠ADC, ∵ ∠ADC = a Rt. ∠ + ∠
CAB. Q. E. D. And in Fig. 15. ſubtracting
equals, the ∠ADB — ∠CDB [e] = a Rt. ∠ — ∠ e 47.
CAB; but ∠ADB — ∠CDB = ∠ADC, ∵ ∠
ADC = a Rt. ∠ — ∠CAB. Q. E. D.

150. THEOREM 15. *In equal Circles,* viz. *Circles* Pl. III.
which have equal Radii, the Angles A, G, *at the Cir-* F.16&17.
cumferences, and ſtanding upon equal Chords BC, EF,
*are equal to each other. And if the Angles at the Cir-
cumferences are equal, the Chords on which they ſtand
will be equal alſo.*

Let D and H be the Centers, and let Lines be drawn as in the Figures. 1. Then, BC being =

EF by the Suppoſition, and BD = CD = EH =
FH being equal Radii, the △s BDC, EHF, have
three Sides of one reſpectively = three Sides of the
[a] 95. other, and ∵ ∠ BDC [a] = ∠ EHF; but the ∠s
[b] 145. A, and G, are [b] Halves of the ∠s BDC, EHF;
and as the Wholes are equal to each other, 'tis manifeſt
their Halves muſt be ſo too, that is, ∠ BAC
= ∠G. Q. E. D.

2dly. If the ∠A = ∠G; as the ∠s A and G,
[b] 145. are the Halves [b] of the ∠s BDC, CHF, reſpectively,
and as the Halves being equal, the Wholes muſt be
ſo too, the ∠BDC muſt be = ∠EHF. But BD =
EH, DC = HF, being equal Radii, the △s BDC,
EHF, have two Sides BD, CD, and ∠ contained
BDC of one = two Sides EH, FH, and ∠ contained
EHF of the other; and conſequently the △s
[a] 95. themſelves [a] equal in all Reſpects; ∵ the Baſe BC =
Baſe EF. Q. E. D.

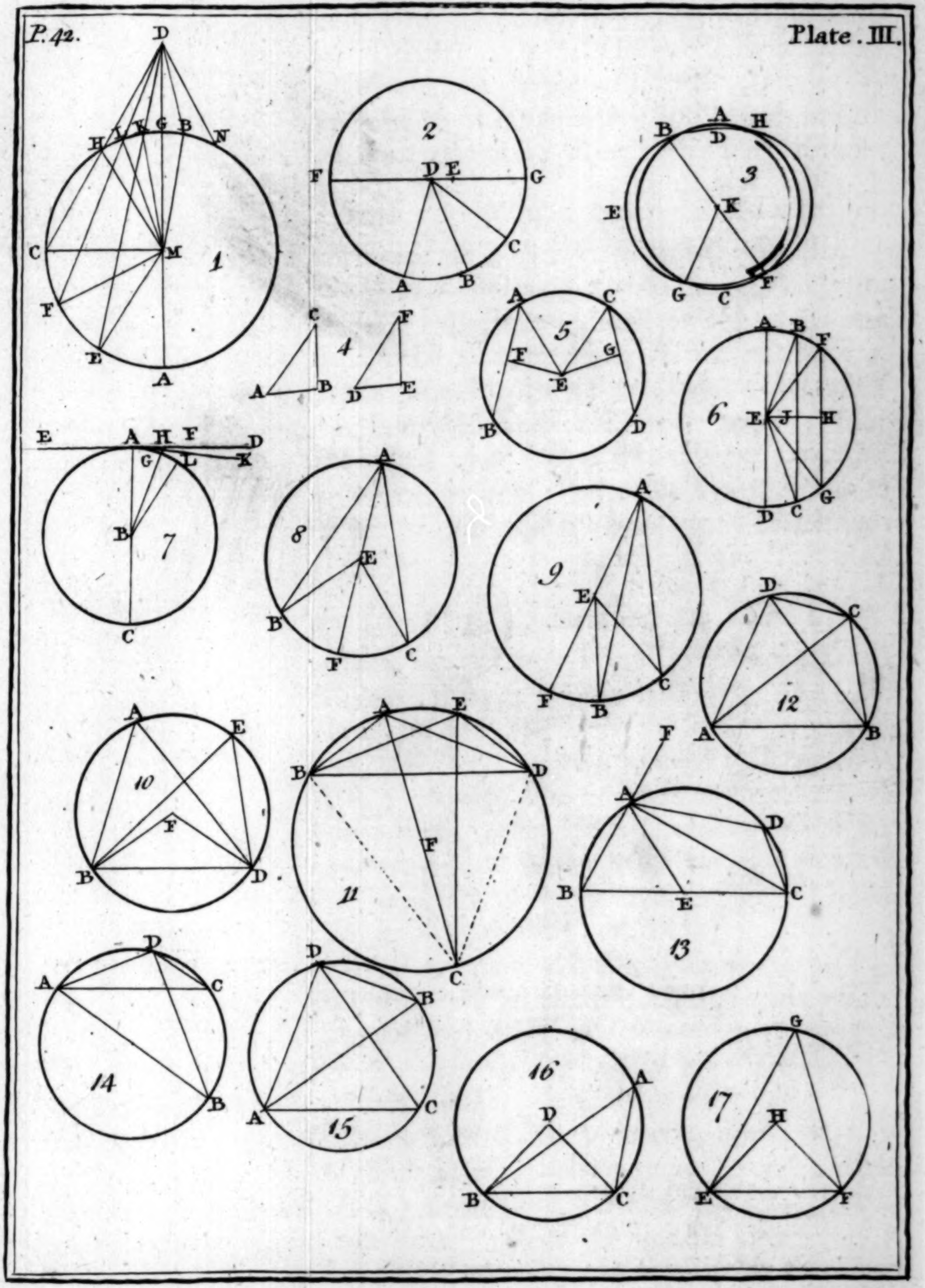
Pl. IV. Fig. 1. 151. THEOREM 16. *Let* AB *be the Diameter of the Circle* ADCB, *and* EF *a Line perpendicular thereto, touching the Circle in the Point* B; *and* BD *a Line cutting the Circle; then I ſay, the Angles made by the touching Line* EF *and cutting Line* BD, *ſhall be equal to the Angles in the alternate Segments*; viz. ∠DBF = ∠ DAB, *and* ∠ DBE = ∠ DCB, *wherever the Points* A *and* C *are taken in the Segments*, DAB, DCB.

For the vertical ∠DAB of the △DAB is ∧ a Rt.
[a] 149. ∠ by the [a] ∠DBA, made by the Baſe DB and Diameter
BA; and ∵ as ABF is a Rt. ∠, muſt be
= ∠DBF. Q. E. D.

Again, the vertical ∠ DCB of the △ DCB exceeds
a Rt. ∠ by the ſaid ∠DBA, and ∵ as ABE
is a Rt. ∠, the ∠ DCB muſt be = (∠ ABE + ∠
ABD) = ∠ DBE. Q. E. D. And the ∠ DAB
[b] 146. continues the [b] ſame wherever the Point A is taken in
the

P. 42.

Plate . III.



the Circumference DAB, as does the ∠DCB whereever the Point C is taken in the Circumference DCB. ∵ this Theorem is true.

BOOK IV.

Of PROPORTION, *containing what is moſt valuable in* Euclid'*s 5th and 6th Books.*

152. *Def.* 1. A Leſſer Magnitude is ſaid to be an *aliquot Part* of a greater Magnitude, when the leſſer meaſures the greater, *i. e.* when the leſſer is contained a certain Number of Times exactly in the greater. Thus, a is an aliquot Part of $3a$, a being contained in $3a$, 3 Times.

153. *Def.* 2. A greater Magnitude is ſaid to be a *Multiple* of a leſſer, when the leſſer meaſures the greater, *i. e.* when the leſſer is contained in the greater, exactly a certain Number of Times. Thus, $3a$ is a Multiple of a, a being contained in $3a$, 3 Times, and $n \times b$ is a Multiple of b, for b is contained n Times in $n \times b$.

154. The 3d *Definition*, "That Ratio is a mutual Relation of two Magnitudes of the ſame Kind to one another in Reſpect of Quantity," is rather metaphyſical than mathematical, and has been the Cauſe of much Controverſy amongſt the modern Mathematicians. Indeed it might well be omitted without any Loſs, and, as Mr. *Simſon* juſtly obſerves, ſeems to be the Addition of ſome unſkilful Editor.

155. *Def.* 4. Theſe Magnitudes are ſaid to have a Ratio to one another, the leſſer of which can be multiplied ſo as to exceed the other.

156. *Def.* 5. The firſt of four Magnitudes is ſaid to have the ſame Ratio to the ſecond which the third has to the fourth, when any Equimultiples whatſoever of the firſt and third being taken, and any Equimultiples whatſoever of the ſecond and fourth: If the Multiple of the firſt be leſs than that of the ſecond, the Multiple of the third is alſo leſs than that of the fourth; or, if the Multiple of the firſt be equal to that of the ſecond, the Multiple of the third is alſo equal to that of the fourth; or, if the Multiple of the firſt be greater than that of the ſecond, the Multiple of the third is alſo greater than that of the fourth.*

157. *Def.* 6. Magnitudes which have the ſame Ratio are called Proportionals.

158. *Def.* 7. Of a greater and leſs Ratio being not uſed in this Eſſay, we have thought proper to omit it, as the Theorems which are obſcurely, (if not inaccurately) demonſtrated by it, may be demonſtrated more clearly by other Means.

159. The ſame may be ſaid of the 8th *Definition, viz.* "That Analogy or Proportion is a Similitude of Ratios," as was remarked concerning the 3d. Definition.

160. *Def.* 9. Proportion conſiſts in three Terms at leaſt.

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* **This is the true geometrical Definition of the Ancients; however it may not be improper to give alſo the *numerical* or *algebraical Definition, viz.* Four Quantities are ſaid to be in direct Proportion to each other, when the Quotient of the firſt by the ſecond is equal to the Quotient of the third by the fourth: Which *Quotients* are called *Ratios.* Or in other Words, *Ratio* is the Number expreſſing how often the Conſequent is contained in the Antecedent; or by which the Conſequent being multiplied, the Product will be equal to the Antecedent.**

161. *Def.* 10. When three Magnitudes are Proportionals, the firſt is ſaid to have to the third the duplicate Ratio of that which it has to the ſecond.

162. *Def.* 11. When four Magnitudes are continual Proportionals, the firſt is ſaid to have to the fourth the triplicate Ratio of that which it has to the ſecond, and ſo forward quadruplicate, &c. increaſing the Denomination each Time by Unity, in any Number of Proportionals.

163. *Def.* 12, of *Compound Ratio.* When there are any Number of Magnitudes of the ſame Kind, the firſt is ſaid to have to the laſt of them the Ratio compounded of the Ratio which the firſt has to the ſecond, and of the Ratio which the ſecond has to the third, and of the Ratio which the third has to the fourth, and ſo forward unto the laſt Magnitude.

For Example, if A, B, C, D, be four Magnitudes of the ſame Kind, the firſt A is ſaid to have to the laſt D the Ratio compounded of the Ratio of A to B, and of the Ratio of B to C, and of the Ratio of C to D; or, the Ratio of A to D is ſaid to be compounded of the Ratios of A to B, B to C, and C to D.

And if A has to B the ſame Ratio which E has to F; and B to C the ſame Ratio which G has to H; and C to D the ſame Ratio that K has to L; then, by this Definition, A is ſaid to have to D the Ratio compounded of Ratios which are the ſame with the Ratios of E to F, G to H, and K to L. And the ſame Thing is to be underſtood when it is more briefly expreſſed by ſaying A has to D the Ratio compounded of the Ratios of E to F, G to H, and K to L.

In like Manner, the ſame Things being ſuppoſed, if M has to N the ſame Ratio which A has to D, then, for Shortneſs Sake, M is ſaid to have to N the

Ratio compounded of the Ratios of E to F, G to H, and K to L.

164. *Scholium.* Nothing has been thought more difficult, or cauſed greater Diſputes in Geometry, than the Definition of compound Ratio given by *Theon* in the 5th Def. of the 6th Book of *Euclid*, *viz.* "A Ratio is ſaid to be compounded of Ratios when the Quantities of Ratios being multiplied by one another, makes a certain Ratio ;" which is faulty, for there can be no Multiplication but by a Number, and ſo the Definition will be ungeometrical. As to the Uſe of compound Ratio, it is introduced into Geometry only to render ſome Demonſtrations more conciſe, or that ſome Propoſitions may be more compendiouſly enunciated. It may be further obſerved, "that in any Magnitudes whatever, "of the ſame Kind, A, B, C, D, &c. the Ratio "compounded of the Ratios of the firſt to the ſe-"cond, of the ſecond to the third, and ſo on to "the laſt, is only a Name or Expreſſion, by which "the Ratio which the firſt A has to the laſt D is "ſignified, and by which at the ſame Time the "Ratios of all the Magnitudes, A to B, B to C, "C to D, from the firſt to the laſt, to one another, "whether they be the ſame or be not the ſame, are "indicated ; as in Magnitudes which are continual "Proportionals, A, B, C, D, &c. the duplicate "Ratio of the firſt to the ſecond is only a Name, or "Expreſſion, by which the Ratio of the firſt A to "the third C is ſignified, and by which, at the "ſame Time, is ſhewn that there are two Ratios "of the Magnitudes from the firſt to the laſt, *viz.* "of the firſt A to the ſecond B, and of the ſecond "B to the third or laſt C, which are the ſame to "one another ; and the triplicate Ratio of the firſt "to the ſecond is a Name, or Expreſſion, by which "the Ratio of the firſt A to the fourth D is ſignified, "and by which, at the ſame Time, is ſhewn that "there are three Ratios of the Magnitudes from the

" firſt to the laſt, *viz.* of the firſt A to the ſecond " B, and of B to the third C, and of C to the fourth " or laſt D, which are all the ſame to one another; " and ſo in the Caſe of any other multiplicate Ra- " tios." And this is the right Meaning of theſe Ratios as uſed by *Euclid*, *Archimedes*, *Apollonius*, and other ancient Geometricians. However, that of *Theon* abovementioned is ſtill retained by moſt Moderns, and indeed may be made Uſe of when we are diſcourſing of Numbers, or of Magnitudes as expreſſed by Numbers, though it would be improper in this on Geometry.

165. *Def.* 13. In Proportionals the antecedent Terms are called homologous to one another, as alſo the Conſequents to one another.

Note. Geometricians make Uſe of the following technical Words to ſignify certain Ways of changing either the Order or Magnitude of Proportionals, ſo as to continue ſtill to be Proportionals.

166. *Def.* 14. *Permutando*, or *alternando*, *by Permutation*, or *alternately:* This Word is uſed when there are four Proportionals, and it is inferred, that the firſt has the ſame Ratio to the third which the ſecond has to the fourth; or that the firſt is unto the third, as the ſecond to the fourth; as is ſhewn in Article 199.

167. *Def.* 15. *Invertendo*, *by Inverſion*; when there are four Proportionals, and it is inferred, that the ſecond is unto the firſt as the fourth to the third. See Art. 198.

168. *Def.* 16. *Componendo*, *by Compoſition*; when there are four Proportionals, and it is inferred, that the firſt, together with the ſecond, is to the ſecond, as the third, together with the fourth, is to the fourth. See Art. 200.

169. *Def.* 17. *Dividendo*, by *Diviſion*; when there are four Proportionals, and it is inferred, that the Exceſs of the firſt above the ſecond is to the ſecond, as the Exceſs of the third above the fourth is to the fourth. See Art. 201.

170. *Def.* 18. *Convertendo*, by *Converſion*; when there are four Proportionals, and it is inferred, that the firſt is to its Exceſs above the ſecond, as the third to its Exceſs above the fourth. See Art. 202.

171. *Def.* 19. *Ex æquali*, (ſcil. *Diſtantia*,) or *ex æquo*, *from Equality of Diſtance*; when there is any Number of Magnitudes more than two, and as many others ſo that they are Proportionals when taken two and two of each Rank, and it is inferred, that the firſt is to the laſt of the firſt Rank of Magnitudes, as the firſt is to the laſt of the others. ———Of this there are the two following Kinds, which ariſe from the different Order in which the Magnitudes are taken two and two.

172. *Def.* 20. *Ex æquali*, *from Equality*: This Term is uſed ſimply by itſelf, when the firſt Magnitude is to the ſecond of the firſt Rank, as the firſt to the ſecond of the other Rank; and as the ſecond is to the third of the firſt Rank, ſo is the ſecond to the third of the other; and ſo forward in Order, and the Inference is as mentioned in the preceding Definition; whence this is called *Ordinate Proportion*.

173. *Def.* 21. *Ex æquali, in proportione perturbata, ſeu inordinata, from Equality, in perturbate or diſorderly Proportion:* This Term is uſed when the firſt Magnitude is to the ſecond of the firſt Rank, as the laſt but one is to the laſt of the ſecond Rank; and as the ſecond is to the third of the firſt Rank, ſo is the laſt but two to the laſt but one of the ſecond Rank; and as the third is to the fourth of the firſt

Rank, ſo is the third from the laſt to the laſt but two of the ſecond Rank, and ſo forward in a croſs Order; and the Inference is as in the 19th Definition.

174. *Def.* 22. *Similar rectilineal Figures* are thoſe which have their ſeveral Angles equal, each to each, and the Sides about the equal Angles Proportionals.

175. *Def.* 23. Two Magnitudes are ſaid to be reciprocally proportional to two others, when one of the firſt is to one of the other Magnitudes, as the remaining one of the laſt two is to the remaining one of the firſt.

176. *Def.* 24. A Line is ſaid to be cut in *extreme and mean Ratio*, when the Whole is to the greater Segment as the greater Segment is to the leſs.

177. *Axiom* 1. *Equimultiples* of the ſame, or equal Magnitudes, are equal to one another; that is, if $A = B$, then any Number of Times $A =$ ſame Number of Times B; ſo $mA = mB$.

178. *Axiom* 2. Theſe Magnitudes of which the ſame, or equal Magnitudes, are Equimultiples, are equal to one another.

179. *Axiom* 3. A *Multiple* of a greater Magnitude is greater than the ſame Multiple of a leſſer.

180. *Axiom* 4. That Magnitude whoſe Multiple is greater than the ſame Multiple of another, is greater than that other Magnitude.

181. *Axiom* 5. Magnitudes having the ſame Ratio to one and the ſame or equal Magnitudes, are equal to each other.

182. *Axiom* 6. Equal Magnitudes have to one and the ſame Magnitude the ſame Ratio : And the ſame unto equal Magnitudes has the ſame Ratio.

183. *Axiom* 7. Magnitudes to which one and the ſame Magnitude has the ſame Ratio, are equal.

184. *Axiom* 8. If two Magnitudes be referred to a third, that is the greateſt which has the greateſt Ratio.

185. *Axiom* 9. Ratios equal to one and the ſame Ratio, or to equal Ratios, are alſo equal to one another.

186. *Axiom* 10. Unequal Magnitudes, compared with one and the ſame or equal Magnitudes, cannot have equal Ratios.

187. THEOREM I. *Magnitudes have the ſame Ratio to one another which their Equimultiples have. Let* a *and* b *repreſent the two Magnitudes, and let* m *be the Number of Times each of theſe are contained in its Multiple ; then,* ma, mb, *repreſent the Equimultiples, and what we affirm is, that* $ma : mb :: a : b$.[*]

For taking any Equimultiples of the firſt and third, *viz.* cma, ca, and any Equimultiples, dmb, db, of the ſecond and fourth ; it is plain that if $cma = dmb$, as they are Equimultiples of ca, db, that
* 178. ca * $= db$; and if $cma \Lambda dmb$, then ca † Λdb ; alſo
† 180. if

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[*] It may be demonſtrated algebraically thus, The Quotient of ma by mb is $\frac{ma}{mb} = \frac{a}{b}$; and the Quotient of a divided by b is alſo $\frac{a}{b}$. $\because ma : mb :: a : b$. Q. E. D.

if *cma* Γ *dmb*, then *ca* Γ *db*. ∵ by Art. 156, as *ma* : *mb* :: *a* : *b*. Q. E. D.

188. THEOREM 2. *If four Quantities*, a, b, c, d, *are proportional*, viz. *as* a : b :: c : d; *then any Equimultiples of the two firſt will have the ſame Ratio as any Equimultiples of the two laſt*, viz. *as* ma : mb :: nc : nd.

For by Art. 187, As, *ma* : *mb* :: *a* : *b*,
And, as, *nc* : *nd* :: *c* : *d*.

But *a* is to *b*, in ſame Ratio as *c* to *d* by the Suppoſition, ∵ by Art. 185, as *ma* : *mb* :: *nc* : *nd*. Q. E. D.

189. THEOREM 3. *Triangles* ABC, ACD, *and Parallelograms* EC, CF, *of the ſame Altitude, are to one another as their Baſes*, BC, CD.* Pl. IV. F. 2.

Let BD be produced both Ways to the Points H, L, and let BG, GH, be each = BC; and DK, KL, each = CD; and let Lines be drawn as in the Figure. Then, becauſe the △s AHG, AGB, ABC, have equal Baſes and the ſame Altitude, they are* = each other; * 109. ∵ whatever Multiple the Baſe HC is of the Baſe BC, the ſame Multiple is the △AHC of the △ABC. By the ſame Reaſon, whatever Multiple the Baſe LC is of the Baſe CD, the ſame Mul-

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* The algebraic Demonſtration may ſtand thus. Let a = Baſe of the Parallelogram EC, b = Baſe of CF, and h = their Height; then ah =* Area of EC, and bh = Area of CF. Now it is plain that as $ah : bh :: a : b$; for the Quotient of the firſt Term by the ſecond is $\frac{ah}{bh} = \frac{a}{b}$; and the Quotient of the third by the 4th is alſo $\frac{a}{b}$; ∵ it will be, as $ah : bh :: a : b$. Q. E. D. And as △s are Halves of the Parallelograms, as the Parallelograms are as their Baſes, their Halves, *viz.* the △s, muſt be in the ſame Ratio alſo. Q. E. D.

tiple is the △ ALC of the △ ADC. And if the Baſe HC be = Baſe CL, the △ AHC will be [a] =
[a] 109. the △ ALC; and if the Baſe HC be Γ CL, the △ AHC Γ ALC; and if HC be Λ CL, the △ AHC
[b] 156. is Λ △ ALC. ∵ [b] as Baſe BC : Baſe CD :: △ ABC : △ ACD. Q. E. D.

Again, ſince we have demonſtrated that the △s ABC, ACD, are to each other as their Baſes BC,
[c] 107. CD, and the Parallelograms EC, CF, are [c] double of the △s; it muſt follow, that as the Halves are in Proportion of their Baſes, their Wholes (*viz.* the
[d] 187. Parallelograms) muſt be [d] ſo too. Q. E. D.

190. Corollary. *From hence it is plain, that △s and Parallelograms which have equal Altitudes, are to one another as their Baſes.*

Pl. IV. F. 3, 4, 5. 191. Theorem 4. *If a Line* DC *be drawn parallel to one of the Sides,* BC, *of a* △ ABC, *it ſhall cut the other Sides, or theſe Sides produced, proportionally,* (viz. BD : DA :: CE : EA, *or* AB : AD :: AC : AE.) *And if the Sides, or the Sides produced, be cut proportionally, the Line* DE, *which joins the Points of Section, ſhall be parallel to the remaining Side of the Triangle,* viz. DE *parallel to* BC.

For join BE, CD, then the △s BDE, CDE, being on the ſame Baſe DE, and between the ſame
[a] 109. ||s DE, BC, are [a] = each other. Hence, as △
[b] 182. BDE : △ ADE :: [b] △ CDE : △ ADE. And if we conſider AD, BD, as the Baſes of the △s AED, BED, the Point E may be conſidered as the Vertex of both, and ſo a ⊥ drawn from E to AB would be common to both; that is, both would have the
[c] 189. ſame Altitude; ∵ As △ BDE : △ ADE :: [c] BD : DA. And by the ſame Reaſon, as △ CDE : △
[d] 185. ADE :: CE : EA; ∵ as BD : DA :: [d] CE : EA. Q. E. D. And, by a like Method of Reaſoning,

ſoning, it may be ſhewn that AB : AD :: AC : AE. Q. E. D.

The ſecond Part of the Theorem may be thus demonſtrated. By the Suppoſition, as BD : DA :: CE : EA; and as BD : DA ::[c] △ BDE : △ ADE; and as CE : EA :: △CDE : △ADE; ∴ As △BDE : ADE ::[d] △CDE : △ADE ∴ the △s BDE, CDE, have the ſame Ratio to the △ ADE, and ∴ the △BDE[e] = △CDE, and ſtands on the ſame Baſe DE: But it is manifeſt that equal △s muſt have equal Areas, and ∴ as the Area of a △ is[f] = $\frac{1}{2}$ its Baſe into its Altitude, and they have the ſame Baſe, they muſt conſequently have the ſame Altitude, otherwiſe their Areas could not be equal; that is, the ⊥ Diſtance of the Points D and E, from the Line BC, are equal, and ∴ DE[g] || to BE. Q. E. D.

[c] 189.
[d] 185.
[e] 181.
[f] 111 & 112.
[g] 96.

192. Theorem 5. *If the Angle* BAC, *of a Triangle* ABC, *be divided into two equal Angles by a Line* AD, *cutting the Baſe, the Segments of the Baſe will have the ſame Ratio that the other two Sides of the Triangle have to one another*; viz. *as* BD : DC :: AB : AC. *And if the Segments of the Baſe have the ſame Ratio which the other Sides of the Triangle have to one another, the Line* AD, *drawn from the Vertex to the Point of cutting, will divide the* ∠BAC *of the Triangle into two equal Parts*; viz. ∠BAD = ∠CAD.

Pl. IV. F. 6.

Firſt. Let BA be produced till AE = AC, and let CE be joined; then, the ∠ACE[a] = ∠E: But the ∠BAC[b] = ∠ACE + ∠E, and as theſe ∠s are equal, each of them muſt be = $\frac{1}{2}$ ∠BAC = ∠DAC, that is, ∠DAC = ∠ACE; ∴ AD, EC, are[c] || to each other, ∴ As BD : DC[d] :: BA : AE. Q. E. D.

[a] 59.
[b] 76.
[c] 73.
[d] 191.

Secondly. By the Suppoſition, BD : DC :: AB : AC; but AC = AE by the Conſtruction; ∴ as BD : DC :: AB : AE; ∴ AD is[d] || to EC;

∴ ∠

e 40. ∵ ∠BAD [e] = ∠E, and ∠DAC [f] = ∠ACE [a] =
f 72. ∠E; ∵ ∠BAD [g] = ∠DAC. Q. E. D.
a 59.
g 45.

193. THEOREM 6. *If four Lines, or Numbers, denoted by the Letters,* a, b, c, d, *are proportional,* viz. *as* a : b :: c : d; *the Rectangle contained by the Extremes is equal to the Rectangle under the Means,* viz. $a \times d = b \times c$. *And if the Rectangle contained by the Extremes be = the Rectangle contained by the Means, theſe four Quantities are proportional; that is, if* $a \times d = b \times c$, *then it will be, as* $a : b :: c : d$.*

Firſt. Since $a \times d$, and $b \times d$, are Equimultiples
a 187. of a and b, we have, as $a \times d : b \times d$ [a] $:: a : b$.
And ∵ ſince by the Suppoſition, $a : b :: c : d$,
b 185. we have $a \times d : b \times d$ [b] $:: c : d$. But $b \times c$, and
c 187. $b \times d$ are Equimultiples of c and d, ∵ $b \times c : b \times$
d 185. d [c] $:: c : d$; ∵ $a \times d : b \times d ::$ [d] $b \times c : b \times d$;
e 181. ∵ $a \times d$ [e] $= b \times c$. Q. E D.

Secondly. It is plain, that $a \times d$, $b \times d$, are E-
f 187. quimultiples of a and b, ∵ $a : b ::$ [f] $a \times d : b \times$
d; and $b \times c$, $b \times d$, are Equimultiples of c, and
d; ∵ $c : d :: b \times c : b \times d$; but $a \times d = b \times c$
by the Suppoſition, ∵ by writing $a \times d$, for its
equal $b \times c$, we have, as $c : d :: a \times d : b \times d$;
but it has been juſt ſhewn that $a : b :: a \times d :$
$b \times d$, ∵ $c : d$ has the ſame Ratio as $a : b$, that
is, as $a : b :: c \; d$. Q. E. D.

194.

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* This *Theorem* may be demonſtrated algebraically thus. Firſt,
a 156. $\frac{a}{b}$ [a] $= \frac{c}{d}$ ∵ multiplying by b we have $a = \frac{bc}{d}$, and this by d gives $da = bc$. Q. E. D. Secondly. If we divide each Side of the Equation $da = bc$ by d, we have $a = \frac{bc}{d}$, and this again by b gives $\frac{a}{b} = \frac{c}{d}$; ∵ by Art. 156, As $a : b :: c : d$. Q. E. D.

194. THEOREM 7. *Equiangular Triangles are ſimilar; that is, the Sides about the equal Angles are Proportionals.*

Let D*a* = AB, D*b* = AC, then the ∠D being = ∠A by the Suppoſition, the △s D*ab*, ABC, are [a] equal in every Reſpect; ∵ ∠D*ab* = ∠ABC; but ∠ABC = ∠DEF by the Suppoſition, ∵ ∠D*ab*[b] = ∠DEF, ∵ *ba*[c] ∥ to EF; ∵ DE : D*a* :: * DF : D*b*. ∵ for D*a*, D*b*, writing their equals AB, AC, the Proportion becomes DE : AB :: DF : AC. Q. E. D.

Pl. IV. F. 7.
[a] 58.
[b] 45.
[c] 40.
* 191.

And if F*d* be taken = CA, and F*c* = CB, it may be ſhewn in the ſame Manner, that FD : CA :: FE : CB. Q. E. D.

By joining theſe two Proportions in one, we have DE : AB :: DF : AC :: FE : CB. Q. E. D.

195. THEOREM 8. *If the outward Angle* CAE, *of a Triangle* ABC, *is biſected by a Line* AD, *cutting the Baſe* BC *produced in* D; *then the Segments* BD, DC, *will have the ſame Ratio as the other Sides of the Triangle*; viz. BD : DC :: BA : AC. *And if* BD : DC :: BA : AC, *the Line* AD *will biſect the outward Angle* CAE, viz. ∠CAD = ∠DAE.

Pl. IV. F. 8.

For let CB be ∥ to DA; then ∠CFA[a] = ∠DAE, and alſo ∠ACF[b] = ∠DAC; but the ∠DAE = ∠DAC by the Suppoſition, ∵ ∠CFA[c] = ∠ACF, ∵ AF[d] = AC. Now, as CF is ∥ to AD by the Suppoſition, it follows, BD : DC[e] :: BA : AF; but it has been juſt ſhewn that AF = AC, ∵ for AF writing its equal AC, the above Proportion becomes BD : DC :: BA : AC. Q. E. D.

[a] 40.
[b] 72.
[c] 45.
[d] 62.
[e] 191.

Secondly. By the Suppoſition, BD : DC :: BA : AC, and FC being ∥ to AD, BD : DC :: BA : AF, ∵ BA : AC[f] :: BA : AF; ∵ AC[g] = AF, and ∵ ∠AFC[h] = ∠ACF; alſo the ∠AFC[a] = DAE,

[f] 185.
[g] 181.
[h] 59.
[a] 40.

[b] 72. DAE, and ∠ACF [b]= ∠CAD, ∵ ∠DAE (=
[c] 45. ACF) [c]= CAD. Q. E. D.

196. THEOREM 9. *If two Triangles,* ABC, DEF,
Pl. IV. *have one Angle* A, *of one, equal to one Angle* D, *of the*
F. 9. *other, and the Sides containing the equal Angles proportional,* viz. AB : AC :: DE : DF, *the Triangles*
are equiangular and ſimilar.

Let A*f* be = D*f*, and A*e* = DE ; and let *fe* be
joined. Then the ∠A being = the ∠D by the
[a] 58. Suppoſition, the △s A*fe*, DFE, are [a] equal in
every Reſpect. By the Suppoſition, AB : AC ::
DE : DF ; or, which is the ſame, by writing
equals, (*viz.* A*e* for DE, and A*f* for DF ;) AB :
[b] 191. AC :: A*e* : A*f*. ∵ *fe* is [b] ∥ to BC, ∵ ∠A*fe*[c]=
[c] 40. ∠ACB, and ∠ A*ef* = ∠ABC ; ∵ the △s A*fe*,
ABC, are equiangular, and conſequently, as the △s
A*fe*, DFE, have been juſt ſhewn to be equal in
every Reſpect, ABC, DEF, are alſo equiangular ;
[d] 194. and ∵ alſo [d] ſimilar. Q. E. D.

197. THEOREM 10. *If two Triangles,* ABC,
Pl. IV. DEF, *have their like Sides proportional, (thus,* AB :
F. 9. AC :: DE : DF ; *and* AB : BC :: DE : FE.)
the Triangles are ſimilar to each other.

Let A*f* = DF, and A*e* = DE, and let *fe* be
joined ; then by the Suppoſition, AB : AC :: DE
: DF ; and by writing equals, *viz.* A*e* for DE,
and A*f* for DF, the Analogy is, AB : AC :: A*e* :
[a] 191. A*f* ; ∵ *fe* is [a] ∥ to BC. Again, by the Suppoſition, AB : BC :: DE : FE ; and the △s ABC,
[b] 196. A*ef*. being [b] equiangular and ſimilar, AB : BC ::
[c] 185. A*e* : *fe* ; ∵ by Equality of Ratios, DE : FE [c] ::
A*e* : *fe* ; but A*e* = DE by the Conſtruction, ∵
writing DE for its equal A*e*, we have, DE : FE ::
DE : *fe*, conſequently FE = *fe*. Hence the three
Sides of the △s A*fe*, DFE, have been proved to be
reſpectively

reſpectively equal to each other, and ∵ the $\triangle$s Afe, DFE, are [d] equal in all Reſpects; and conſequently, as the $\triangle\, Afe$ is ſimilar to the $\triangle$ABC, the $\triangle$DEF muſt be ſo too. Q. E. D.

[d] 95.

198. Theorem 11. *If four Quantities are proportional, they will alſo be Proportionals when taken inverſely*; viz. *if* $a : b :: c : d$, *then alſo*, $b : a :: d : c$.

For the Rectangle of the Extremes being [*] = the Rectangle of the Means, $a \times d = b \times c$; or, which is the ſame, $b \times c = a \times d$, ∵ $b : a$ [*] $:: d : c$. Q. E. D.

[*] 193.

199. Theorem 12. *If four Quantities be proportional, they will alſo be proportional when taken alternately*; *that is*, *if* $a : b :: c : d$, *then* $a : c :: b : d$.

For the Rectangle of the Extremes being [*] = Rectangle of the Means, $a \times d = b \times c$, or, which is the ſame, $a \times d = c \times b$, ∵ $a : c$ [*] $:: b : d$. Q. E. D.

[*] 193.

200. Theorem 13. *If four Lines, denoted by* a, b, c, d, *are Proportionals*, viz. $a : b :: c : d$; *they will alſo be compoundedly proportional*; *that is*, $a + b : b :: c + d : d$.[†]

Let AB $= a$, BC $= b$, AE $= c$, ED $= d$; then is AC $= a + b$, and AD $= c + d$. Becauſe

Pl. IV. Fig. 10.

[†] Algebraically demonſtrated thus. For the Product of the Extremes, $\overline{a+b} \times d = ad + bd$, and the Product of the Means, $b \times \overline{c+d} = bc + bd$; but $a : b :: c : d$, by the Suppoſition; ∵ $ad = bc$, conſequently $ad + bd = bc + bd$, or, which is the ſame, $\overline{a+b} \times d = b \times \overline{c+d}$, and ∵ by Art. 193. $a + b : b :: c + d : d$. Q. E. D.

by the Suppoſition, $a : b :: c : d$, that is, AB :
[a] 191. BC :: AE : ED; BE, CD, are [a] || to each other,
and ∵ AC : BC :: AD : DE, that is, $a + b$:
$b :: c + d : d$. Q. E. D.

201. THEOREM 14. *If four Lines, denoted by* a, b, c, d, *are Proportionals,* viz. a : b :: c : d, *they will alſo be Proportionals by Diviſion,* viz. a — b : b :: c — d : d.[†]

Pl. IV. Let AC $= a$, CB $= b$, AD $= c$, DE $= d$;
Fig. 11. then, AB $= a - b$, AE $= c - d$; by the Suppo-
ſition, $a : b :: c : d$, that is, AC : CB :: AD :
[a] 191. DE, ∵ BD, DC, are [a] || to each other, and ∵
AB : BC :: AE : ED; that is, $a - b : b :: c$
$- d : d$. Q. E. D.

202. THEOREM 15. *If four Lines, denoted by* a, b, c, d, *are Proportionals,* viz. a : b :: c : d, *then they will alſo be Proportionals by Converſion, that is,* a : a — b :: c : c — d.*

Let AC $= a$, CB $= b$, &c. as in the laſt Theorem; then BC, CD, being there ſhewn to be || to

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[†] This may be demonſtrated analytically thus. The Product of the Extremes, $\overline{a-b} \times d = ad - bd$, and the Product of the Means $b \times \overline{c-d} = bc - bd$; but $ad = bc$ by Art. 193, conſequently $ad - bd = bc - bd$; or, which is the ſame, $\overline{a-b} \times d = b \times \overline{c-d}$; ∵ $a - b : b :: c - d : d$. Q. E. D.

* Algebraically thus. The Product of the Extremes $a \times \overline{c-d} = ac - ad$, and of the Means $\overline{a-b} \times c = ac - bc$; but it has been before ſhewn that $ad = bc$; conſequently $ac - ad = ac - bc$; and ∵ their Equals are equal to each other, that is, $a \times \overline{c-d} = \overline{a-b} \times c$, or Product of the Extremes = the Product of the Means, and ∵ theſe Quantities are proportional by Art. 193, as aſſerted in the Theorem. Q. E. D.

each other, we have AC : AB[a] :: AD : AE, that [a] 191.
is, $a : a - b :: c : c - d$. Q. E. D.

203. THEOREM 16. *If four Lines, denoted by* a, b, c, d, *are Proportionals*, viz. a : b :: c : d, *then it will alſo be*, $a + b : a - b :: c + d : c - d$. *This is uſually cited by the Word* mixtly. *

Let AE $= a$, EC and EG each $= b$, AD $= c$, Pl. IV.
DB and DF each $= d$, then AC $= a + b$, AG $= a$ Fig. 12.
$- b$, AB $= c + d$, and AF $= c - d$. Since by the Suppoſition, $a : b :: c : d$, that is, AE : EC :: AD : DB; and AE : EG :: AD : DF; BC
and GF are[a] ∥ to ED, ∵ AC : AG :: AB : [a] 191.
AF, that is, $a + b : a - b :: c + d : c - d$. Q. E. D.

204. SCHOLIUM. What has been, or may hereafter be demonſtrated, concerning Lines being Proportionals, muſt, it is manifeſt, hold good in any other Magnitudes; for Lines may be aſſumed, having the ſame Ratio to each other as the Magnitudes themſelves; and if the Ratios by which the Theorems are deduced are the ſame, the Concluſions muſt certainly be ſo likewiſe.

205. THEOREM 17. *If three Lines are Proportionals, the Rectangle under the Extremes is equal to*

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* Analytically thus. The Product of the Extremes $\overline{a+b} \times \overline{c-d} = ac + bc - ad - bd$; and the Product of the Means $\overline{a-b} \times \overline{c+d} = ac - bc + ad - bd$. But it is known that $ad = bc$ by the Suppoſition and Art. 193; ∵ $+ bc - ad$ in one Equation, and $- bc + ad$ in the other, vaniſh, being equal to Nothing; whence the Equations become $\overline{a+b} \times \overline{c-d} = ac - bd$, and $\overline{a-b} \times \overline{c+d} = ac - bd$ alſo; conſequently $\overline{a+b} \times \overline{c-d} = \overline{a-b} \times \overline{c+d}$, that is, the Product of the Extremes = Product of the Means, and ∵ the Quantities proportional, Q. E D.

the Square on the Mean; *that is, if* $a : b :: b : c$; *then* $a \times c = b^2$.

[a] 193. For, the Rectangle of the Extremes being[a] = the Rectangle of the Means, $a \times c = b \times b$; but the Rectangle $b \times b$ is a Square, ∵ $a \times c = b^2$. Q. E. D.

Note. This follows ſo eaſily from Art. 193, that it might have been given as a Corollary thereto, but was not at that Time thought of.

206. THEOREM 18. *The Rectangles under proportional Lines are Proportionals. Thus, if* $a : b :: c : d$, *and* $e : f :: g : h$, *then we affirm that* $a \times e : b \times f :: c \times g : d \times h$.*

[a] 153.
[*] 187.
[†] 185.

For, $a \times e$, and $b \times e$, being[a] Equimultiples of a and b, we have $a \times e : b \times e$[*] $:: a : b$; but $a : b :: c : d$ by the Suppoſition, ∵ by Equality of Ratios, $a \times e : b \times e$[†] $:: c : d$. Again, $c \times g$, $d \times g$, are Equimultiples of c, d, ∵ $c \times g : d \times g :: c : d$; ∵ by Equality of Ratios, $a \times e : b \times e :: c \times g : d \times g$. Alſo $b \times e$, $b \times f$, are Equimultiples of e, f, ∵ $b \times e : b \times f :: e : f$; but $e : f :: g : h$, by the Suppoſition, ∵ by Equality of Ratios, $b \times e : b \times f :: g : h$.

Once again; $d \times g$, $d \times h$, are Equimultiples of g and h, ∵ $d \times g : d \times h :: g : h$. ∵ $b \times e : b \times f :: d \times g : d \times h$, by Equality of Ratios. By Alternation the Analogy (above found) $a \times e : b \times e ::$

$c \times g$

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[*] 193. * This may be thus demonſtrated by *Algebra, viz.* By the Suppoſition, as $a : b :: c : d$; and $e : f :: g : h$; hence the Product of the Extremes being * = the Product of the Means, we have $ad = bc$, and $eh = fg$; ∵ by multiplying each Side of the firſt Equation by the correſpondent Term of the other Equation, we get $aedh = bfcg$, (becauſe equal Things multiplied by equal Things muſt produce equal Products,) or, which is the ſame, $ae \times dh = bf \times cg$; ∵ (by Art. 193.) as $ae : bf :: cg : dh$, Q. E. D.

$c\times g : d\times g$, becomes $a\times e : c\times g :: b\times e : d\times g$; and the Analogy juſt found, taken alternately, is $b\times e : d\times g :: b\times f : d\times h$; $\because$ by Equality of Ratios, $a\times e : c\times g :: b\times f : d\times h$. Q. E. D.

207. THEOREM 19. *The Squares upon four proportional Lines are proportional*; viz. *if* $a : b :: c : d$; *then we affirm that* $a^2 : b^2 :: c^2 : d^2$.

For, if $a : b :: c : d$, and $a : b :: c : d$, then by the laſt Theorem, $a\times a : b\times b :: c\times c : d\times d$. Q. E. D.

208. THEOREM 20. *If four Squares are Proportionals, their Sides will be ſo too*; *that is, if* $a^2 : b^2 :: c^2 : d^2$, *then we affirm that* $a : b :: c : d$.

For let $a : b :: c : e$, then *per* laſt Theorem, $a^2 : b^2 :: c^2 : e^2$, $\because$ by Equality[a] of Ratios, $c^2 : d^2 :: c^2 :$ [a] 185.
e^2, $\because$ d^2 * $= e^2$. $\because$ $d = e$, $\because$ by writing d for its * 183.
Equal e, we have $a : b :: c : d$. Q. E. D.

†‡† We may perhaps treat more fully of the Doctrine of Proportionals in a future *Eſſay*, in a new and algebraic Manner; our Intention at this Time being only to give ſo much as is in ſome Degree neceſſary for the preſent Occaſion: In doing which we have been obliged to deviate greatly from *Euclid*'s Method; and indeed, after all that has or can be done, to reſtore the genuine Demonſtrations of *Euclid*, that Method will be found generally too obſcure for Learners at their firſt ſetting out. And we hope thoſe who are prejudiced to Antiquity will excuſe us in ſaying, that, as Mathematics are now improved, we cannot ſee any Neceſſity of adhering to *Euclid*'s Demonſtrations of the Doctrine of Proportionals. And to ſpeak plainly, we think many of the Propoſitions are in a Manner uſeleſs to an Algebraiſt, as many Theorems which are with Difficulty

demonſtrated by them, may be eaſily inveſtigated by the analytic Art.

Notwithſtanding what Truth has obliged us to remark, we may venture to affirm, that we hold ***Euclid*** in the higheſt Veneration, and, if we are not miſtaken, more juſtly than thoſe who are ſo much bigotted to him, as to condemn any Author, who ſhall dare to deviate from him in the leaſt Article. The Cenſures of ſuch will therefore give us no Concern.

209. THEOREM 21. *If in a right-angled Triangle* ABC, *right-angled at* A, *a Perpendicular* AD *be let fall from the right Angle, to the oppoſite Side* BC; *the Triangles on each Side of it ſhall be ſimilar to the whole Triangle, and to one another.* Pl. IV. F. 13.

For the △ADC being right-angled at D, the ∠ CAD is [a] = a Rt. ∠ — ∠C, and the △ABC being Rt. ∠d at A, the ∠B = a Rt. ∠ — ∠C; ∵ ∠ CAD [b] = ∠B; and ∠C being common to both △s, and ∠s ADC, BAC, Rt. ∠s, the △s ABC, ADC, are equiangular, and ∵ [c] ſimilar. In the ſame Manner it may be ſhewn, that the △s BAD, ABC, are ſimilar; and the △s ADC, ABD, being each ſimilar to the △ABC, they muſt conſequently be ſimilar to each other. Q. E. D.

[a] 83. [b] 45. [c] 194.

210. COROLLARY 1. *Hence theſe Analogies.*

1. AB : BC :: AD : AC.
2. BD : DA :: DA : DC. *
3. BC : AB :: A : BD.
4. BC : AC :: AC : DC.

211. COROLLARY 2. *If from any Point* A, *in the Circumference of a Circle, the Perpendicular* AD *is let*

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* This is all we mean when we ſay AD is a geometric mean Proportional between BD and DC.

let fall on the Diameter BC, *and the Chords* AB, AC, *drawn from the Point* A *to the Extremities of the Diameter* BC ; *as the Angle in a Semicircle is a* * *right* $\angle$, * 142.
*it plainly follows, that the Rectangle of the two Chords is equal to the Rectangle under the whole Diameter and that Perpendicular ; alſo that the Rectangle under the two Segments of the Baſe is equal to the Square of the Perpendicular ; and laſtly, that the Square of either Chord is equal to the Rectangle under the whole Diameter and adjacent Segment.**

For in four proportional Quantities, the Rectangle of the Extremes being [a] = the Rectangle of the Means, from the Analogies in Art. 210 we have, [a] 193.

$$\left.\begin{array}{l} 1.\ AB \times AC = BC \times AD. \\ 2.\ BD \times DC = DA^2. \\ 3.\ BC \times BD = AB^2. \\ 4.\ BC \times DE = AC^2. \end{array}\right\}\ \text{Q. E. D.}$$

212. THEOREM 22. *If two Lines cut one another within a Circle, (as in* Pl. IV. Fig. 14.) *the Rectangle of the Segments of one of them is equal to the Rectangle of the Segments of the other,* viz. AE × EC = BE × ED. *And if two Lines cutting a Circle, being produced, interſect each other without the Circle, (as in* Pl. IV. Fig. 15.) *then alſo will* AE × EC = BE × ED.

For let BC, AD, be joined ; then the $\angle$s C, and D, ſtanding on the ſame Arc AB are [a] equal to each [a] 146.

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* From hence may be deduced a very eaſy Demonſtration of the celebrated Propoſition generally cited by 47 *Euclid's 1ſt Book*, *viz.* our 115th Article. For, by this, $BD \times BC = AB^2$, and $DC \times BC = AC^2$; $\because BD \times BC + DC \times BC$ (by the 46 Art.) $= AB^2 + AC^2$; or, which is the ſame, $\overline{BD + DC} \times BC = AB^2 + AC^2$; that is, ($BC \times BC$, or) $BC^2 = AB^2 + AC^2$. Q. E. D.

[b] 69. each other; and the ∠BEC [b] = ∠AED, the △s BEC, AED, have two ∠s of one reſpectively = [c] 80. two ∠s of the other, and ∵ the third ∠A [c] = the third ∠B; conſequently, the △s are equiangular, [d] 194. and [d] ∵ ſimilar. Hence, AE : ED :: BE : EC; ∵ [e] 193. AE × EC [e] = BE × ED.

Pl. IV. Fig. 16. 213. THEOREM 23. *If from any Point* C, *without a Circle, two Lines* CA, CB, *are drawn in ſuch Manner, that* CB *touches the Circle, and* CA *cuts it; then will* CA × CD = BC^2.

[a] 151. For let BD, BA be joined; then the ∠DBC [a] = A, and the ∠C common to both △s, ∵ the △s DBC, CBA, have two ∠s in one = two ∠s in the other, ∵ the remaining ∠BDC in one = the remaining ∠CBA in the other, ∵ the △s are equi- [b] 194. angular, and conſequently [b] ſimilar, ∵ AC : CB :: [c] 193. CB : CD; ∴ AC × CD [c] = CB × CB. Q. E. D.

Pl. IV. F. 17. 214. THEOREM 24. *In equal Circles, Angles, whether at the Centers or Circumferences, have the ſame Ratio with the Circumferences on which they ſtand. Thus, let* G *and* H *be the Centers of the two Circles; then we affirm, that, as the Circumference* BC : *the Circumference* EF, :: *the* ∠ BGC : ∠ EHF *and* :: ∠ BAC : ∠ EDF.

For if we take any Number of Circumferences, CK, KL, each = BC; and any Number whatever FM, MN, each = EF; and join GK, GL, HM, HN; then, becauſe the Circumferences BC, CK, KL, are all equal, the ∠s BGC, CGK, KGL, will be = each other, (for 'tis manifeſt they would exactly cover each other;) ∵ whatever Multiple the Circumference BL is of the Circumference BC, the ſame Multiple is the ∠BGL of the ∠BGC. And by the ſame Method of Reaſoning the Circumference EN is the ſame Multiple of the Circumference EF, as the ∠EHN is of the ∠EHF. And if the Circumference BL be = the Circumference EN, the ∠ BGL will be = ∠EHN, (as they would exactly cover each other;) and if the Circumference BL be Γ EN, the ∠BGL will be Γ ∠EHN, and if BL be Λ EN, the ∠ BGL Λ ∠ EHN. Here, then, are four Magnitudes, *viz.* the two Circumferences BC, EF, and the two ∠s BGC, EHF; and of the Circumference BC, and of the ∠BGC, have been taken any Multiples whatever, *viz.* the Circumference BL, and ∠BGL; and of the Circumference EF, and ∠EHF, any Equimultiples whatever, *viz.* the Circumference EN, and ∠EHN; and it has been proved, that if the Circumference BL be either Γ, =, or Λ the Circumference EN, the ∠BGL will be reſpectively Γ, =, or Λ the ∠ EHN; ∵ the Circumference BC : Circumference EF* :: ∠BGC : ∠EHF. Q. E. D. * 156.

As to the ∠s BAC, EDF, at the Circumferences, they are the [b] Halves of the ∠s at the Centers BGC, EHF; ∵ as the Wholes have been ſhewn above to be in Proportion to the Circumferences BC, EF, their Halves muſt be ſo too. Q. E. D. [b] 145.

215. Corollary 1. *The* ∠BGC *in the Center* : 4 Rt. ∠s :: *the Circumference or Arc* BC *on which it ſtands* : *the whole Circumference of the Circle.*

216. Corollary 2. *Hence the Circumferences* DE, BC, *of unequal Circles, which ſubtend equal Angles at the Centers, as* ∠s DAE, BAC, *are ſimilar*; viz. *as Arc* DE : *the whole Circumference of the Circle* DEF :: BC : *the Circumference of the Circle* BCG. *And if they are ſimilar, they will ſubtend equal Angles.* Pl. IV. F. 18.

For 1ſt. ∠DAE : 4 Rt. ∠s [a] :: DE : the Circumference of the whole Circle DEF; and (∠BAC, or, which is the ſame,) ∠ DAE : 4 Rt. ∠s :: Arch BC : the Circumference of the whole Circle [a] 215.

BCG.

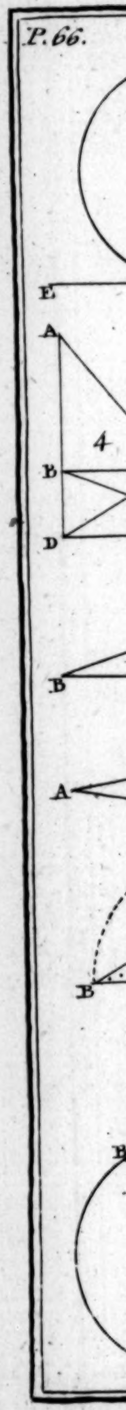
BCG. ∵ by Equality of Ratios, Arc DE : Circumference of the Circle DEF :: Arc BC : Circumference of the Circle BCG. Q. E. D.

[a] 215. Secondly. As ∠ DAE : 4 Rt. ∠s[a] :: the Arc DE : the Circumference DEF ; and ∠ BAC : 4 Rt. ∠s :: Arc BC : the Circumference of Circle BCG ; but, by the Suppoſition, the Arc DE : the whole Circumference of the Circle DEF :: BC : the whole Circumference of the Circle BCG ; ∵ by Equality of Ratios, ∠DAE : 4 Rt. ∠s :: ∠BAC : 4 Rt. ∠s ; ∵ ∠DAE[b] = ∠BAC. Q. E. D.

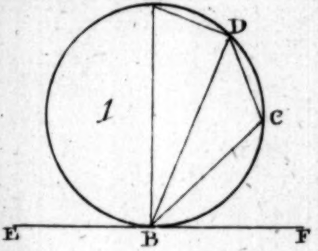
[b] 181.

217. *Scholium.* Mathematicians, for the more eaſy comparing the Proportion which Angles bear to each other, ſuppoſe the Circumference or Periphery of every Circle to be divided into 360 equal Parts, which they call Degrees, (ſo that 180 Degrees are a Semicircle, or 2 Rt. ∠s, and 90 Degrees a Quadrant, or Rt ∠ ;) each Degree is ſuppoſed to be divided into 60 equal Parts, called Minutes, and each Minute into 60 equal Parts, called Seconds, &c. —— Hence it is plain, that in unequal Circles, as well as in equal, equal Angles are ſubtended by an equal Number of Degrees. For, as a Degree is $\frac{1}{360}$ Part of the Circumference of any Circle, the Length of a Degree will increaſe, or decreaſe, in the ſame Proportion as the Circumferences of the Circles ; and ſo the Arch, repreſenting any Number of Degrees in one Circle, will be to the whole Circumference, as the Arch repreſenting the ſame Number of Degrees in any other Circle, to the whole Circumference of that other Circle, and ∵ will, by the laſt Corollary, ſubtend equal ∠s. Hence appears the Reaſon of the Method of denoting an ∠ by ſaying it contains ſo many Degrees.

BOOK



P. 66. Plate IV.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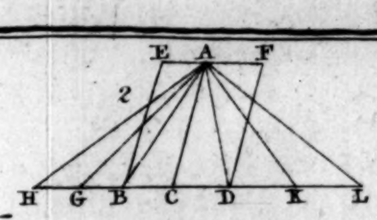
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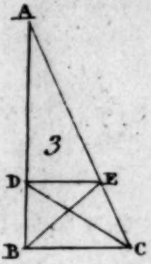
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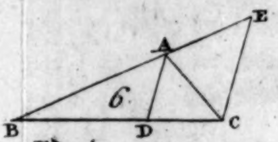
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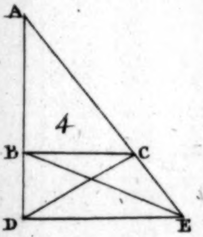
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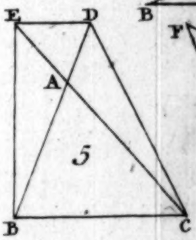
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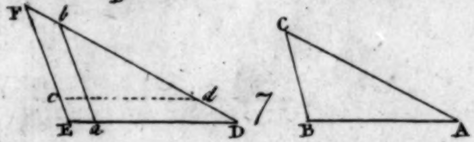
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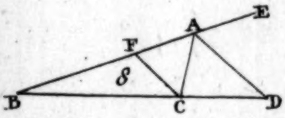
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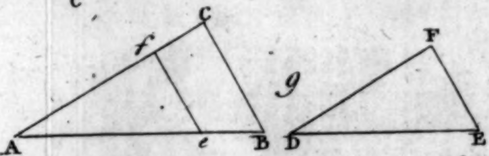
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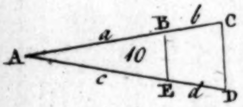
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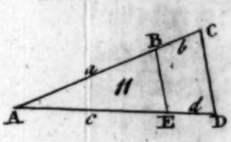
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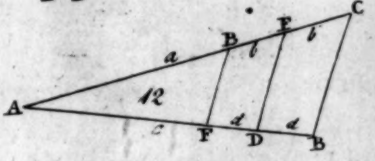
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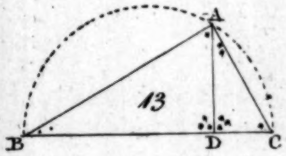
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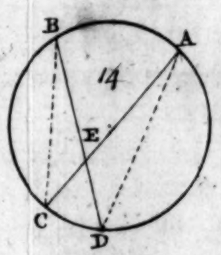
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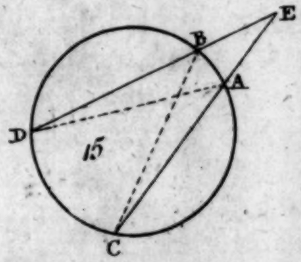
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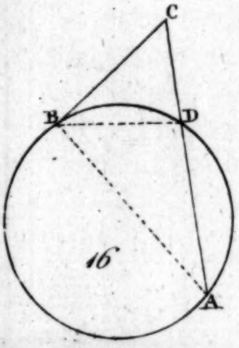
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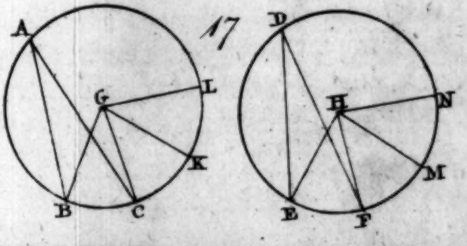
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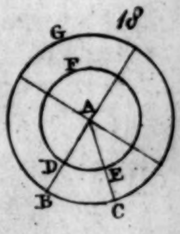
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BOOK V.

Of PROBLEMS.

218. IT will now be convenient for the Learner to procure a Caſe of Pocket Inſtruments, for his more eaſy performing the following Problems.

The Caſe we would recommend ought at leaſt to contain 2 Pairs of Compaſſes, a Drawing-Pen and Point, a Pencil and Point, Parallel-Ruler, Ivory Plotting-Scale, Ivory Sector, and Braſs Protractor. To theſe may alſo be added, a Pricking or Dotting-Wheel-Point, a Protracting-Pin, a Pair of Bows, Tracing-Point, Feeder, &c.

219. *Compaſſes* are ſo well known to every one, that it would be loſt Labour to deſcribe them; however, it may be of Uſe to ſome Perſons to hint, that one of the Compaſſes hath one Leg to take out, that in its Stead may be occaſionally uſed, the Drawing-Pen-Point, a Pencil-Point, and Dotting-Wheel-Point. ——— In chooſing Compaſſes take Notice, that in opening and ſhutting, the Legs move not by Starts, but that the Motion in the Joint be * eaſy and regular; and that the Points are of an equal Length, &c. For leſſening the irregular Friction, (if any,) the Head of the Compaſſes may be warmed, and a little || yellow Wax melted and poured between the Joints.

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* And for this End let Part of the Joint be Steel, for Braſs and Steel will wear more equable than either ſingly. Alſo let the Pin or Axle of the Joint be a Steel Screw, by Means of which you can make your Compaſſes to move in the Joint ſtiffer or eaſier at Pleaſure.

|| Or, if the Wax is too ſtiff of itſelf, a little Wax and Oil mixed together.

The Solder for cementing the Steel Points to the Braſs, is generally made with twice as much Silver as Copper; which muſt be melted in a Crucible, and, when cold, hammered to about the Thickneſs of a Card. Solder is alſo frequently made by melting 3 Parts of Copper with one of Zink.

In Order to make the Solder run the better between the Parts to be ſoldered, Borax, bruiſed very fine, is laid on the Parts to be cemented, and melted together with the Solder.

To one Point of the ſmaller Compaſſes is ſometimes fixed in the Shank, a Spring, which, by Help of a Screw, when you have opened the Compaſs nearly to the required Diſtance, moves it very regularly and gradually, without any ſtarting, to the Point intended, for the more accurately taking off any required Diſtance. Thus, holding the Compaſſes in the left Hand, (with the Screw turned towards the right,) if with the other you turn the Screw backward, the Spring Point will approach nearer to the other Point; but if you turn the Screw forward, the Spring Point will be farther diſtant from the other: However, after the Learner has been ſome Time uſed to a Pair of Compaſſes, he will not ſtand in much Need of this Contrivance.

The leſſer Compaſſes are uſed in transferring the Meaſures of Diſtances from one Place to another, or in deſcribing Circles, or Parts of Circles. ——— The larger Compaſſes are chiefly intended for deſcribing Circles, or Parts of Circles, either obſcurely by the Steel Point, in Ink by the Drawing-Pen-Point, in Lead by the Pencil Point, or dotted by the Dotting-Wheel-Point; for either of theſe, as before obſerved, may be fixed in the Shank of the Compaſſes. The *Dotting-Wheel-Point* is not abſolutely neceſſary, for dotted Lines are better made by the Drawing Pen or Point, (though ſomething ſlower,) for it is but ſeldom the Dotting-Wheel-Point is ſo well made as to perform accurately.

As almoſt every Learner, at his firſt Beginning, handles a Pair of Compaſſes very aukwardly, it may be proper to caution him by all means to endeavour to make Uſe of them with the right Hand only, never uſing the left, unleſs in a Caſe of Neceſſity, which after ſome Practice he will ſeldom have Occaſion for.——In uſing the Compaſſes be always careful to keep them nearly upright.

The *Bow* is a ſmall Kind of Compaſſes, one of whoſe Legs is as common, the other is a Drawing Pen Point: Its Uſe is to deſcribe Arcs or Circles of ſhort Radii, which could not be ſo readily and accurately done by the large Compaſſes and Drawing-Pen-Point.

The *Drawing Pen Point* and *Pencil Point* (or Crayon Point, as it is frequently called,) in the beſt Caſes have generally a Socket fitted to them, that they may fill up but one of the Holes or Partitions of the Caſe. Theſe Points, and alſo the Dotting-Wheel-Point, have a Joint in them juſt under that Part which locks into the Shank of the Compaſſes, that the Part below the Joint may ſtand perpendicular, or nearly ſo, to the Paper, or Plane, on which the Arc is to be deſcribed; otherwiſe it will not be ſmooth.

Of the larger Compaſſes, thoſe are eſteemed the beſt, whoſe moveable Points are locked in by a Spring and Catch fixed in the Shank; for if this Spring be well effected, the Point is thereby kept firm and ſteady; the contrary of which frequently happens when the Point is kept in by a Screw in the Shank.

If, inſtead of the larger Compaſſes being made with ſhifting Points, there were two Pairs put into the Caſe, to one of which the Ink Point was fixed, and to the other the Pencil Point, this, as Mr. *Robertſon* juſtly * obſerves, would ſave the Trouble of changing the Points in the Compaſs at every Time they were uſed, and the additional Expence, or Bulk of the Caſe, would be very little.

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* In his Treatiſe of Math. Inſtruments, &c.

The *Black-lead Pencil* is of great Uſe to deſcribe the firſt Sketch of a drawing before it is marked with Ink; becauſe any falſe or ſuperfluous Lines may be rubbed out with a Piece of ſtale Bread.

Black-Lead is produced in many Countries; but the beſt yet diſcovered is found in the North of *England*; it is dug out of the Ground in Lumps, and ſawed into Scantlings proper for Uſe. If it will not bear paring to a Point with a ſharp Penknife, it is not fit for drawing on Paper. There are three Sorts of Black-lead; the ſofteſt is proper for taking rough Sketches, the middling for drawing of Landſcapes and Ornaments, and the hard for drawing mathematical Figures.

The *Feeder* is a thin flat Piece of Braſs, or Silver, commonly fixed to a Cap that is put on the Pencil: Its Uſe is either to put Ink between the Blades of the Drawing Pen, or to paſs it between the Points when the Ink by drying does not flow freely.

The *Drawing-pen* is uſed only for drawing right Lines: It conſiſts of two Blades with Steel Points, fixed to a Handle; the Blades being a little bent, by Means of a Screw, cauſe the Steel Points to approach as near together as you pleaſe, ſo that a Line may be drawn ſtronger or finer at Pleaſure. When you uſe this Inſtrument put the Ink between the Blades, either with a common Pen or the Feeder, and by moving the Screw bring the Points to a proper Diſtance; then hold the Pen a little inclined, but take Care that both Points touch the Paper, by which Means you may draw a neat Line equally broad in all Parts; which is not eaſily done by a common Pen. ——— Before you put the Pen into the Caſe after uſing, it will be neceſſary to clean the Points from the Ink, otherwiſe they will be inclined to ruſt, &c. If one of the Blades is made to move on a Joint, as the *Bow* generally is, they would be more eaſily cleaned.

The *Protracting-pin* is a ſmall Piece of ſharp Steel much reſembling the Point of a Needle, fixed into the lower End of the Top Part of the Handle of the *Drawing-pin* ; it ſcrews for Preſervation into the lower Part of the Handle: Its Uſe is to ſet off Angles from the Protractor, (as will be ſhewn hereafter,) and to mark off the Interſection of Lines, that the Places of croſſing may be the better ſeen.

220. We think it needleſs to deſcribe the *Tracing Point* in this Place, as it will come in more properly in another ; and the Conſtruction of the other Inſtruments will be ſhewn, as they occaſionally become neceſſary or uſeful. We ſhall therefore, in this Place, only obſerve, that as a Ruler muſt be very ſtrait to draw right Lines with, it may be examined by drawing a right Line with it, and then turning the Ruler about End for End, apply the ſame Side to the Line, and, if the Ruler exactly coincides with the Line, the Ruler is ſtrait, but if not, you will ſee where the Fault is. A good Eye will alſo diſcover whether a Ruler is ſtrait by only looking over the Edge of it.

221. PROBLEM I. *To draw a Line from a given Point* C, *equal to a given Line* AB. P.V.F.1.

Take a Pair of Compaſſes, and placing one Point in A, open the other Point to B ; then will the Diſtance between the Points of the Compaſſes Legs be = the given Line AB ; ∵ ſetting one Leg in the Point C, with the other make another Point D, and by placing the Edge of a Ruler to touch the Points C, D, draw the right Line CD, and it is manifeſt the Thing is done ; *viz.* CD = AB.

222. PROBLEM 2. *From a greater Line*, AB, *to cut off a Part equal to a leſſer Line* C. Fig. 2.

This may be eaſily done by opening the Compaſſes equal to the Length of the Line C, (as ſhewn in the 1ſt Problem,) and laying that Diſtance from A to D on the Line AB, ſo is AD = the Line C. Which was to be done.

Fig. 3. 223. PROBLEM 3. *About a given Point*, A, *to deſcribe a Circle whoſe Radius ſhall be equal to a given Line* B.

Open the Compaſſes = the Line B; then reſting the Point of one Leg in A, carry the other about from where it falls, as C, till it arrives again at that Point, and it is manifeſt the required Circle will be deſcribed by the moving Point.

N. B. In deſcribing of Circles be careful not to preſs the Paper, either at the Center or under the moveable Point, with more than juſt the Weight of the Compaſſes, to avoid making great Holes or Blots, which frequently happen to young Perſons at their firſt beginning to practice.

Fig. 4. 224. PROBLEM 4. *To make a Triangle whoſe Sides ſhall be reſpectively equal to three given Lines.*

Let the three given Lines be A, B, C; take DE
[a] 221. [a] = C, then about D, as a Center, with a Radius =
[b] 223. B; deſcribe Part of [b] a Circle, *viz.* Arc *ab*, and take DE = C, and about E, with a Radius = A, deſcribe Part of the Circumference of a Circle, *viz.* the Arc *dc*; then from the Point of their Interſection draw the Lines FD, FE, and the △DFE will be that required; for by the Conſtruction, DE = C, DF = B, EF = A.

Note. A + B muſt be r C, otherwiſe the Arcs will not interſect, and ſo the Problem would be impoſſible.

225. PROBLEM 5. *To biſect a given Angle* BAC, *that is, to divide it into two equal Angles.* Pl. V. F. 5.

CONSTRUCTION. With any Radius AD, ſweep the Arc DE; then with any Radius Γ $\frac{1}{2}$ the Diſtance of the two Points D, E, about the Points D and E, as Centers, ſweep the Arcs *cd*, *ab*, and from the Point of Interſection F, draw the Line AF, and it is done; *viz.* the $\angle FAC = \angle FAB$.

DEMONSTRATION. AD being $= AE$, and $DF = EF$, by the Conſtruction, and AF common to both Δs DAF, EAF, the Δs are themſelves [a] equal in all Reſpects; $\because \angle DAF = \angle EAF$. Q. E. D. [a] 95.

226. PROBLEM 6. *To biſect a given finite Line* AB, *that is, to divide it into two equal Parts.* Pl. V. F. 6.

CONSTRUCTION. About the Points A and B, as Centers, with any Radius Γ $\frac{1}{2}$ AB, deſcribe Parts of Circles, *viz.* the Arcs *ab*, *cd*, and join the Points of Interſection E, F; and the Line EF will cut the Line AB in G, the Point which was required.

DEMONSTRATION. For by the Conſtruction, $AE = AF = BE = BF$, and EF common; $\because$ the Δs EAF, EBF, have 3 Sides of one reſpectively $=$ 3 Sides of the other, and $\because$ the Δs are themſelves [a] equal in all Reſpects, and $\because \angle AEG = \angle BEG$. Hence the Δs AEG, BEG, have two Sides AE, EG, of one $=$ two Sides BE, EG, of the other, and the contained $\angle$ AEG of one $=$ the contained $\angle$BEG of the other, $\because$ the Baſe AG [b] $=$ the Baſe BG. Q. E. D.

[a] 95.

[b] 58.

227. COROLLARY. *This Problem ſerves alſo to raiſe a Perpendicular on the Middle of a Line.*

For we have juſt ſhewn that the Point G is in the Middle of the Line, *viz.* AG = BG; alſo AE = BE, and EG common; ∵ the △s AEG, EBG,
a 95. are [a] equal in all Reſpects, and ∵ ∠AGE = ∠
c 14. BGE; ∵ EG [c] ⊥ to AB. Q. E. D.

Pl. V. F. 7. 228. Problem 7. *To draw a Perpendicular to a given Line* AB, *from any Point* C *in that Line.*

Construction. From the Point C ſet off any Diſtance towards A, as CD; and from the ſame Point the ſame Diſtance CE towards B. Then with any Extent of the Compaſſes r CD, putting one Leg in D, with the other ſweep the Arc *ab*, and with the ſame Extent of the Compaſſes, placing one Leg in E, with the other ſweep the Arc *cd*; then from the Point of Interſection F, draw the Line FC, and it is the ⊥ which was required.

Demonstration. CD being = CE, and DF = EF, by the Conſtruction, and FC common, the △s DCF, ECF, have 3 Sides of one reſpectively = the three Sides of the other; and ∵ the △s themſelves are equal in all Reſpects, and conſequently the ∠DCF = ∠ECF. Q. E. D.

Pl. V. F. 8. 229. Problem 8. *To draw a Perpendicular to a given Line* AB, *of an unlimited Length, from a given Point* C, *without it. Or, in other Words, to let fall a Perpendicular from a Point above to a Line beneath.*

Construction. With any Radius of a ſufficient Length to interſect the Line AB, with one Leg of the Compaſſes in C, with the other ſweep the Arc DE; then with any Extent of the Compaſſes r $\frac{1}{2}$ DE, one Leg reſting in D, with the other ſweep the Arc *ab*; in like Manner, putting one Leg in E, with the other ſweep the Arc *cd*, then from the Point of Interſection, F, draw a Line to C, and it will be the ⊥. Q. E. I.

Demonstration. For DC being = CE, and DF = EF, by the Conſtruction, and CF common, the △s DCF, ECF, have 3 Sides of one reſpectively = 3 Sides of the other, and ∵ they are[a] = each other in all Reſpects: Conſequently the ∠ DCG = ∠ ECG. Now DC being = EC, GC common, and the contained ∠DCG = the contained ∠ECG, as hath been juſt ſhewn, the △s DGC, EGC, are[b] = each other in all Reſpects; and conſequently the ∠DGC = ∠EGC. Q. E. D. [a] 95. [b] 58.

Otherwiſe thus.

Construction. Join A, C, and[a] biſect the Line AC in E; then with the Radius EA, or EC, ſweep the Semicircle CDA, and where it interſects the Line AB in D, draw the Line CD, and it will be that required. [a] 226. Pl. V. Fig. 9.

Demonstration. For the ∠ ADC in a Semicircle is a[*] Rt. ∠. Q. E. D. [*] 148.

230. Problem 9. *At a given Point* A, *in a given Line* AB, *to make an Angle equal to a given Angle* EDF. Pl. V. Fig. 10.

Construction. About the Points A and D, with any Radius, deſcribe the Arcs *ab*, *cd*, then take with the Compaſſes the Diſtance between the two Points, *d*, *c*, which ſet off from *b* to *a*, on the Arc *ab*, and thro' the Point *a* draw the Line AC; then is the ∠BAC = ∠EDF.

Demonstration. For let the Points *d*, *c*, and *b*, *a*, be joined; then by the Conſtruction the 3 Sides of the △ *b*A*a* are reſpectively = the 3 Sides of the △

*d*D*c*, and ∵ the $\triangle$s themſelves are equal in all Reſpects; conſequently the $\angle A = \angle D$. Q. E. D.

231. PROBLEM 10. *To conſtruct and ſhew the Uſe of a Protractor, that is, a Semicircle divided into Degrees, for ſetting off or meaſuring Angles.*

Pl. V. Fig. 11.
[a] 228.

CONSTRUCTION. Let ADB be a ſemicircular Piece of Braſs. Now to divide the Circumference ADB into 180 equal Parts, called Degrees, we proceed thus. On the Center C $\perp$ AB draw [a] CD, ſo will the $\angle$ ACD or BCD be a Rt. $\angle$, or the Arc AD = 90 d. = BD. Then take the Radius AC in the Compaſſes, and ſet it from A to E, and from B to F, then is AE, EF, FB each = 60 Degrees; and ſetting the ſame Diſtance from D to G, and from D to H, the Arcs DG, DH, are each = 60 d. and ſo AG, GE, ED, DF, FH, HB, each = 30 Degrees. Hence, by dividing each of theſe Arcs into 3 equal Parts, (by a Trial or two, or by Art. 241.) the Circumference will be divided into every 10 Degrees; which by taking the Halves will be divided into every 5 Degrees, and by a Trial or two the 5th Part of theſe laſt Diviſions may be taken, and ſo the Semicircle divided into every ſingle Degree, as repreſented in the Figure.

[a] 60.
[b] 85.

DEMONSTRATION. The only Thing requiring a Demonſtration in this Conſtruction is, that the Radius is = the Chord of 60 Degrees; or, that the Compaſſes being opened = the Radius, one Leg reſting in the Point A, the other will fall on the Circumference in E, the Point of 60 Degrees. For ſuppoſe A, E, joined, then by the Conſtruction, the $\triangle$ACE would be equilateral, and ∵ [a] equiangular; ∵ the $\angle ACE = {}^{b}\frac{1}{3}$ of 2 Rt. $\angle$s, that is, $\frac{1}{3}$ of 180 Degrees = 60 Degrees. Q. E. D.

232. *The 1ſt Uſe of the* PROTRACTOR *is, to make an Angle equal to any Number of Degrees.*

For Example, at the Point C, on the Line IC, to make an ∠ = 60 Degrees. Lay the Diameter AB on the Line IC, ſo that the Center of the Protractor may be at the angular Point C; then againſt 60 Degrees, (*viz.* the Point E,) make a * Dot, and, taking off the Protractor, draw the Line CK, to paſs thro' the Point E, and the ∠ KCI will be that required.

233. *Secondly. To meaſure an Angle.*

This is only the Reverſe of the laſt Article, and is done by laying the Protractor as there directed, and it will be ſeen by Inſpection what Number of Degrees is cut by the Line KC. —— All this is very plain.

234. PROBLEM II. *To conſtruct and ſhew the Uſe of a Line of Chords.* Pl. V. Fig. 12.

Definition. A Line of Chords is only the Chords taken from a Quarter of a Circle and ſet off on a Rt. Line.

Hence this CONSTRUCTION. Set one Point of the Compaſſes in the Point A, Fig. 11, and open the other Leg to any Number of Degrees, for Example Sake, to G, 30 Degrees; then that Diſtance laid from *a* towards *c*, on the Line *ac*, Fig. 12. will give the Point *b* for 30 Degrees. After the ſame

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* For this there is in the Drawing Pen a Point like a Pin, called a *Protracting Pin.* Another Kind of Protractor is made by transferring the Degrees from the Arch to the Sides of a Scale, or Ruler, as is intended to be fully deſcribed in another *Eſſay.*

Manner the other Degrees as far as 90 are laid on the Line *ac*, which then is called a Line of Chords. This is too manifeſt to need a Demonſtration.

235. *The Uſe of a Line of Chords* is very eaſy. For 1ſt. *To make an Angle equal to any Number of Degrees*, the Chord of 60 Degrees being = the Radius, as demonſtrated in the laſt Problem, with that Extent, (commonly called a Sweep of 60) deſcribe an Arc of a Circle of a ſufficient Length; then take off from the Scale (Fig. 12.) the Chord of the Number of the Degrees the ∠ is to contain, and ſet off that Diſtance on the Arc; then draw a Line from the angular Point, as in Art. 232, &c. *To meaſure an Angle being only the Reverſe of this*, we ſhall omit particular Directions as unneceſſary.

Note. If the ∠ is more than 90 Degrees, firſt ſet off the Chord of 90 Degrees on the Arc, and from the Point where that falls, ſet off the Chord of the Exceſs of the ∠ above 90 Degrees.

Pl. V. 236. PROBLEM 12. *To draw a Line through a*
Fig. 13. *given Point* A, *parallel to a given Line* BC.

CONSTRUCTION, &c. Take any Point D in the
Line BC, and join the Points A, D; then make
[a] 230. the ∠ EAD [a] = ∠ ADC, and the Line EF will be
[b] 73. [b] || to BC. Q. E. I. & Q. E. D.

Pl. V. 237. PROBLEM 15. *At any given Diſtance to*
F. 14. *draw a Line parallel to another given Line* AB.

CONSTRUCTION. In the Line AB take any
[a] 228. Point C, and erect CE [a] ⊥ to AB; then on it ſet
off the given Diſtance from C to G, and make the
[b] 230. ∠ KGC [b] = ∠ BCG, then will the Line IK be [c] ||
[c] 40. to AB. Q. E. I. & Q. E. D.

238. *Scholium.* This and the laſt Problem may be performed ſomething more expeditiouſly, though not ſtrictly geometrical, by the following Methods.

Problem 12th *may be thus conſtructed.* Pl. V. Fig. 15.

Setting one Leg of the Compaſſes in the Point A, open the other ſo as, being turned about, it may touch, but not cut the Line BD (as repreſented by the Arc *ab*;) then take any Point C in the Line BD, and placing one Point of the Compaſſes in it, with the ſame Extent of the Compaſſes, with the other Leg deſcribe the Arc *cd*; then lay a Ruler ſo as to touch the Arc *cd* and the Point A, and draw the Line EA; which will be that required.

The 13th Problem *may be thus conſtructed.*

With a Radius = the given Diſtance, one Leg of the Compaſſes being in C, (a Point taken any where in the Line AB,) with the other deſcribe the Arc *ab*: Again, take another Point D in the Line AB, and putting one Leg of the Compaſſes in it, with the other Leg and the ſame Radius deſcribe the Arc *cd*; then applying a Ruler to touch theſe two Arcs, draw the Line EF for that required. Theſe two Methods of drawing parallel Lines are grounded on the * Theorem that parallel Lines are every where equidiſtant. Pl. V. Fig. 16. * 96.

239. Problem 14. *To conſtruct a parallel Ruler for the more ready drawing parallel Lines.*

Construction. Let AB, CD, be two Rulers, very ſtraight, and every where of an equal Breadth, joined together by two Braſs or Silver Pieces, EG, FH, of equal Length, moveable about the Pins E, F, G, H. Note, the Diſtance between the Pins Pl. V. Fig. 17.

EF muſt be = the Diſtance between the Points, G, H; then will the Figure repreſent a parallel Ruler.

Demonstration. For whatever Diſtance we
open the Rulers AB, CD, to, they will always be
|| to each other. For, EG being = FG, and EF
= GH, by the Conſtruction, and EH common, the
△s EGH, EFH, have 3 Sides of one reſpectively
= the 3 Sides of the other, and ∵ the △s are equal
[a] 95. in [a] all Reſpects; conſequently the ∠ GHE = ∠
[b] 73. FEH, and ∵ AB [b] || to CD. Q. E. D.

240. Figure 18th repreſents another Method of making a parallel Ruler. The Braſs Pieces EH, GF, are of equal Length, joined together exactly in the Middle by a Pin I, about which they are moveable, in the ſame Manner as a Pair of Sciſſars. The Points F and H are moveable about the Pins F, H; and the Ends, E and G, move in Slits made in the Ends, A and C, of the Rulers.

Demonstration. That the Rulers, AB, CD,
being opened to any Diſtance, will always be || to
each other, will appear by conſidering that FI, HI,
EI, GI, are = each other, by the Conſtruction,
and the ∠EIF [a] = ∠GIH, and ∵ the △sEIF, GIH,
[a] 69. [b] 58. are [b] equal in all Reſpects; and conſequently the ∠
[c] 73. FEI = ∠IHG, and ∵ the Rulers AB, CD [c] || to
each other. Q. E. D.

Scholium. *The Manner of Uſing a parallel Ruler,* when a Line is to be drawn parallel to a given Line, at a Diſtance not exceeding that to which the Ruler will open, ſeems too evident to need any Deſcription: But when the given Point thro' which the parallel Line is to be drawn, is at a greater Diſtance from the given Line than the Ruler can be opened, in ſuch Caſe it may be convenient to hint, that having placed the Edge of one of the Rulers, ſuppoſe CD, on the given Line, and opened the

Ruler as much as may be towards the given Point, keep the Ruler AB, which is neareſt that Point, firm in its Poſition, and then bring the other Leg CD cloſe to it; where ſtopping it, move forward the Ruler AB, and thus proceed, alternately moving each Ruler carefully, (ſo as to continue always parallel to each other,) until the Ruler AB touches the given Point; when the required Line may be drawn by the Edge of that Ruler.

241. PROBLEM 15. *To triſect an Angle* ACB, *that is, to make an Angle equal to one third of a given Angle.* Pl. V. F. 19.

CONSTRUCTION. About C, as a Center, with any Radius deſcribe the Semicircle DEH, and produce BH at Pleaſure. Then prick a fine Pin in the Point E, and on the Edge of a Ruler GI ſet off GF = the Radius of the Semicircle; then move the Ruler ſo that the End G may always be in the Line BH produced, and the Edge of the Ruler alſo touch the Pin E; and obſerve when the Point F on the Edge of the Ruler touches the Circumference of the Semicircle; then ſtop the Ruler in that Poſition, and draw the right Line GE; then will the ∠ FCH be $\frac{1}{3}$ of the ∠ ECD, or, which is the ſame, the Arc HF = $\frac{1}{3}$ of the Arc DE. Q. E. I.

Note. This Problem is impoſſible to be conſtructed by a Method ſtrictly geometrical, either by right Lines, or Circles; which is our Reaſon for giving the above mechanical Method.

DEMONSTRATION. Let the Number of Degrees contained by the ∠ FGC be denoted by *a*; then, FG being = FC by the Conſtruction, the ∠ FCG = ∠ FGC = *a* Degrees; ∵ ∠ FCG + ∠ FGC = *a* + *a* = 2*a*. But the ∠ EFC being the external ∠ of the △GFC is [b] = the two internal and oppoſite ∠ s, ∵ = 2*a*; but FC = EC by the Conſtruction, ∵ the △FCE is Iſoſceles, and ∵ ∠ EFC [b] = ∠ FEC = 2*a*

[a] 59.

[b] 76.

[c] 78. $= 2a$ by the above. Now the $\angle$FCE [c] $=$ *180°. *minus* the Sum of the two $\angle$s EFC, and FEC, that is, $= 180° - 4a$. But the Arcs HF, FE, are the Meaſures of the $\angle$s GCF, and FCE, reſpectively; $\because$ the Arc HE$=$FE $+$ Arc HF $= 180° - 4a + a$, that is, $= 180° - 3a$ (for one a being to be added, and $4a$ ſubtracted, it is manifeſtly the ſame if we only ſubtract $3a$.) Now a Semicircle being $=$ 180° the Arc ED muſt be $=$ 180° *minus* Arc HE, that is, $=$ 180° *minus* $\overline{180° - 3a}$, that is, $= 180° - 180° + 3a$, (to ſubtract a Negative being the ſame as to add an Affirmative) $= 3a$; Hence ED $=$ 3 Times HF, or, which is the ſame, the $\angle$FCG$= \frac{1}{3}$ of $\angle$ACB. Q. E. D.

Pl. V. Fig. 20. 242. PROBLEM 16. *To deſcribe a Parallelogram that ſhall be equal to a given Triangle* ABC, *and have one of its Angles equal to a given Angle* D.

[a] 226. CONSTRUCTION. Biſect [a] BC in E, (and join AE) and at the Point E, in the Line EC, make the
[b] 230. $\angle$ CEF [b] $= \angle$D; then through the Point A draw
[c] 236. AG [c] || to EC, and alſo CG || to EF; then is EFGC the Parallelogram which was required.

DEMONSTRATION. The $\angle$ ECF being $= \angle$D, CG || to EF, and FG || to EC, by the Conſtruction, the Figure EFGC is a [d] Parallelogram, and one
[d] 97. of its $\angle$s CEF $=$ the given $\angle$D. And becauſe BE
[e] 109. $=$ EC, the $\triangle$ AEB [e] $= \triangle$ AEC, and the $\triangle$ AEC and Parallelogram EG, ſtanding on the ſame Baſe and between the ſame Parallels, the Parallelogram
[f] 106. is [f] double of the $\triangle$AEC, and conſequently $=$ the whole $\triangle$ABC. Q. E. D.

Pl. V. Fig. 21. 243. PROBLEM 17. *To apply a Parallelogram to a given Line* AB, *which ſhall be equal to a given Triangle* C, *and have one of its Angles equal to a given Angle* D.

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* By 180° is ſignified 180 Degrees.

Construction. Make the Parallelogram BEFG = the △C, and having the △ EBG [a] = ∠D, ſo that BE be in the ſame Line with AB. Then produce FG to H, and through A draw AH [b] || to GB or FE, and join the Points H, B, and produce FE, HB, till they meet each other, and let K be the Point where they meet: Draw KL || to EA or FH, and produce GB, HA, to M and L; then will ABML be the Parallelogram which was required.

[a] 242.
[b] 236.

Demonstration. For KL being || to FH, and FK || HL, by the Conſtruction, the Figure FKLH is a [c] Parallelogram, whoſe Diameter is HK, and AG, ME, are Parallelograms about HK, and LB, BF are the Complements; ∵ BL [d] = BF; but BF = △C by the Conſtruction, ∵ BL [e] = △C, and ∠ EBG = D by the Conſtruction, and ∠ ABM [f] = ∠EBG, ∵ ∠EBG [e] = ∠ ABM; conſequently the Parallelogram BL is = the △C, and has one ∠ABM = the ∠D. Q. E. D.

[c] 97.
[d] 113.
[e] 45.
[f] 69.

244. Problem 18. *To deſcribe a Parallelogram equal to a given rectilineal Figure* ABCD, *and having an Angle equal to a given rectilineal Angle* E. Pl. VI. Fig. 1.

Construction. Join BD, and make the Parallelogram FH [a] = the △ ADB, and having the ∠ HFK = the ∠E; and to the Line GH apply the Parallelogram GM [b] = the △ BDC, having the ∠ GHM = ∠E; then will the Figure FLMK be the Parallelogram required.

[a] 242.
[b] 243.

Demonstration. For the ∠s FKH, GHM, being each = ∠E, by the Conſtruction, it is manifeſt KM is one Rt. Line, and ∵ the Parallels to it, FG, GL, paſſing through the ſame Point G, will be one Rt. Line FL; and GM being a Parallelogram, LM is || to GH, and, for the ſame Reaſon, FK ||

to GH, and conſequently LM, FK are Parallels, and ∵ the Figure FLMK a Parallelogram, having the ∠ FKM = ∠E; and the Parallelogram FH being = △ ADB, and Parallelogram GM = △ BDC, the whole Parallelogram FLMK muſt be = the whole Figure ADCB. Q. E. D.

245. Scholium I. After the ſame Manner it is manifeſt a Parallelogram may be deſcribed, equal to any given rectilineal Figure, how many ſoever the Sides are; for Example, if the given Figure contained one △ more, it would be only to apply a Parallelogram to the Line LM equal to that third △, and having one ∠ = ∠E, by Prob. 17.

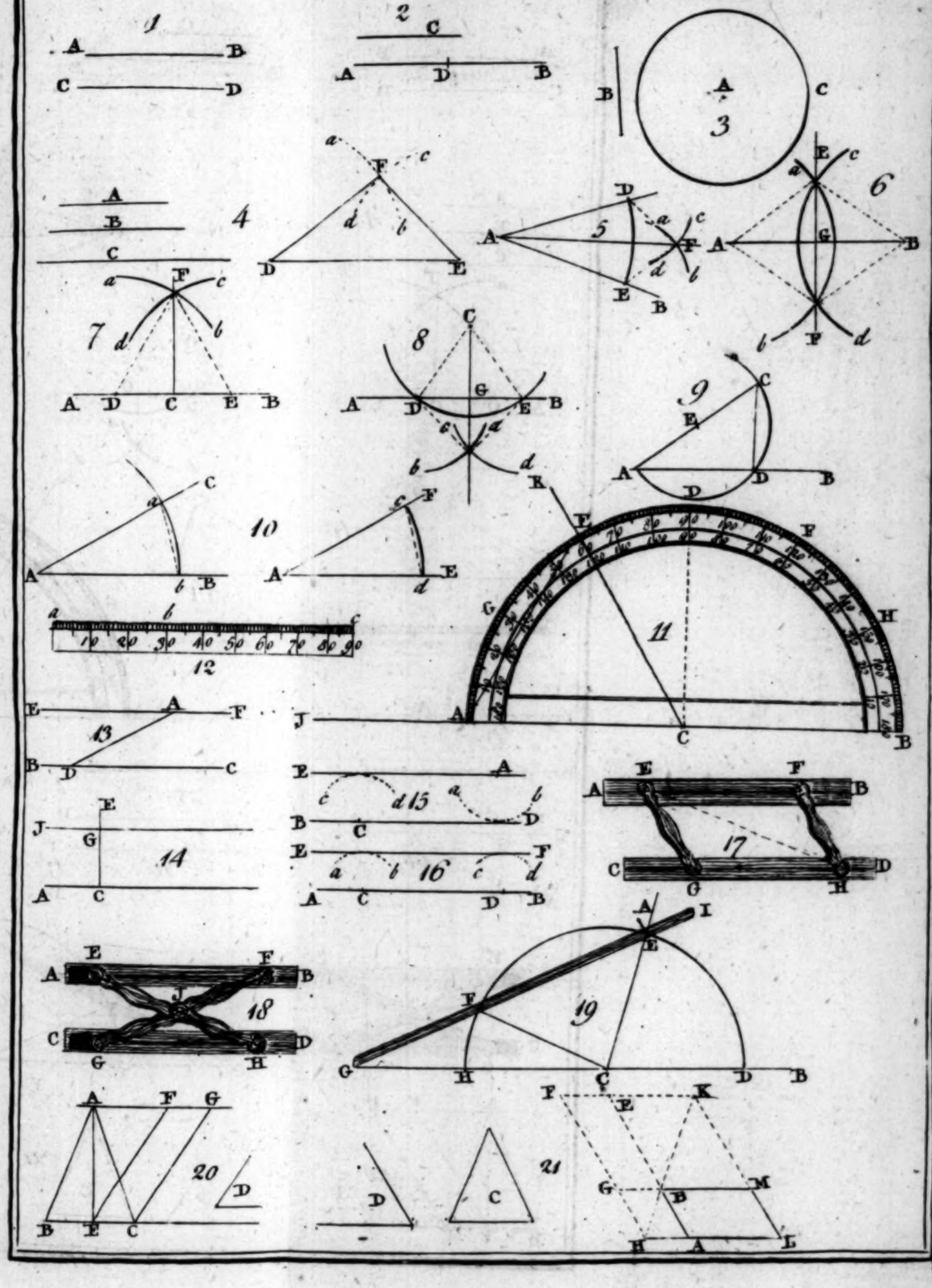
Pl. VI. Fig. 2. 246. Scholium 2. *By this Problem may be eaſily found a Parallelogram equal to the Exceſs of one right-lined Figure given, above another right-lined Figure given.* Thus, if to any Line, CD, a Parallelogram DF is applied = A, and to the ſame Line CD a Parallelogram DH = B, it is manifeſt their Difference, or the Parallelogram GF will be = the Difference between the Figures A and B.

Scholium 3. *Hence alſo it is manifeſt how to find a Parallelogram = the Sum of any two rectilineal Figures.* For if the Parallelogram DH be made = A, and on GH the Parallelogram GF = B, and have an ∠ HGE = ∠CDG, the Parallelogram DF muſt be = Parallelogram DH + Parallelogram GF, or, which is the ſame, A + B = Parallelogram DF.

Pl. VI. Fig. 3. 247. Problem 19. *To deſcribe a Rectangle whoſe Sides ſhall be equal to two given Lines* AB, EF.

Construction. On one of the given Lines AB, at one Extremity A, erect the Perpendicular AD; then take AB in the Compaſſes, and ſetting one Leg in D, with the other deſcribe the Arc *ab*; after the ſame

P. 84. Plate V.


1
A B
C D
2
C
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4
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B
C
a F c
d b
D E
5
A D a c F d b E B
6
E a c
A G B
b F d
7
a F c
d b
A D C E B
8
C
A D G E B
c a
b d
9
C
E
A D B
D
10
a C
A b B
c F
A d E
12
a b c
10 20 30 40 50 60 70 80 90
11
K E F
G H
A C B
J
13
E A F
B D C
14
E
J G
A C
15
E A
c d a b
B C D
16
E F
a b c d
A C D B
17
A E F B
C G H D
18
A E F B
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19
A I
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G H C D B
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A F G
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21
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ſame Manner, with the Extent of EF, and one Leg in B, with the other ſweep the Arc *cd*, and let C be the Point of Interſection; then join DC, BC, and the Thing is done.

DEMONSTRATION. The oppoſite Sides being equal by the Conſtruction, the oppoſite $\angle$s are alſo [a] equal; and $\because$ the $\angle$ DAB being a Rt. $\angle$ by the Conſtruction, the other $\angle$s muſt be ſo too: And $\because$ the Figure ABCD is a [b] Parallelogram, and is that required, becauſe its Sides are by the Conſtruction = the given Lines. Q. E. D.

[a] 98.
[b] 35.

248. SCHOLIUM. *Hence we have an eaſy Method for conſtructing a Square*; for if EF and AB are equal, the Rectangle will be a [c] Square.

[c] 34.

249. PROBLEM 20. *To make a Square equal to any Number of given Squares*, e. g. *equal to three given Squares, whoſe Sides are equal to the Lines* A, B, C.

Pl. VI. F. 4.

CONSTRUCTION. Draw two indefinite Lines ED, DF, at Rt. [d] $\angle$s to each other, and take DG = A, and DH = B, and join GH, (and the Line DH will be the Side of a Square = the two Squares whoſe Sides are A and B.) Again, take DI=GH, DK = C, and join KI, then will KI be the Side of a Square = the three given Squares. Q. E. I.

[d] 228 or 232.

DEMONSTRATION. For GH^2 [e] $= GD^2 + DH^2$; but DG = A, and DH = B, by the Conſtruction, and $\because$ $DG^2 = A^2$, and $DH^2 = B^2$, conſequently GH^2 [f] $= A^2 + B^2$. Again, $KI^2 =$ [a] $DK^2 + DI^2$; but $DI^2 = (GH^2 =) A^2 + B^2$ by the above, and $DK^2 = C^2$, by the Conſtruction, conſequently, $KI^2 =$ [b] $A^2 + B^2 + C^2$. Q. E. D. ——— After the ſame Manner may a Square be conſtructed = any Number of given Squares.

[e] 115.
[f] 45.
[a] 115.
[b] 45.

Pl. VI. Fig. 5. 250. Problem 21. *To make a Square equal to the Difference of two given Squares, whoſe Sides are* AB *and* CD.

Construction. On one Extremity B, of the ſhorteſt Line AB, raiſe a ⊥ BE, then taking CD in the Compaſſes, put one Leg in A, and with the other deſcribe the Arc *ab*, interſecting BE in F; draw AF, and it will be the Side of the required Square.

Demonstration. For $AB^2 + BF^2$ [c] $= AF^2$,
[c] 115. ∵ taking AB^2 from both Sides, we have BF^2 [d] $=$
[d] 47. $AF^2 - AB^2$; but AF = CD by the Conſtruction,
[e] 45. ∵ $AF^2 = CD^2$, conſequently BF^2 [e] $= CD^2 - AB^2$.
Q. E. D.

Pl. VI. Fig. 6. 251. Problem 22. *To make a Trianlge equal to any given quadrilateral Figure* ABCD.

Construction. Draw the Diagonal BD, and ∥ to it draw CE interſecting AD produced in E; then join EB, and the △ ABE will be that which was required.

Demonstration. For the △s DCB, DEB, being on the ſame Baſe BD, and between the ſame Parallels, (or, which is the ſame Thing, having
[a] 109. equal Altitudes,) are [a] = each other; ∵ △ DEB + △DBA = △DCB + △DBA; but the △DCB + △DBA = the whole quadrilateral Figure ABCD, and △DEB + △DBA = whole △EBA, ∵ the △ EBA = the Quadrilateral ABCD. Q. E. D.

Pl. VI. Fig. 7. 252. Problem 23. *To make a Triangle equal to any given five-ſided Figure* ABCDE.

Construction. Join EC, AC, and draw DF ∥ to EC, and BG ∥ to CA, meeting AE produced,

duced, in the Points F and G, reſpectively ; then draw CF and CG, and the $\triangle$FCG will be that required.——-This is ſo very much like the laſt Problem that it needs no Demonſtration.

253. PROBLEM 24. *To deſcribe a Circle through three given Points, (provided they are not in a right Line.)* Pl. VI. Fig. 8.

CONSTRUCTION. Let A, B, C, be the three Points given ; draw the Lines AB and BC, and [a] biſect them by the Lines DE, FG, and the Point of their Interſection H will be the Center of the Circle ; ∵ put one Leg of the Compaſſes in H, and open the other Leg to one of the given Points, and with that Extent ſweep a Circle, and the Circumference will paſs thro' the three given Points.

[a] 226.

DEMONSTRATION. Suppoſe AH, BH, CH, joined ; then DB being = DA, and DH common, and the contained $\angle$ ADH = the contained $\angle$ BDH by the Conſtruction, the Side AH [b] = Side BH ; and by the ſame Reaſoning BH = CH, ∵ AH = BH = CH ; conſequently H is the [c] Center of a Circle whoſe Circumference will paſs thro' the Points A, B, and C. Q. E. D.

[b] 58.

[c] 136.

254. SCHOLIUM. *By this it is manifeſt, the Center of any given Circle may be found,* by aſſuming any three Points in its Circumference, and proceeding as above. *By this alſo, when we have a Segment of a Circle given, we may compleat the whole Circle.*——— *This Problem alſo ſhews how to deſcribe a Circle about a given Triangle.*

255. PROBLEM 25. *To draw a Tangent to a Circle, from a given Point* D *without it.* Pl. VI. Fig. 9.

CONSTRUCTION. Let B be the Center of the Circle ; join BD, which biſect in [a] C ; then about C, as

[a] 226.

as a Center, with the Radius CB, or its equal CD, deſcribe a Semicircle BAD, and draw DA from D to A, the Point where the Circumference of the Semicircle interſects that of the Circle, and DA will be the Tangent required.

DEMONSTRATION. For if BA be joined, the ∠
[b] 148. BAD is a [b] Rt. ∠, being in a Semicircle ; and ∵
[c] 141. DA is a [c] Tangent to the Circle at the Point A.
Q. E. D.

256. SCHOLIUM. *If it was required to draw a Tangent from a given Point* A, *in the Circumference of the Circle*, it would be only to erect a ⊥ AD to the Radius BA at the Point A.

Pl. VI. Fig. 10. 257. PROBLEM 26. *Upon a given Line* AB, *to deſcribe a Segment of a Circle, which ſhall contain a given Angle* C.

[d] 230. CONSTRUCTION. Make the ∠ DAB [d] = ∠C ;
[e] 228. and on AD, at the Point A, erect [e] the ⊥ AI, and
[a] 226. [a] biſect AB by the Line FH ; then let the Point G,
where the Lines AI, FH, interſect, be a Center, about which, with the Radius GA, deſcribe the Circle ABE, and the Segment AEB will be that required.

DEMONSTRATION. Suppoſe B, G, joined ; then
AF being = BF by the Conſtruction, FG com-
[b] 227. mon, and contained ∠ AFG [b] = contained ∠ BFG,
[c] 58. the Side AG is [c] = Side BG, ∵ a Circle deſcribed
about G, as a Center, will paſs thro' the Points A
and B ; let E be the Point where the Circle inter-
[d] 151. ſects AI, and join BE, then the ∠ AEB [d] = BAD ;
but the ∠ BAD = ∠C, by the Conſtruction, ∵ the
[e] 45. ∠ AEB [e] = ∠C ; that is, the Segment AEB contains
an ∠ = the given ∠C. Q. E. D.

258. SCHOLIUM. If the given ∠ is a Rt. ∠, we have only on AB to deſcribe a Semicircle; becauſe the ∠ in a Semicircle is a [f] Rt. ∠. [f] 148.

259. PROBLEM 27. *To cut off a Segment from a given Circle* ABC, *which ſhall contain an Angle equal to a given Angle* D. Pl. VI. Fig. 11.

CONSTRUCTION. To any Point B, in the Circumference, draw [a] the Tangent EF, and make the ∠ FBC [b] = the given ∠ D; then is the Segment BAC that required. [a] 256. [b] 230.

DEMONSTRATION. For the ∠ FBC [c] = BAC, and the ∠ FBC alſo = ∠ D, by the Conſtruction, ∴ ∠ BAC [d] = ∠ D. Q. E. D. [c] 151. [d] 45.

260. PROBLEM 28. *To * inſcribe in a given Circle,* BCD, *a Line equal to any given Line,* A, *leſs than the Diameter of the Circle.*

CONSTRUCTION. About any Point B, in the Circumference, as a Center, with a Radius = the given Line A, deſcribe the Arc *ab*, interſecting the Circumference in C; join BC, and the Thing is done. This needs no Demonſtration, it being manifeſt from the Conſtruction that BC = A. Pl. VI. Fig. 12.

261. PROBLEM 29. *In a given Circle* ABC, *to ‡ inſcribe a Triangle equiangular to a given Triangle* DEF. Pl. VI. Fig. 13.

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* A Line is ſaid to be inſcribed, or placed in a Circle, when the Extremities of it are in the Circumference of the Circle.

‡ A rectilineal Figure is ſaid to be inſcribed in a Circle when all the angular Points of the inſcribed Figure are upon the Circumference of the Circle.

CONSTRUCTION. To any Point A, in the Cir-
[e] 256. cumference, [e] draw the Tangent GAH, and make
[a] 230. the ∠ HAC [a] = ∠ DEF, and the ∠ GAB = ∠
DFE, and join BC; then is the △ ABC that required.

[b] 151. DEMONSTRATION. For ∠ HAC [b] = ∠ ABC,
and ∠ HAC = ∠DEF by the Conſtruction, ∵ ∠
[c] 45. ABC [c] = ∠ DEF: By the ſame Reaſoning, ∠
ACB = DFE. ∵ the remaining ∠BAC = the remaining ∠ EDF, and ſo the △s ABC, DEF, are
[d] 80. equiangular. Q. E. D.

Pl. VI. F. 14. 262. PROBLEM 30. *About a given Circle* ABC, *to* † *deſcribe a Triangle equiangular to a given Triangle* DEF.

CONSTRUCTION. Produce EF both Ways to G
and H; from the Center K draw a Radius KB, and
[e] 230. make the ∠ KBA [e] = ∠ DEF, and ∠ BKC = ∠
DFH, and thro' the Points A, B, C, draw the Tangents LAM, MBN, NCL, and the △ LMN will be that required.

DEMONSTRATION. The ∠s MAK, MBK,
[a] 142. being [a] Rt. ∠s, and the four ∠s of any quadrilate-
[b] 87. ral Figure being [b] = four Rt. ∠s, the ∠AMB is =
[c] 64. 2 Rt. ∠s − ∠BKA; but ∠DEF [c] = 2 Rt. ∠s —
∠DEG, and ∠ BKA = ∠ DEG by the Conſtruc-
[d] 45. tion, ∵ ∠AMB [d] = 2 Rt. ∠s — ∠DEG; ∵ ∠
[e] 45. AMB [e] = ∠DEF; and by the ſame Reaſoning,
∠BNC = ∠DFE. Hence we have ſhewn that the two ∠s LMN, LNM, are reſpectively = the two ∠s DEF, DFE, and, conſequently, the remaining
[f] 80. ∠MLN [f] = the remaining ∠EDF; and ∵ the ∠s
LMN, DEF, are equiangular. Q. E. D.

263.

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† A rectilineal Figure is ſaid to be deſcribed about a Circle when each Side of the circumſcribing Figure touches the Circumference of the Circle.

263. Problem 31. *To * inſcribe a Circle in a* Pl. VI.
given Triangle ABC. Fig. 15.

Construction. Biſect [g] the ∠s ABC, BCA, [g] 225.
by the Lines BI, CH, and let D be the Point of In-
terſection, and draw DF [h] ⊥ to BC; then about D, [h] 229.
as a Center, with a Radius = DF, deſcribe a Circle
FGE, and it will be that required.

Demonstration. Let DE be ⊥ to BA, and
DG ⊥ to AC; then the ∠s BED, BFD, being
[a] Rt. ∠s, are = each other; and the ∠ EBD [a] 14.
= ∠ FBD by the Conſtruction, and the Side BD
common, ∵ the △s EBD, FBD, are = [b] each [b] 94.
other in all Reſpects, and conſequently ED = DF.
By the ſame Reaſon DG = DF; ∵ ED, DF, DG,
are = each other, and conſequently the Circle de-
ſcribed about the Center D with the Radius DF,
will paſs thro' the Points FEG, or, in other Words,
will touch the Sides of the given Triangle in the
Points E, F, and G. Q. E. D.

264. Problem 32. *To inſcribe a Square in a* Pl. VII.
given Circle ABCD. F. 1.

Construction. Through the Center E draw
the Diameter AC, and [c] ⊥ to it the Diameter AC, [c] 226.
and join BA, AD, DC, BC, then is the rectilineal
Figure ABCD the Square required.

Demonstration. BE being = ED, AE com-
mon, and the ∠ AEB [d] = ∠ AED, the Side AB [d] 14.
will be [e] = Side AD; and, by the ſame Reaſon, AB [e] 58.
= BC, and AD = DC; ∵ AB [a] = BC = AD = [a] 45.
DC; that is, the rectilineal Figure ABCD is equi-
lateral. And BD being a Diameter, the ∠ BAD is

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* A Circle is ſaid to be inſcribed in a rectilineal Figure when the Circumference touches each Side of the rectilineal Figure.

[b] 148. an ∠ in a Semicircle, and ∵ a Rt. [b] ∠ : By the
ſame Reaſon the ∠s ABC, BCD, CDA, are each
of them Rt. ∠s. Hence we have ſhewn that the
rectilineal Figure ABCD is equilateral, and its ∠s
[c] 34. all Right ∠s. Conſequently it is a [c] Square. Q. E. D.

Pl. VII. F. 2. 265. PROBLEM 33. *To deſcribe a Square about a given Circle* ABCD.

CONSTRUCTION. Draw the Diameters AC, BD,
[d] 226. [d] at Rt. ∠s to each other, and thro' the Points A,
[e] 256. B, C, D, draw the [e] Tangents GF, GH, HK,
KF; then will the Figure FGHK be the required
Square.

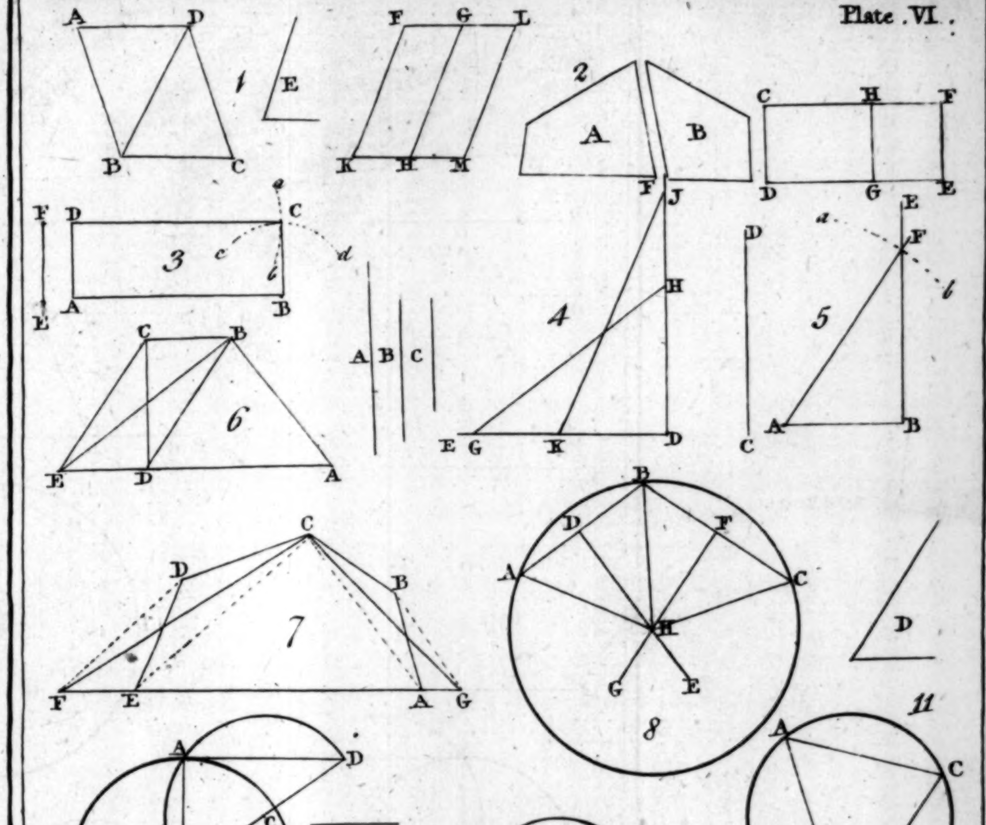
DEMONSTRATION. For the ∠s at A, B, C, D,
are right ∠s by the Conſtruction, and becauſe the
∠s GBE, AED, are each of them Rt. ∠s, they
[f] 40. are equal to each other, and ∵ GH [f] || to AC;
[g] 74: and by the ſame Reaſon FK || to AC, ∵ GH [g] ||
to FK. After the ſame Manner it may be ſhewn
that GF is || to HK; hence the Figure FGHK is
[a] 97. a [a] Parallelogram, and its oppoſite Sides and ∠s are
equal; alſo GC, AK, FB, BK, are Parallelograms,
and ∵ their oppoſite Sides are equal; hence GF =
BD in the Parallelogram FB, and GH = AC in the
Parallelogram GC; but AC = BD being each the
Diameter of the Circle, ∵ GF = GH; and the
oppoſite Sides of the Figure FGHK having been
ſhewn above to be equal, it follows that GF = GH
= HK = FK, or that it is equilateral: It is alſo
rectangular, for the ∠s GAE, AEB, EBG, being
[b] 87. all Rt. ∠s, the ∠ AGB muſt be alſo a [b] Rt. ∠ : After the ſame Manner it may be ſhewn that the ∠s
at the Points H, K, F, are Rt. ∠s; ∵ the Figure GFKH has been proved to be both equilateral
[c] 34. and rectangular; ∵ it is a [c] Square. Q. E. D.

Pl. VII. F. 2. 266. PROBLEM 34. *To inſcribe a Circle in a given Square* FGHK.

CON-

P. 92.

Plate. VI.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Construction. [d] Biſect the Sides GF, GH, [d] 226.
by the Lines BD, AC; then about the Point of Interſection E, with the Radius EA, ſweep the Circle ABCD, and it will be that which was required.

Demonstration. The ∠s AGB, EBH, being both Rt. ∠s by the Conſtruction, are = each other,
and ∴ BE [a] || to GA; by the ſame Reaſon AE || [a] 40.
to GB; ∴ the Figure AGBE is a [b] Parallelogram, [b] 97.
and ∴ its oppoſite Sides are equal, *viz.* AE = GB, and BE = GA: But GB, GA, being the Halves of equal Lines, *viz.* of GF, GH, (by the
Conſtruction) are [c] = each other, ∴ AE = GB = [c] 51.
BE = GA, or the Figure is equilateral; it is alſo rectangular, for the ∠s EAG, AGB, GBA, being Rt. ∠s by the Conſtruction, the other ∠ AEB muſt
be a [d] Rt. ∠ alſo; and ∴ the quadrangular Fi- [d] 87.
gure AGBE a [e] Square. After the ſame Manner [e] 34.
AFDE, DKCE, CHBE, are ſhewn to be Squares = the former; ∴ EA = EB = EC = ED, and conſequently, a Circle deſcribed about the Center E, with the Radius EA, will paſs thro' the Points A,
B, C, D, and [f] touch the Sides of the given Square [f] 141.
in thoſe Points, (as the ∠s at thoſe Points are Rt. ∠s.
Q. E. D.

267. **Problem** 35. *To deſcribe a Circle about a given Square* ABCD. Pl. VII. F. 1.

Construction. Draw the Diagonals AC, BD; then, about the Point of Interſection E, as a Center, with the Radius EA, deſcribe a Circle, and it will be that required.

Demonstration. AB being = DC, AD com-
mon, and ∠BAD = ∠ADC, BD [a] = AC; and [a] 58.
EA [b] = EC, and ED [b] = EB. But the whole Dia- [b] 14.
gonals AC, BD, have been juſt ſhewn to be equal,

∴

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CONSTRUCTION. [d] Biſect the Sides GF, GH, d 226.
by the Lines BD, AC; then about the Point of
Interſection E, with the Radius EA, ſweep the
Circle ABCD, and it will be that which was required.

DEMONSTRATION. The ∠s AGB, EBH, being
both Rt. ∠s by the Conſtruction, are = each other,
and ∵ BE[a] ∥ to GA; by the ſame Reaſon AE ∥ a 40.
to GB; ∵ the Figure AGBE is a [b] Parallelogram, b 97.
and ∵ its oppoſite Sides are equal, *viz.* AE =
GB, and BE = GA: But GB, GA, being the
Halves of equal Lines, *viz.* of GF, GH, (by the
Conſtruction) are [c] = each other, ∵ AE = GB = c 51.
BE = GA, or the Figure is equilateral; it is alſo
rectangular, for the ∠s EAG, AGB, GBA, being
Rt. ∠s by the Conſtruction, the other ∠ AEB muſt
be a [d] Rt. ∠ alſo; and ∵ the quadrangular Figure d 87.
AGBE a [e] Square. After the ſame Manner e 34.
AFDE, DKCE, CHBE, are ſhewn to be Squares
= the former; ∵ EA = EB = EC = ED, and
conſequently, a Circle deſcribed about the Center E,
with the Radius EA, will paſs thro' the Points A,
B, C, D, and [f] touch the Sides of the given Square f 141.
in thoſe Points, (as the ∠s at thoſe Points are Rt. ∠s.
Q. E. D.

267. PROBLEM 35. *To deſcribe a Circle about a* Pl. VII.
given Square ABCD. F. 1.

CONSTRUCTION. Draw the Diagonals AC, BD; then, about the Point of Interſection E, as a Center, with the Radius EA, deſcribe a Circle, and it will be that required.

DEMONSTRATION. AB being = DC, AD common,
and ∠BAD = ∠ADC, BD [a] = AC; and a 58.
EA [b] = EC, and ED [b] = EB. But the whole Diagonals b 14.
AC, BD, have been juſt ſhewn to be equal,

∵

$\because$ their Halves muſt be equal alſo, that is, $EA = ED = EC = EB$; and conſequently a Circle deſcribed about the Point E, with a Radius $= EA$, muſt paſs thro' the Points A, B, C, and D. Q. E D.

268. Before we proceed any further, it may be proper to give ſome Definitions concerning *Polygons.*

A *Polygon* is generally defined to be a Plane Figure terminated by more than four Sides. ——— According to its Etymology it is a Figure having many Angles; which is in Fact the ſame as the above Definition; for when a Figure has many Sides it muſt have many Angles, *et contrà*; the Number of Sides and Angles being always equal. — When it is both *equilateral* and *equiangular*, it is called *regular*, otherwiſe *irregular*.

A Polygon of	5	Sides is called a	Pentagon
	6		Hexagon
	7		Heptagon
	8		Octagon
	9		Enneagon
	10		Decagon
	11		Endecagon
	12		Dodecagon
	&c.		&c.

Note. In what follows we would be underſtood to treat of regular Polygons only.

269. The Angle made by any two contiguous Sides of a Polygon is called the *Angle of the Polygon.* And the Angle made by two Lines drawn from the Center of the Polygon, to the extreme Points of the ſame Side of the Polygon, is called the *Angle at the Center.*

270. A LEMMA: *Lines biſecting the Angles of the Polygon, will interſect each other in one Point within the Polygon, and may be called Radii, for they will be equal to each other.* Pl. VII. Fig. 3.

For the $\angle$s of the Polygon being [a] equal, their Halves muſt be ſo too: $\because$ the $\angle ABS = \angle BAS$, $\because$ SA [b] $= SB$; and by the ſame Reaſon $SB = SC$, &c. $\because$ $AS = SB = SC$, &c. Q. E. D.

[a] 268.
[b] 62.

271. COROLLARY 1. *Hence, Lines drawn from the Center of a Polygon to the Extremities of each Side, will divide the Polygon into as many Iſoſceles Triangles, equal to each other in all Reſpects, as the Figure has Sides.*

272. COROLLARY 2. *Hence alſo, each Angle at the Center* (as all the Angles at that Point are [c] = 4 Rt. $\angle$s, or 360 Degrees,) *muſt be* = 360 *Degrees divided by the Number of Sides.*

[c] 68.

273. COROLLARY 3. *Hence alſo it follows, that the Angle of the Polygon is* $= 180^\circ -$ *the* $\angle$ *at the Center.*

For $\angle ABS + \angle SAB$ [a] $= 180^\circ - \angle$ at the Center; but $\angle ABS = \angle SAB$ by the [b] above, $\because$ ($\angle ABS + \angle ABS$, or, which is the ſame) $2\angle ABS = 180^\circ - \angle$ at the Center; but $2\angle ABS$ is $= \angle$ of the Polygon, $\because$ the $\angle$ of the Polygon $= 180^\circ - \angle$ at the Center.

[a] 78.
[b] 270.

274. From the two laſt Corollaries the following Table is eaſily conſtructed.

Name

Name of the Polygon.	Angle at Center.	Angle of the Polygon.
Pentagon	72° 00′	108° 00′
Hexagon	60 00	120 00
Heptagon	51 $25\frac{5}{7}$	128 $34\frac{4}{7}$
Octagon	45 00	135 00
Enneagon	40 00	140 00
Decagon	36 00	144 00
Endecagon	32 $43\frac{7}{11}$	147 $16\frac{4}{11}$
Dodecagon	30 00	150 00

Pl. VII. Fig. 3. 275. PROBLEM 36. *In a given Circle to inſcribe a Polygon of a given Number of Sides.*

CONSTRUCTION. Draw a Radius AS, and make the ∠ASB = the ∠ at the Center; draw AB, and it will be one of the Sides of the Polygon. Take AB in the Compaſſes, and ſet off that Diſtance from B to C, from C to D, &c. then will the Figure ABC, &c. be the required Polygon. This is manifeſt.

276. COROLLARY. *In a Hexagon, the Side of the Polygon is equal to the Radius,* viz. AB = AS; for in that Caſe the ∠s ASB, ABS, SAB, are each = 60°, and ∵ the △ ASB equilateral. *Note*, we ſhall defer ſaying more on the Conſtruction of Polygons till we treat of the Conſtruction and Uſe of an excellent Inſtrument called a *Sector*.

Pl. VII. Fig. 4. 277. PROBLEM 37. *From a given Line,* AB, *to cut off any Part required; for Example, a Fifth of* AB.

CONSTRUCTION. Draw an indefinite Line AC, making any ∠ with AB; on which ſet off any Diſtance A1, ſet off the ſame from 1 to 2, from 2 to 3, from 3 to 4, from 4 to C; then have we ſet off as many equal Parts on AC as AB is ſuppoſed to be divided into. Join B5, and ∥ to it, draw 1*a*, then will A*a* be $\frac{1}{5}$ Part of AB.

Demonstration. BC, *a*1, being ∥ to each other, as C1 : A1 [a] :: B*a* : A*a*, ∵ compoundedly C1 + A1 : A1 [b] :: B*a* + A*a* : A*a*, that is, CA : A1 :: BA : A*a*; conſequently, A*a* is the ſame Part of AB as A1 is of CA. Q. E. D.

278. Scholium. By this Problem a Line AB is divided into any Number of equal Parts. For Example Sake, A*a* being $\frac{1}{5}$ Part of AB, if we ſet the Diſtance A*a* from *a* to *b*, from *b* to *c*, and from *c* to *d*, the ſame would be = *d*B, and ſo the Line AB divided into five equal Parts. And after this Manner Scales may be conſtructed. But for this Purpoſe, *viz. to divide a given Line into any Number of equal Parts*, the following way is better adapted. For Example Sake, let it be required to divide the Line AB (ſee Plate VII. Fig. 20.) into five equal Parts.

Construction, From the Point A draw the indefinite Line AC, making any Angle at Pleaſure with the given Line AB; and ∥ to AC, draw the indefinite Line BD: Then taking any convenient Diſtance in your Compaſſes, ſet it from A to 1, from 1 to 2, 2 to 3, &c. alſo ſet the ſame Diſtance from B to *a*, *a* to *b*, *b* to *c*, &c. then the Line being to be divided into 5 equal Parts, join B5, and you cannot miſtake in joining the other correſpondent Points, *a*4, *b*3, *c*2, *d*1; and theſe Lines will divide the given Line AB into 5 equal Parts, *viz.* A*f* = *fg* = *gh* = *hi* = *i*B.

Demonstration. It is manifeſt from the Conſtruction, that the Lines *a*4, *b*3, &c. are all of them ∥ to B5, and to each other, and therefore the △s A1*f*, A2*g*, &c. are ſimilar to each other; ∵ as

[a] 191.

[b] 200.

A1 : Af :: A2 : Ag ; but if we denote Ai by m, and Af by n, A2 will be = 2m, and ſo the Analogy become $m : n :: 2m$: Ag ; but it is manifeſt Ag muſt be = 2n, for $m : n$ has the ſame Ratio as $2m : 2n$; ∵ Ag = 2n, and conſequently $fg = n$. Again, as A1 : Af :: A3 : Ah, that is, $m : n :: 3m$: Ah, conſequently, Ah muſt be = 3n, to preſerve the ſame Ratio : But Ag has been juſt ſhewn to be = 2n, ∵ gh (= Ah − Ag muſt be) = n alſo. After the ſame Manner hi and iB may be proved to be equal to n, and ſo Af, fg, hi, iB = one and the ſame Quantity, and conſequently equal to each other ; or, in other Words, the Line AB is divided into five equal Parts, in the Points f, g, h, i, and B. Q. E. D.

N. B. We ſhall treat of the Conſtruction and Uſe of diagonal Scales in our Eſſay on Trigonometry.

279. PROBLEM 38. *To find a third Proportional to two given Lines.*

Pl. VII. Fig. 5. CONSTRUCTION. Let AB, AC, be the two given Lines, ſo placed as to contain any Angle. Produce AC, AB, at Pleaſure ; in AB produced take BD = AC ; join BC, and thro' D draw DE || to it ; then will CE be the third Proportional required.

* 191. DEMONSTRATION. For as AB : AC* :: BD : CE ; but BD = AC by the Conſtruction, ∵ it is, as AB : AC :: AC : CE. Q. E. D.

Pl. VII. Fig. 6. 280. PROBLEM 39. *To find a fourth Proportional to three given Lines* A, B, C.

CONSTRUCTION. Draw two indefinite Lines, Da, Db, making any Angle. In Db take DG = A, and GE = B ; and in Da take DH = C ; join GH, and draw EF ∥ to it; then will HF be the fourth Proportional. Q. E. I.

DEMONSTRATION. For as DG : GE [a] :: DH : HF; or, which is the ſame, A : B :: C : HF. Q. E. D. [a] 191.

281. PROBLEM 40. *To find a mean Proportional to two given Lines,* a, b. Pl. VII. Fig. 7.

CONSTRUCTION. Draw an indefinite Line A*d*, in which take BD = *a*, DC = *b*, and upon BC deſcribe a Semicircle; and on the Point D raiſe the ⊥ DA, to cut the Circumference of the Semicircle in A, and it will be the mean Proportional required.

DEMONSTRATION. Join BA, CA, then it is manifeſt from Art. 210, that DA is a mean Proportional between BD and DC, or, which is the ſame, between the Lines *a* and *b*. Q. E. D.

N. B. This ſhews alſo how to make a Square = a given Rectangle. For if BD, DC, be the Sides of the Rectangle, DA is the Side of a Square = it: For by Art. 211, $BD \times DC = DA^2$.

282. PROBLEM 41. *To divide a Line* BC *according to extreme and mean Proportion.* Pl. VII. Fig. 8.

CONSTRUCTION. Draw BE ⊥ to BC and = $\frac{1}{2}$ BC; and about E as a Center, with the Radius EB, deſcribe a Circle, and thro' the Points C, E, draw the Line CA, cutting the Circumference of the Circle in the Points D and A; then make CF = CD, and the given Line BC will be divided by the Point F, into extreme and mean Proportion, as was required.

DEMONSTRATION. By Art. 213, $CA \times CD = BC^2$; ∵ BC : CA [a] :: CD : BC, whence inverſe- [a] 194.

ly,

[b] 198. ly, CA : BC [b] :: BC : CD; this dividedly is
[c] 201. CA — BC : BC [c] :: BC — CD : CD; which in-
[d] 198. verſely is, BC : CA — BC [d] :: CD : BC — CD. By the Conſtruction CF = CD, and DA, being a Diameter, is = twice BE, or = BC, by the Conſtruction; ∵ CA = CF + BC; hence, by writing CF for its equal CD, and CF + BC for its equal CA, in the laſt Analogy, we have BC : CF + BC — BC :: CF : BC — CF = BF, or, which is the ſame, (as + BC — BC deſtroy each other,) BC : CF :: CF : BF. Q. E. D. (See Art. 176.)

283. Problem 42. *To make a Square equal to any given right-line Figure.*

Construction. By Problem 18, find a Rectangle = the given Figure; then, by Problem 40, find a Square = that Rectangle.

284. Scholium. When the given Figure has not more than five Sides, perhaps the eaſieſt Way to find a Rectangle = the given Figure, is, firſt to find, by Problem 22, or 23, a △ = that Figure; and then a Rectangle = that △ may be found by Prob. 16.

285. Problem 43. *To find two Lines which ſhall have the ſame Ratio to each other as any two given rectilineal Figures.*

Construction. Take two Lines at Pleaſure = each other, and on them make Rectangles = the reſpective given Figures; then are the Rectangles
[a] 190. in the ſame [a] Ratio as their Baſes, (their Altitudes being equal by the Conſtruction;) conſequently, the given Figures muſt be in the ſame Ratio as the Baſes of the Rectangles.

286. **Problem** 44. *Upon a given Line* FG *to deſcribe a rectilineal Figure, ſimilar, and ſimilarly ſituated, to a given rectilineal Figure* ABCDE. Pl. VII. Fig. 9.

Construction. Join BE, BD, and make the $\angle GFK = \angle A$, and $\angle FGK = \angle ABE$; then make $\angle GKI = \angle BED$, and $\angle KGI = \angle EBD$. Again, make $\angle GIH = \angle BDC$, and $\angle IGH = \angle DBC$, and the Figure FGHIK will be that required. After the ſame Manner, by proceeding from $\triangle$ to $\triangle$, a Figure may be made ſimilar to another of any Number of Sides.

Demonstration. The $\angle GFK$ being $= \angle BAE$, and $\angle FGK = \angle ABE$ by the Conſtruction, the remaining $\angle FKG$ muſt be [a] $=$ the remaining $\angle AEB$, and $\because$ the $\triangle$s GFK, BAE, equiangular, and conſequently [b] ſimilar: After the ſame Manner the $\triangle$ GKI is ſhewn to be ſimilar to the $\triangle$ BED, and the $\triangle$GIH ſimilar to the $\triangle$BDC; conſequently the Polygons FGHIK, ABCDE, which are compoſed of the ſimilar $\triangle$s ſimilarly ſituated, muſt be ſimilar to each other. Q. E. D.

[a] 80.

[b] 194.

287. **Scholium.** When we treat of *Surveying*, we intend to give ſome practical Methods for ſolving this Problem in a more expeditious Manner.

BOOK VI.

Of the Application of Geometry to ſome practical Problems for meaſuring Heights, with the Addition of ſome pleaſant Problems and Paradoxes for the Amuſement of young Minds.

Pl. VII. Fig. 10. 288. Prob. I. *To find the Height of an acceſſible Object by Means of two Poles, of an unequal Length.*

Solution. Let AB repreſent the $\perp$ Object, whoſe Height is to be found, above the Plane DB. —— Let ED, the ſhorteſt Pole, be erected $\perp$ at any Point D, (which may be done by a Plumb Line, ſuch as Maſons commonly uſe in building Walls upright,) then, fixing your Eye at the upper End of it E, let another Perſon carry along the longeſt Pole, till you ſee its Top appear in a direct Line with A, the Top of the Object, where let it be ſtuck down; then meaſure the Diſtance DC, and CB. It is evident the $\triangle$s EFG, EAH, are ſimilar, and $\because$ we may ſtate by the Golden Rule, as EG (or DC) : FG (the Difference of the Lengths of the two Poles) :: EH (or DB) : HA. And AH + HB (the Height of the ſhorteſt Pole) gives BA the Height of the Object. Q. E. I.

Pl. VII. Fig. 11. 289. A Lemma. *If a Ray of Light*, EC, *falls on a reflecting Surface (as a Piece of Looking-Glaſs)* C, *and is reflected to the Eye at* A, *the Angles* ACB, ECD, *are equal to each other.*

This is demonſtrated by the Writers on Optics, but muſt here be taken for granted, till we treat of that Science.

290. PROBLEM 2. *To find the Height of an acceſſible Object by Means of any plain reflecting Surface, ſuch as a Looking Glaſs, or Bowl of Water.*

SOLUTION. Let ED repreſent the Height of the Object ⊥ to the Ground BD; at any Place on the Ground C, place the reflecting Surface; then go directly backward in the Line DC produced to B, till E the Top of the Object is ſeen by Reflection in the reflecting Surface at C, by the Eye at A; then meaſure the Height of the Eye BA, the Diſtances BC, and CD. Then the ∠ ACB being = ∠ ECD by the Lemma, and the Angles ABC, CDE, each Rt. ∠s, the remaining ∠ BAC = the remaining ∠ DEC, and ſo the △s ABC, EDC, equiangular, and ∵ ſimilar. Hence we may ſtate by the Golden Rule, as CB : BA :: CD : DE, the Height of the Object required.

N. B. In our *Eſſay on Decimal Arithmetick*, Art. 60, is given a Method of finding the Height of an Object by its Shadow. ——— We ſhall treat more fully, and illuſtrate Methods more accurate in Practice, after we have treated of the Elements of Trigonometry.

291. PROBLEM 3. *To deſcribe a Figure called an Oval.* Pl. VII. Fig. 12.

CONSTRUCTION. Draw a Line AB, about B, with any Radius BA ſweep a Circle AECK, with the ſame Extent of the Compaſſes about C, as a Center, ſweep a Circle BHDI; let E and F be the Points where the Circles interſect each other; join F, B, and produce FB to G; in like Manner draw FH, EK, and EI; then ſetting one Leg of the Compaſſes in F, with the Radius FG deſcribe the Arc GH; laſtly, one Point of the Compaſſes being placed in E, with the other deſcribe the Arc KI; then is the Figure AGHDIK called an *Oval.*

Pl. VII. Fig. 13. 292. Problem 4. *To deſcribe a Figure called by ſome an Helix, or Spiral Line.*

Conſtruction. Draw a Line AB, and about any Point a, with any Radius ac, ſweep the Semicircle ced; then take the Point b ſo that $bc = ba$, and about it, as a Center, with the Radius bd, deſcribe the Semicircle dgf; then about the Center a, with the Radius af, deſcribe a Semicircle fmn, and thus proceed at Pleaſure, making the Points a and b alternately the Center of the Semicircle to be deſcribed.

Pl. VII. F.14.&15. 293. Problem 5. Paradox 1. *Having a round Table, to make it into a ſquare Table, and have a Hole the Middle, without Waſte.*

Solution. Let the round Table be cut, as repreſented by the croſs Diameters in Figure 14, then place the four Parts together, as ſhewn in Figure 15, and the Thing is done.

294. Problem 6. Paradox 2. *Having a round Table, to change it into two Oval-like Tables, and have a Hole in the Middle of each, without Waſte of Timber.*

Pl. VII. F.16&17. Solution. Let the Table be cut into eight Pieces, as repreſented in Figure 16, then with the four Parts, 1, 2, 3, 4, may be made an Oval-like Table, as ſhewn in Figure 17; and with the other four Parts may be made another ſimilar and equal to Figure 17. Q. E. I.

295. Problem 7. Paradox 3. *A Gentleman having a Hole exactly a Foot ſquare, to try the Skill of his Joiner, has ordered him to fill up that Hole with a*

Piece of Board in Length 16 Inches, and Breadth 9 Inches, with a ſtrict Charge to cut the Piece into two Parts only: How may the Joiner perform his Taſk?

Solution. Let the Board, Figure 18, be cut as repreſented by the ſtrong Lines *ab*, *bc*, *cd*, *de*, *ef*; then put the two Parts A*abcdef*B and *a*CD*fedcb* together, as repreſented in Figure 19, and the Thing is done. Pl. VII. F.18.&19.

296. Problem 8, Paradox 4. *To deſcribe Circles of different Radii, with the ſame Extent of the Compaſſes.* Pl. VIII. Fig. 1.

Construction. With any Extent of the Compaſſes, and one Leg in A, with the other deſcribe a Circle: then, taking a little Solid, (ſuppoſe a Die,) reſting one Leg of the Compaſſes on it as a Center B, with the other deſcribe a Circle on the Paper, and it is manifeſt it will be of leſs Radius than the other.

297. Problem 9, Paradox 5. *To deſcribe an oval-like Figure at one Sweep of the Compaſſes.*

Construction. Take a round Solid, (ſuppoſe a round Ruler,) and on it roll a Piece of Paper; then, having opened the Compaſſes to any convenient Extent, fixing one Leg at any Point aſſumed on the Ruler, turn the other round as if you were to deſcribe a Circle: Then take off the Paper, and the Figure ſo deſcribed will repreſent a Kind of Oval, being longer one Way than the other.

298. Problem 10, Paradox 6. *To cut a Hole in a Card, large enough for a Man to paſs through.*

Solution. Let ABCD repreſent the Card. In, or near the Middle, cut a Slit, as repreſented by the Line *ma*, each End of it almoſt cloſe to the Edge Pl. VIII. Fig. 2.

of the Card; then cut, as ſhewn by the Line *a1*; again, from 2 to 3, 4 to 5, 6 to 7; and thus proceed until you have cut the Line *mn*, and one Half of the Card is finiſhed. This being done, and the other Part cut in the ſame Manner, on opening the Card, you will find a Hole ſufficiently large for a Man to paſs through. *N. B.* The nearer the Lines *a*1, 2, 3, &c. are to each other, the larger will the Hole be.

299. **Problem II.** *To deſcribe an Ellipſis of any given Length and Breadth.*

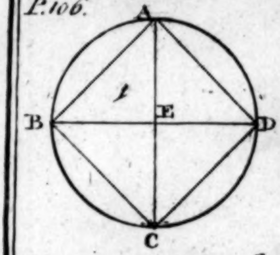
Pl. VIII. Fig. 3. Construction. Draw the Line AC = the given Length, and biſect it by the Perpendicular DB, making ED, EB, each $= \frac{1}{2}$ the given Breadth; then is AC called the *Tranſverſe Axis*, and BD the *Conjugate Axis*. Take EA or EC in the Compaſſes, and ſetting one Foot in B, with the other deſcribe two Arcs interſecting the Tranſverſe in the Points F and G; which Points are called the *Foci*. Then take two Pins, and placing one in the Point A, and the other in C, join them by a ſilk or other convenient String: This done, remove the Pins from the Points A and C to the Foci F and G; then take a Pencil in your Hand, and with it keep the String always extended, as repreſented at H; then if you begin at A and carry your Hand from A towards D, and from thence to C, the Point of the Pencil will deſcribe the Half of the Curve ADC: And in the ſame Manner, beginning again at A, proceed towards B, and from thence to C, and you will have deſcribed the other Half ABC; and the Figure ABCD ſo deſcribed is called an Ellipſis.

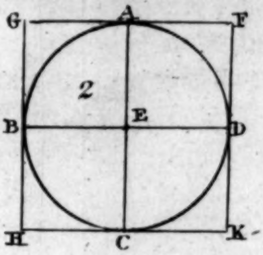
Otherwiſe without a String, thus.

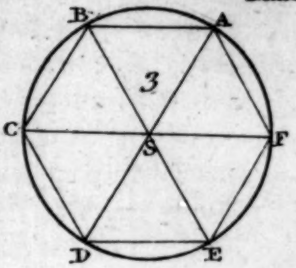
Open the Compaſſes to any Diſtance leſs than the tranſverſe Axis AC, and with it about the Focus F, as a Center, deſcribe the Arcs *ab*, *ef*; and with the ſame Extent of the Compaſſes about the Focus G, as a Center, the Arcs *ik*, *no*. This being done, ſet the

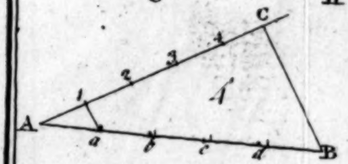
P. 106.

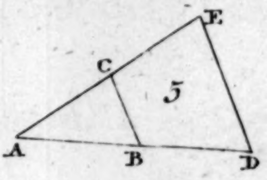
Plate. VII.

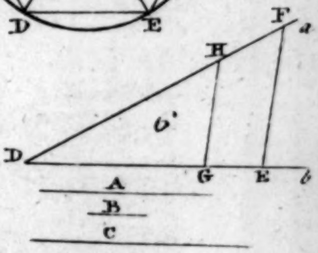


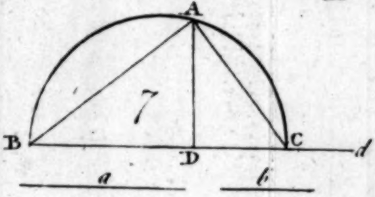


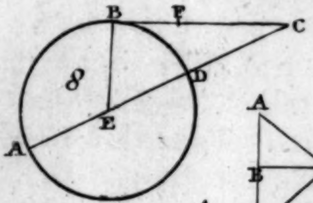


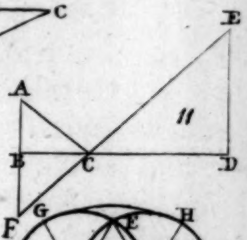


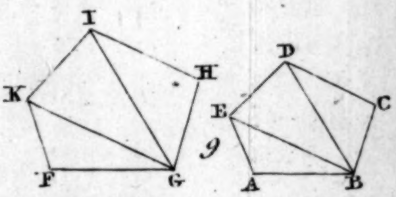


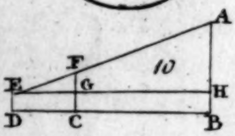


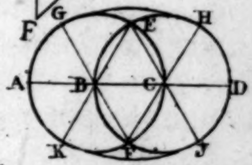


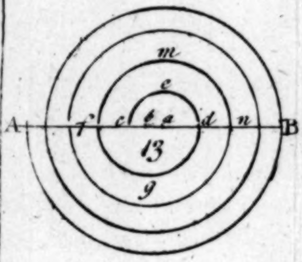


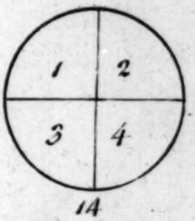












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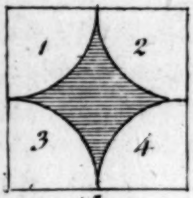

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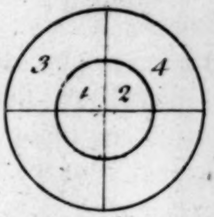

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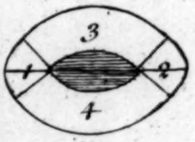

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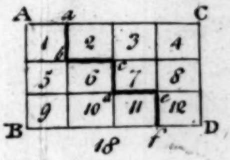

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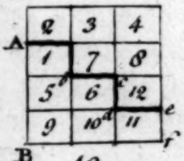

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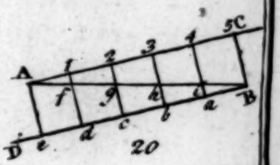

the Diſtance (to which the Compaſſes are opened) from A towards C on the tranſverſe Axis, *viz.* to I: Then, taking the remaining Part of the Tranſverſe IC in the Compaſſes, about G as a Center, deſcribe the Arcs *cd*, *gb*: and with the ſame Radius, about F as a Center, deſcribe the Arcs *lm*, *pq*; then will the Points of their Interſections *r*, *v*, *s*, *t*, be 4 Points in the Curve of the Ellipſis. After the ſame Manner, by taking different Radii at Pleaſure, as many Points may be found in the Curve as may be ſufficient to join by an even Hand, to form the required Ellipſis.

Scholium. This Figure is one of the Sections of a Cone, and therefore is intended to be more particularly treated of hereafter; but as it is in daily Uſe amongſt Joiners, Coopers, and other Mechanics, very few of whom can be ſuppoſed to ſtudy *Conic Sections*, we thought it would be of Service to ſome of them, to give the moſt eaſy Way of deſcribing it in this Place.

300. Problem 12. *To deſcribe a Figure reſembling a Hen's Egg.*

Pl. VIII. Fig. 4.

Construction. From a Scale of equal Parts ſet off 32 from A to G, and 43 from G to H. Make the ∠AGF and ∠DHC each = 40°. Produce FG until GI is = 21 (on the ſame Scale of equal Parts.) Then about G as a Center, with the Radius GA deſcribe the Arc AF; and about I as a Center, with the Radius IF, deſcribe an Arc FE until the Chord FE is = the Radius IF. Again, through the Point I draw the Line EK to interſect DH produced, in L: Then, about L as a Center, with the Radius LE deſcribe the Arc ED. Laſtly, about H as a Center, with the Radius HD deſcribe the Arc DC, and the Curve AFEDC will repreſent Half of the Figure required. The other Half may be thus deſcribed, *viz.* About G as a Center deſcribe the Arc AM = AF; and about H as a Center, the Arc CP = CD. Produce MG, and make MN = FI; then about N as a Center, deſcribe the Arc MB until its Chord MB is = the Radius NM. Produce BN, and make BO = EL; then about O as a Center deſcribe the Arc BP; and the Figure ABCEA will be a very natural Repreſentation of a Hen's Egg.

N. B. If the Numbers are ſet off from a Scale of 40 or 45 to an Inch, (in the Plate annexed we have uſed that of 45,) the Figure will be about the Size of a real Egg, that from which we deduced this Conſtruction being 42 to an Inch.

Scholium. Many more Problems might have been given in this Chapter, but as they are intended chiefly for the Amuſement of young Minds, perhaps theſe which have been already given, may by ſome Perſons be thought more than ſufficient.

A N

AN
APPENDIX,
CONTAINING
Some ADDITIONAL NOTES, *a* GEOMETRICAL SPECULUM, *and an* INDEX *to the whole Volume.*

301. A *Note* on the 6th, 7th, 8th, 9th, 10th, 11th, and 12th Articles.

In theſe Articles we have given the Definitions in the Order in which they ſtand in *Euclid,* becauſe we ſet out with a Reſolution to follow him as nearly as we conveniently could, conſiſtent with our Plan; though at the ſame Time we cannot but acknowledge, that as they have all their Exiſtence in a Solid, the moſt natural Way ſeemed to be, firſt to define a Solid, and then to ſhew how they exiſt in it; which will be extremely natural and eaſy, and take away the Objections commonly made againſt *Euclid*'s Definitions, by ſuch Perſons as are not uſed to Abſtraction: And ſince this Eſſay was ſent to the Preſs, we find Profeſſor *Simſon* has moſt elegantly illuſtrated this, in the 2d. Edition of his *Euclid,* and therefore, as we cannot give a better, take the Illuſtration in his own Words, *viz.*

" It is neceſſary to conſider a SOLID, *that is a Magnitude which has Length, Breadth, and Thickneſs,* in Order to underſtand aright the Definitions of a Point, Line, and Superficies; for theſe all ariſe from a Solid and exiſt in it. *The Boundary, or Boundaries, which contain a Solid, are called* SUPERFICIES, *or the Boundary which is common to two Solids which are contiguous or which divides one Solid into two contiguous Parts, is called a* SUPERFICIES. Thus, if BCGF be one of the Boundaries which contain the Solid ABCDEFGH, or which is the common Boundary of this Solid and the Solid BKLCENMG, and is therefore in the one as well as the other Solid, it is called a Superficies, and has no Thickneſs. For if it have any, this Thickneſs muſt either be a Part of the Solid AG, or of the Solid BM, or a Part of the Thickneſs of each of them. It cannot be a Part of the Thickneſs of the Solid BM, becauſe if this Solid be removed from the Solid AG, the Superficies BCGF, the Boundary of the Solid AG, remains ſtill the ſame as it was. Nor can it be a Part of the Thickneſs

Pl. VIII.
Fig. 5.

Thickneſs of the Solid AG, becauſe if this be removed from the Solid BM, the Superficies BCGF, the Boundary of the Solid BM, does nevertheleſs remain. Therefore the Superficies BCGF has no Thickneſs, but only Length and Breadth.

The Boundary of a Superficies is called a Line, *or a Line is the common Boundary of two Superficies that are contiguous, or which divides one Superficies into two contiguous Parts.* Thus, if BC be one of the Boundaries which contain the Superficies ABCD, or which is the common Boundary of this Superficies and of the Superficies KBCL, which is contiguous to it, this Boundary BC is called a Line, and has no Breadth. For if it have any, this muſt be Part either of the Breadth of the Superficies ABCD, or of the Superficies KBCL, or Part of each of them. It is not Part of the Breadth of the Superficies KBCL, for if this Superficies be removed from the Superficies ABCD, the Line BC, which is the Boundary of the Superficies ABCD, remains the ſame as it was. Nor can the Breadth that BC is ſuppoſed to have be a Part of the Breadth of the Superficies ABCD, becauſe if this be removed from the Superficies KBCL, the Line BC, which is the Boundary of the Superficies KBCL, does nevertheleſs remain. Therefore the Line BC has no Breadth. And becauſe the Line BC is in a Superficies, and that Superficies has no Thickneſs, as was ſhewn; therefore a Line has neither Breadth or Thickneſs, but only Length.

The Boundary of a Line is called a Point, *or a Point is the common Boundary or Extremity of two Lines that are contiguous.* Thus, if B be the Extremity of the Line AB, or the common Extremity of the two Lines AB, KB, this Extremity is called a Point, and has no Length. For if it have any, this Length muſt either be Part of the Length of the Line AB, or of the Line KB. It is not Part of the Length of KB, for if the Line KB be removed from AB, the Point B, which is the Extremity of the Line AB, remains the ſame as it was: Nor is it Part of the Length of the Line AB; for if AB be removed from the Line KB, the Point B, which is the Extremity of the Line KB, does nevertheleſs remain. Therefore the Point B has no Length. And becauſe a Point is in a Line, and a Line has neither Breadth nor Thickneſs, therefore a Point has no Length, Breadth, nor Thickneſs. And in this Manner the Definitions of a Point, Line, and Superficies are to be underſtood."

302. The 8th Definition, which we have omitted, is not only *uſeleſs*, but *obſcure*; and as Mr. *Simſon* juſtly obſerves, ſeems to be the Addition of ſome leſs ſkilful Editor.

303. (Art. 57.) Having by Experience found that in printing the common Characters for *greater* and *leſs*, they are frequently turned the wrong Way, and ſo when it ſhould be *greater* have *leſs*, and the contrary. In Order to remove the Confuſion occaſioned hereby, we have introduced the Greek G (Γ) for *greater than*, and the Greek L (Λ) for *leſs than*; which entirely prevent ſuch Accidents, and at the ſame Time carry with them the Ideas

we have Occaſion to expreſs, and therefore are no Burden to the Memory.

304. (Art. 94.) The laying of one Figure on another has been objected to as a mechanical Conſideration, depending on no Poſtulate: But for our Parts we muſt acknowledge that we ſee no Reaſon why this Method of Demonſtration ſhould not be uſed, as its Evidence cannot be denied by any Perſon: And as there is no Occaſion for the actual placing of one Figure on the other, but only to conceive them in the Mind to be ſo placed, it does not appear to us to be more mechanical than the conceiving of a Line to be drawn, or a Circle deſcribed. And *Euclid* and all other Geometricians make Uſe of it in demonſtrating our 58th Article, which indeed does not ſeem capable of any other Method. As to its being grounded on no Poſtulate, that Objection may be eaſily removed, by making a fifth Poſtulate (after Article 44.) viz. *Grant that one Figure may be laid on another*; and certainly no Perſon who grants the others, will deny this, it being as manifeſt as either of them. Thus much we thought ourſelves obliged to remark, as the Objection was made by a late learned Geometrician, for whoſe Memory we have the greateſt Eſteem.

305. (Art. 115.) As this Theorem is one of the moſt valuable in Geometry, it may perhaps be expected that we ſhould give an Example or two of its Uſe. By this it is that any two Sides of a right-angled Triangle being given, we find the other Side: This admits of two Caſes.

CASE I. *The two ſhort Sides being given, to find the longeſt.*

From the *Theorem* this *Rule* is eaſily deduced, *viz.* * The Square Root of the Sum of the Squares of the two ſhort Sides will be equal to the longeſt Side.

EXAMPLE. Let DC repreſent Half the Breadth of a Houſe, and be = 15 Feet, and CE the Height of the Roof = 10 Feet; it is required to find the Length of the Rafter DE. Pl. VIII. Fig. 10.

The Solution by the above Rule is as follows.

$$\begin{array}{rrrr}
15 & 10 & 225 & \dot{3}2\dot{5}(18 \\
15 & 10 & 100 & 1 \\ \hline
75 & CE^2 = 100 & DC^2 = 325 & 28)125 \\
15 & & & 224 \\ \hline
DE^2 = 225 & & & \\ \hline
\end{array}$$

Hence the Length of the Rafter is ſomething more than 18 Feet.

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* DEMONSTRATION. Let DC be expreſſed by b, CE by p, and DE by h, then by the Theorem, $b^2 + p^2 = h^2 \because$ extracting the Square Root of each Side of the Equation, we get $\sqrt{b^2 + p^2} = h$. Q. E. D.

After the ſame Manner the Side of a Square being given, its Diagonal may be found nearly true: We ſay nearly, becauſe it does not admit of an exact Solution; for if we ſuppoſe the Side = 1, the Sum of the Squares of both Sides will be = 2, the Square Root of which is 1.41 nearly, but cannot be found quite accurate, and therefore, though a finite Quantity, cannot be accurately expreſſed in Numbers. Such Quantities are called *incommenſurate.*

CASE 2. *When the longeſt Side and one of the ſhort Sides are given, to find the other we have this* RULE. From the Square of the longeſt Side ſubtract the Square of the given ſhort Side, and the Square Root of the Remainder will be the ſhort Side required.*

But by Article 121. this may be worded ſo as to be more ready in Practice, *viz.*

Multiply the Sum of the two given Sides by their Difference, and extract the Square Root of their Product.

EXAMPLE. A Ladder 50 Feet long reached the Top of a Houſe when its Foot was 30 Feet from the Bottom of the Wall; it is required to find the Height of the Houſe.

The Operation will ſtand thus, by the firſt Rule.

50	30	2500	1600(40
50	30	900	16
2500	900	1600	

By the other Rule thus.

50	80	1600(40
30	20	16
Sum 80	1600	

Difference 20

Anſwer, the Houſe is 40 Feet high.

306. (BOOK 2. Art. 117.) This Definition is according to *Euclid*, but every Perſon knows, that *two* right Lines cannot contain a Space, that is, cannot incloſe any Thing, as it at leaſt requires *three* right Lines for that Purpoſe; and therefore this muſt not be taken in a ſtrict Senſe: But when a Rectangle or right-angled Parallelogram is ſaid to be contained by any two Lines, *Euclid* muſt only be underſtood to denote a Rectangle made on one of the Lines, and having an Altitude equal to the other.

307.

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* DEMONSTRATION. By the Theorem $b^2+p^2=h^2$, $\therefore$ taking p^2 from both Sides of the Equation, we have $b^2=h^2-p^2$ $\therefore$ by extracting the Square Root we have $b=\sqrt{h^2-p^2}$. Q. E. D.

307. (Book 3. Art. 144.) This Remark is alſo ſupported by Mr. *Simſon* (who is undoubtedly one of rhe beſt Judges in theſe Matters) for in Page 301 of the ſecond Edition of his *Euclid* he has this Note, *viz.* " In this we have not followed the *Greek*, nor the *Latin* Tranſlation literally, but have given what is plainly the Meaning of the Propoſition, without mentioning the Angle of the Semicircle, or that which ſome call the cornicular Angle, which they conceive to be made by the Circumference and the ſtraight Line which is at right Angles to the Diameter, at its Extremity; which Angles have furniſhed Matter of great Debate between ſome of the modern Geometers, and given Occaſion of deducing ſtrange Conſequences from them, which are quite avoided by the Manner in which we have expreſſed the Propoſition. In like Manner we have given the true Meaning of Prop. 31. Book 3. without mentioning the Angles of the greater or leſſer Segments. Theſe Paſſages *Vieta* with good Reaſon ſuſpects to be adulterated, in the 386th Page of his *Oper. Math.*"

308. (Book 4, Definition 1.) We have here to the Word *Part*, as it is in *Euclid*, prefixed *aliquot*, as it ſeems to be wanting to expreſs *Euclid*'s Meaning; for a leſſer Magnitude may be Part of a greater, and yet not meaſure the greater; *e.g.* the Side of a Square is leſs than the Diagonal, and therefore from the Diagonal may be taken a Part = the Side; but this will not meaſure the Diagonal, being incommenſurate to each other by Article 305.

309. We have before obſerved in Scholium to Article 208, that *Euclid*'s Method of treating the Doctrine of Proportion is too obſcure for young Students, at their firſt Beginning to read theſe Subjects; and conſequently this is a ſufficient Reaſon (as we write for Learners) for attempting to demonſtrate the principal Theorems in another Manner. But as ſome may perhaps expect a Proof of our Aſſertion, that it is too difficult, it may be proper to obſerve, that this evidently appears from the great Diſputes which have been concerning the Principles on which the *Euclidian* Method is grounded amongſt the greateſt Geometricians: For if they are of ſo difficult a Nature that the learned themſelves cannot agree about them, it muſt follow they cannot be proper for initiating Youth, as Things cannot be well too plain for them. Mr. *Thomas Simpſon*, one of the beſt, as well as one of the lateſt Writers on Geometry, thus expreſſes himſelf on this Subject in Page 274, " That there is ſomething very ingenious and ſubtle in the Doctrine of Proportions as deliver'd in *Euclid*'s 5th Book, cannot be denied. All that I contend for is, that the Principles on which it is built are *obſcure*, and not ſo firmly eſtabliſhed as to authorize its Partiſans to aſſume that great Superiority they lay Claim to, in Point of geometrical Strictneſs." In Page 270 he thus remarks on the Definition which we have omitted (ſee our 158 Art.) *viz.* " I would ſeriouſly aſk the Contemners of *the vulgar and confuſed Notion* of

"Proportions, if a Definition, by which it cannot be known "whether the Ratio of the firſt to the ſecond of four given Magnitudes may not at the ſame Time be both greater and leſs "than that of the third to the fourth, is really calculated to "afford thoſe very accurate Ideas they pretend to?" And he even goes ſo far as to affirm that "it ſeems ſufficiently plain "from *Euclid*'s own Authority, that he himſelf was not entirely "pleaſed with his own Performance on this Head; or that he "was convinced (at leaſt) that it had not every Advantage: "For otherwiſe it will be very difficult to account for his having "demonſtrated many Things in his 7th Book, by another Method, whoſe Demonſtrations had been actually given before, in "the 5th, under a different Form. For theſe Reaſons, when I "ſee the extravagant Commendations that have been laviſhed on "this 5th Book of *Euclid*, I am no farther convinced by them "than that great Men may ſometimes launch out too far in Behalf of Opinions which they have adopted."

Thus much ſeemed neceſſary for us to extract in our own Vindication; but as we are determined not to enter into any Diſpute on this Subject, we ſhall proceed no further on this Head.

310. We ſhall conclude our Obſervations on this Book with the Theorem hinted at in the 116th Article, viz. *In any Triangle,* ABC, *(the Angle at the Vertex being the greateſt Angle, or at leaſt =*
Pl. VIII. *either of the Angles of the Triangle) make the Angle* ACD $=\angle$B,
F.6,7,&8. *and* $\angle$BCE $=\angle$A; *then will the Sum of the Squares on the two Sides containing the vertical Angle* C, *be equal to the Sum of the Rectangles contained by the whole Side ſubtending that Angle and Segments,* AD, BE, that is, $AC^2+BC^2=AD\times AB+BE\times AB$.

N. B. Fig. 6. repreſents the Caſe when the $\angle$C is obtuſe, Fig. 7. when acute, but greater than either of the others, that is, when it is acute, but greater than 60 Degrees. Fig. 8. when the Angle is a right Angle. Fig. 9. when that Angle is not greater, but equal to either of the other Angles, *viz.* $=\frac{1}{3}$ of 2 Right Angles, or 60 Degrees. Theſe Figures ſhew all the Varieties poſſible, and the Truth of the Theorem in all theſe Varieties will appear from one general DEMONSTRATION, *viz.* The $\triangle$s ABC, ACD, BCE, are ſimilar by the Conſtruction. $\therefore$ Comparing the Triangles ACD, ABC, together, we have $AC : AD :: AB : AC$, $\therefore AC^2 =$* $AD\times AB$.
* 193. Again, by the ſimilar $\triangle$s BCE, ABC, we have $BC : BE :: AB : BC$ $\therefore BC^2=BE\times AB$. $\therefore$ by Addition of Equals, $AC^2+BC^2=AD\times AB+BE\times AB$. Q. E. D.

311. COROLLARY 1. When the Angle C is a Rt. $\angle$, the
Fig. 8. Points D and E will coincide (as in Fig. 8.) and hence if on AB we make a Square, it is manifeſt that $AD\times AB+BE\times AB=$ the Rectangle AG + the Rectangle BG, or, which is the ſame, = the Square on AB, and $\therefore AC^2+BC^2=AB^2$; which is the famous *Pythagorean* Theorem in our 115 Article.

312. COROLLARY 2. When the $\angle$ C is $=60°$, the $\triangle$ is
Fig. 9. known to be equiangular and equilateral, and the Points A and E, B and D, coincide; and ſo $AD\times AB+BE\times AB$ is the ſame

ſame as $AB^2 + AB^2$, or twice the Square on AB. But this is alſo known to be true from other Principles; for as the three Sides are equal, it is manifeſt their Squares muſt be ſo too, and conſequently the Sum of any two of them muſt be = twice the other.

313. **Corollary** 3. From the firſt Corollary it evidently appears, that "*If a Line be divided into any two Parts, the Rectangles contained by the whole and each of the Parts are together = the Square of the Whole,* viz. $AG + BG = AH$, or $AB \times AD + AB \times BD = AB^2$. This is the 2d. of *Euclid*'s ſecond Book. Fig. 8.

314. **Corollary** 4 Hence alſo, *if there be two right Lines,* AB, AF, *and one of them,* AB, *is divided into two or more Parts, the Rectangle contained by the two Lines is = the Sum of the Rectangles contained by the undivided Line* AF, *and the ſeveral Parts of the divided Line* AB. Thus it is manifeſt from the Figure that the Rectangle AG + the Rectangle BG = Rectangle AH, or, which is the ſame, $AD \times AF + DB \times$ (BH, or its Equal) $AF = AB \times AF$.

This is the 1ſt. of *Euclid*'s ſecond Book.

315. **Book** 5. Art. 228. It ſometimes happens in Practice, that the Point C is ſo very near B, the End of the Line, that a ſufficient Diſtance cannot be ſet from C towards B, nor the Line by ſome Circumſtance admit of being produced longer; as for Inſtance, the Point C may be almoſt cloſe to the Edge of the Paper; and conſequently in ſuch Caſe *Euclid*'s Method is not eaſy in Practice: Therefore it may be neceſſary to give other Methods of Conſtruction peculiarly adapted to this **Problem**. *To draw a Perpendicular to a Line* AB *from any Point* C, *either in or very near the End* B *of that Line, ſo as to be impracticable by the* Euclidian *Method.* Pl. VIII. Fig. 10.

This may be done ſeveral Ways.

1ſt. From a Scale of equal Parts take off 3, and ſet it from C to D; then from the ſame Scale take off 4, and placing one Point of the Compaſſes in C, with the other deſcribe the Arc *ab*: Again, take off 5, and one Point of the Compaſſes being in D, with the other deſcribe the Arc *cd*, and through the Point of Interſection E draw a Line CF, and it will be the Perpendicular required. For if D, E, be joined, the △ DCE will be a Right ∠△, right-angled at C, by Art. 305.

N. B. Any other Numbers having the ſame Ratio to each other, will do the ſame Thing; as for Example, 6, 8, and 10; or 30, 40, and 50, &c.

Or thus.

Above the Line AB aſſume any Point D at Pleaſure, and about it as a Center, with the Radius DC, deſcribe a Circle (or a ſufficient Part of it,) and through the Point of Interſection E, and Center D, draw the Diameter EF; then through the Points C, F, draw the Line CG, and it will be the Perpendicular required. For the ∠ ECF in a Semicircle is a Right Angle by Art. 148. Pl. VIII. Fig. 11.

There are many other Methods, but theſe are ſufficient.

316. Before we conclude our Notes on this Book, it may be of Uſe to ſome Perſons *to ſhew one of the eaſieſt and beſt Methods of trying whether a Carpenter's Square is truly made*; *viz.* apply the Square to the ſtraight Edge of a Plank AB, as repreſented by
Pl. VIII. Fig. 12. the ſhaded Square, and by it draw the Line CD; then taking it off, turn it about, and apply it as ſhewn by the unſhaded Square, and if, when brought to the Point C, it agrees with the Line CD before drawn, the Square is true, if not, the Difference will be double the Error of the Square.

317. Book VI. Art. 289. It is a Maxim made Uſe of by ſome modern Philoſophers, that *Nature*, or rather the God of *Nature*, does every Thing in the *ſhorteſt Way*, and indeed it is a Principle which ſeems to follow from the very Idea of every Thing being ordered by *Infinite Wiſdom*, and as far as it has been applied, has always been found agreeable to the Nature of Things. From this Principle, by a fluxionary Proceſs, the late Writers on Optics ſhew *that the Angle of Incidence muſt be equal to the Angle of Reflection*, which was the Reaſon of our ſaying, "that it muſt be taken for granted until we treat of that "Science:" But recollecting that in reaſoning on *Hadley*'s Octant in 1752, we thought on a very eaſy geometrical Way of demonſtrating it, we ſhall here give it, with which we ſhall conclude our Notes.

Demonstration. Produce AB to F, making $BF = BA$, and join CF. By the Suppoſition $EC + CA$ is to be a *Minimum*. By the Conſtruction $BF = BA$, the $\angle$s ABC, FBC, each Rt. $\angle$s, and BC common, and therefore by Art. 58. the $\triangle$s are equal in all Reſpects, $\because$ $FC = CA$. Hence, when $EC + CA$ is a *Minimum*, its Equal $EC + FC$ muſt be ſo too; but it is evident that $EC + CF$ is a *Minimum* when EF is a Right Line, the neareſt Diſtance to any two Points being a right Line. But when EF is a Rt. Line, the $\angle ECD = \angle FCB$ by Art. 69. but the $\triangle$s ABC, FBC, have been ſhewn to be equal in all Reſpects, $\because$ $\angle FCB = \angle ACB$; $\because$ by Axiom 1, Art. 45, $\angle ECD = ACB$. Q. E. D.

Scholium. Hence it appears that Plane Geometry has in ſome Caſes the Advantage over Fluxions, as this is very conciſe and evident, but the fluxionary Proceſs ſomething tedious and affected with Surds.

As in this Eſſay, for Conveniency of Demonſtration, the Theorems (as in *Euclid*) relating to the ſame Subjects are ſome in one Place, ſome in another; and as they are in continual Uſe in almoſt every Branch of the Mathematics, we ſhall here collect them together in a ſhort View, or as it may be properly called, *A* Geometrical Speculum, which will be found of the greateſt Uſe to all young Beginners.

N. B. The Numbers prefixed refer to the Articles in which they are demonſtrated.

I. *Of the Properties of Lines interſecting or making Angles with each other.*

THEOREM 1.

64. A Line ſtanding on another Line making Angles with it, the Sum of thoſe Angles is equal to two right Angles.

2.

65. If one of the Angles (in the laſt Article) be a right one, the other is alſo a right Angle; but if one be acute, the other is obtuſe.

3.

66. If ſeveral Lines ſtand on the ſame Line at the ſame Point, the Angles which they make are together equal to two right Angles.

4.

67. Two Lines croſſing each other make four Angles, which taken together are equal to four right Angles.

5.

68. All the Angles made about a Point are = 4 Rt. ∠s.

6.

69. If two Lines interſect each other, the oppoſite ∠s are equal.

7.

289 If a Ray of Light, falling on a reflecting Surface, is reflected to the Eye, the ∠ of Incidence is = the Angle of Reflection. See Art. 317.

II. *Of Parallel Lines.*

1.

72. If a Line croſſes two Parallel Lines, the alternate ∠s are = each other.

2.

73. If a Line, falling upon two other Lines, makes the alternate ∠s equal to each other, the Lines will be parallel.

3.

74. If 2 Lines be each of them parallel to another Line, they are parallel to each other.

4.

75. If a Line is perpendicular to another Line, it is alſo perpendicular to all the ∥s of that other.

5.

96. Two ∥ Lines, though infinitely continued, could never meet.

III. *Of Rectangles of Lines, or Products of Quantities.*

1.

119. If a Line be divided into any two Parts, the Square on the whole Line is = the Squares on the two Parts + twice the Rectangle contained by the Parts.

2.

120. The Square on any Line is = 4 Times the Square on Half that Line.

3.

121. A Rectangle made by the Sum and Difference of two Lines is = the Difference of the Squares on the ſaid Lines.

4.

313. If a Line be divided into any two Parts, the Rectangles contained by the Whole and each of the Parts are together = the Square of the Whole, *viz.* Pl. VIII. Fig. 8. $AB \times AD + AB \times BD = AB^2$.

5.

314. If there be two Rt. Lines, AB, AF, and one of them, AB, is divided into two or more Parts, the Rectangle contained by the two Lines is = the Sum of the Rectangles contained by the whole Line AF and the ſeveral Parts of the divided Line AB. Thus, $AD \times AF + BD \times AF = AB \times AF$.

IV. *Of the Properties of* TRIANGLES *with Reſpect to their Angles and Sides.*

1.

59. The ∠s at the Baſe of an Iſoſceles △ and oppoſite to the equal Sides are = each other; and a Line biſecting the Angle oppoſite the Baſe divides the Baſe into two equal Parts, and is perpendicular thereto.

2.

60. Every equilateral △ is alſo equiangular.

3.

62. If 2 ∠s of a △ be equal, the Sides ſubtending the equal Angles will be equal.

4.

63. Every equiangular △ is alſo equilateral.

5.

70. Any two Sides of a △ are together greater than the third Side.

6.

76. If any Side of a △ be produced, the external ∠ is = the 2 internal oppoſite ∠s.

7.

77. The external ∠ is Γ either of the internal oppoſite ∠s.

8.

78. The 3 interior ∠s of every △ are = 2 Rt. ∠s.

9.

79. The 3 ∠s of every △ taken together are = the 3 ∠s of any other △ taken together.

10.

80. If in one △ 2 of its ∠s taken together are = 2 ∠s of another △ taken together, the remaining ∠ of one will be = the remaining ∠ of the other.

11.

81. If 2 △s have one ∠ of one = one ∠ of the other, the Sum of the remaining ∠s of one = the Sum of the 2 remaining ∠s of the other.

12.

82. If in any △ one of its ∠s is a Rt. ∠, the Sum of the other 2 is a Rt. ∠.

13.

83. If in a Rt. ∠ △ one of the acute ∠s is given, the other is = the Difference between the given ∠ and a Rt. ∠.

14.

84. When the Angle contained between the equal Sides of an Iſoſceles △ is a Rt. ∠, the other 2 ∠s are each half a Rt. ∠.

15.

85. An equilateral △ hath each ∠ = $\frac{2}{3}$ of 2 Right ∠s = 60 Degrees.

16.

86. If in any △ one of its ∠s is either a Rt. or an obtuſe ∠, the other two will be each of them acute.

17.

89. The greater Side of every △ is oppoſite to the greater ∠.

18.

90. In every △ the greater ∠ is oppoſite to the greater Side.

19.

91. If two △s have the ſame common Baſe, the 2 Sides of the inſcribed △ taken together are leſs than the two Sides of the circumſcribing △; but the vertical ∠ of the inſcribed greater than the vertical ∠ of the circumſcribing △.

20.

92. If 2 △s have 2 Sides of one = 2 Sides of the other, each to each, but the contained ∠ of one Γ the contained ∠ of the other, that which has the greateſt contained ∠, will have the greateſt Baſe.

21.

93. If 2 △s have 2 Sides of one = 2 Sides of the other, each to each, but the Baſe of one Γ the Baſe of the other; the ∠ contained by the equal Sides of that which has the greateſt Baſe ſhall be Γ the ∠ contained by the equal Sides of the other.

22.

96. In any Right ∠ △ the Square upon the Side ſubtending the Rt. ∠ is = both the Squares upon the Sides containing the Rt. ∠.

See another Demonſtration in Note to Art. 211.

23.

310. In any △ ABC (Pl. VIII. Fig. 6, 7, 8, and 9.) (the ∠ at the Vertex being the greateſt, or at leaſt = either of the Angles of the △) make the $\angle ACD = \angle B$, and $\angle BCE = \angle A$; then will the Sum of the Squares on the two Sides containing the vertical ∠ C, be = the Sum of the Rectangles contained by that whole Side ſubtending that ∠, and Segments AD, BE, that is, $AC^2 + BC^2 = AD \times AB + BE \times AB$.

N. B. The Theorems relating to the letting fall a Perpendicular from the vertical Angle to the Baſe of any Triangle, being chiefly of Uſe in Algebra, are referred to our intended Eſſay on that Art.

V. *Of the Equality of △s in every Reſpect*; viz. *when all their Angles and Sides are reſpectively equal to each other.*

1.

58. If 2 △s have 2 Sides of the one = the 2 Sides of the other, each to each, and the ∠s contained by thoſe equal Sides equal, the △s will be equal in all Reſpects.

2.

94. If 2 △s have two ∠s of one = 2 ∠s of the other, each to each, and one Side of one equal to one Side of the other, the △s are equal in all Reſpects.

3.

95. If 2 △s have the 3 Sides of one = the 3 Sides of the other, each to each, the △s will be equal in all Reſpects.

VI. *Of the Similarity of Triangles and Proportionality of their Sides*, &c.

1.

191. If a Line be drawn ∥ to one Side of a △, it ſhall cut the other Sides proportional. And if the Sides, or Sides produced, are cut proportionally, the Line which joins the Points of Section ſhall be parallel to the remaining Side of the △.

2.

192. If the vertical ∠ of a △ be divided into 2 equal ∠s by a Line cutting the Baſe, the Segments of the Baſe will have the ſame Ratio that the other two Sides of the △ have to one another. And if the Segments of the Baſe have the ſame Ratio which the other Sides of the △ have to one another, the Line drawn from the Vertex to the Point of cutting will divide the vertical Angle into 2 equal Parts.

3.

194. Equiangular △s are ſimilar, that is, the Sides about the equal ∠s are proportional.

4.

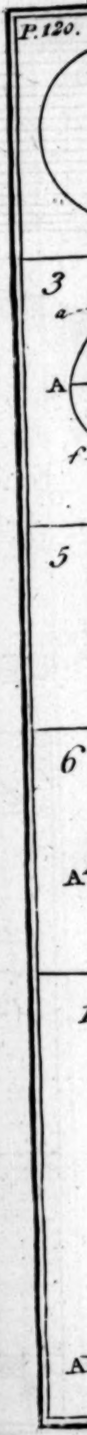

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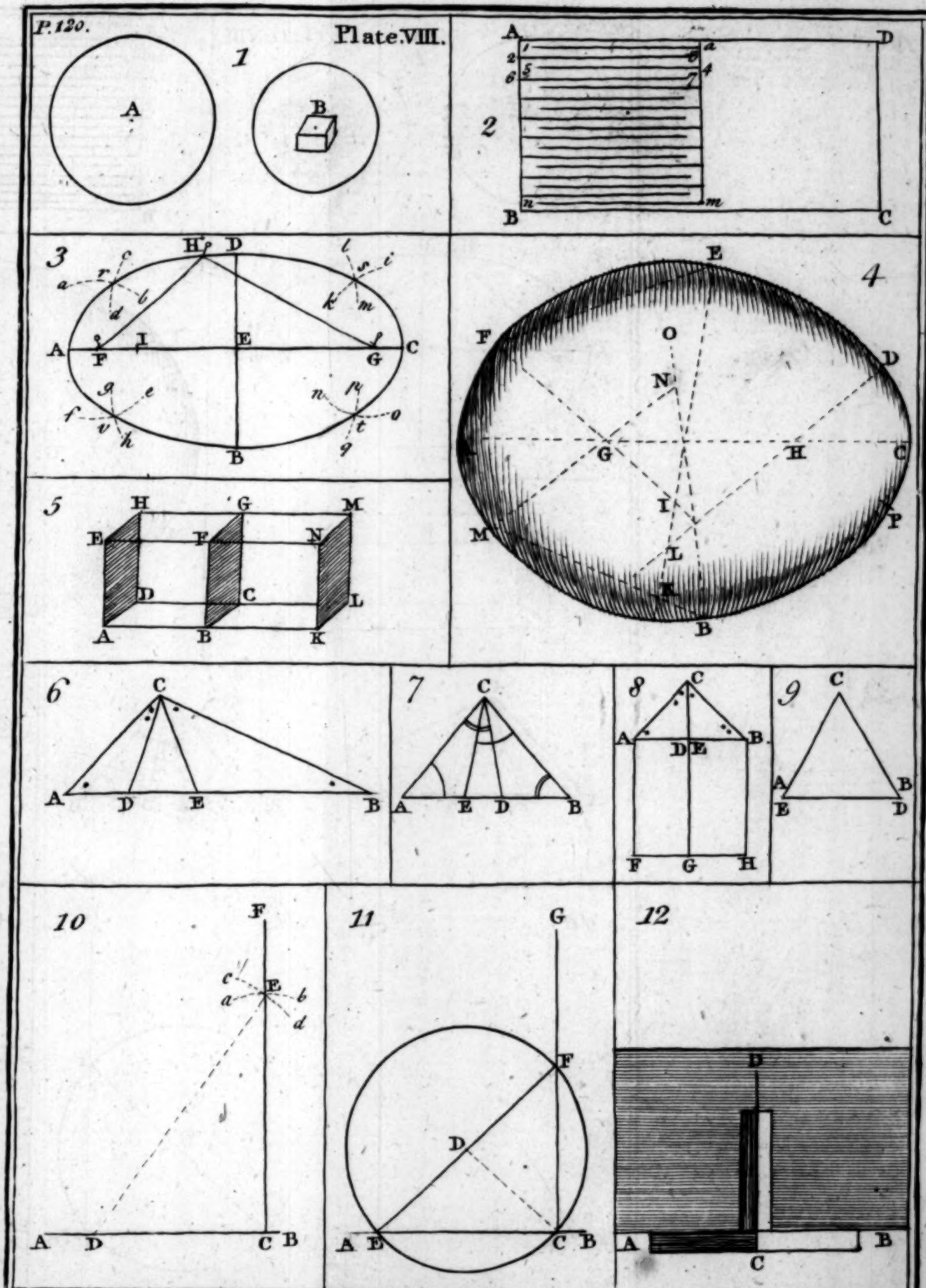
Plate. VIII.



4.

195. If the outward ∠ of a △ is biſected by a Line cutting the Baſe produced, then the Segments between the dividing Line and the Extremities of the Baſe have the ſame Ratio which the other Sides of the △ have to one another. And if the Segments of the Baſe produced have the ſame Ratio which the other Sides of the △ have, the Line drawn from the Vertex to the Point of Section divides the outward ∠ of the △ into 2 equal Parts.

5.

196. If 2 △s have one ∠ of one = one ∠ of the other, and the Sides containing the ∠s proportional, the △s are equiangular and ſimilar.

6.

197. If 2 △s have their like Sides proportional, the △s are ſimilar to each other.

7.

198. If in a Rt. ∠ △ a Perpendicular be let fall from the Rt. ∠ to the oppoſite Side, the △s on each Side of it ſhall be ſimilar to the whole △ and to each other.

VII. *Of Parallelograms, and the Compariſon of △s and Parallelograms, with Reſpect to their Areas, or Equality of Surface.*

1.

97. The oppoſite Sides and ∠s of Parallelograms are = each other ; and the Diameter (or Diagonal) biſects them; that is, divides them into 2 equal Parts.

2.

99. Figures which are between the ſame Parallels have equal Altitudes.

3.

100. Rectangles contained under equal Sides are equal.

4.

101. If 2 Squares have the Side of one = the Side of the other ; the Squares are = each other.

5.

102. Parallelograms upon the ſame Baſe, and between the ſame ||s are equal.

6.

103. Parallelograms having the ſame Baſe and equal Altitudes are equal.

7.

104. Every Parallelogram is = a Rectangle having the ſame Baſe and equal Altitudes.

8.

105. Parallelograms having equal Baſes and equal Altitudes are equal.

9.

106. If a △ and Parallelogram ſtand upon the ſame Baſe and are between the ſame Parallels, the △ is ½ of the Parallelogram.

10.

107. If a Parallelogram and △ have equal Baſes and are between the ſame ||s, or, which is the ſame Thing, have equal Altitudes, the △ is half of the Parallelogram.

11.

108. If a △ and Rectangle have equal Baſes and Altitudes, the △ is Half of the Rectangle.

12.

109. If 2 △s have equal Baſes and Altitudes, they are equal to each other.

13.

111. The Area of any Parallelogram is = the Product of the Number of Parts into which the Baſe is ſuppoſed to be divided by the Number of like Parts contained in its Altitude.

14.

113. The Complements of Parallelograms which are about the Diagonal of any Parallelogram are = each other.

15.

114. The Diagonals of any Parallelogram biſect each other.

16.

189. Triangles and Parallelograms of the ſame Altitude are to one another as their Baſes.

17.

190. Triangles and Parallelograms which have equal Altitudes are to one another as their Baſes.

VIII. *Of Polygons in general.*

1.

87. All the interior ∠s of any rectilineal Figure are together with 4 Rt. ∠s = twice as many Rt. ∠s as the Figure has Sides.

2.

88. All the exterior ∠s of any rectilineal Figure are together = 4 Rt. ∠.

3.

98. All quadrilateral Figures, whoſe oppoſite Sides are equal, have their oppoſite ∠s equal.

4.

114. The Diagonals of any Parallelogram biſect each other.

IX. *Of regular Polygons.*

1.

270. Lines biſecting the Angles of the Polygon will interſect each other in one Point within the Polygon, and may be called Radii, for they will be equal to each other.

2.

271. Lines drawn from the Center of a Polygon to the Extremities of each Side will divide the Polygon into as many Iſoſceles △s, equal to each other in all Reſpects, as the Figure has Sides.

3.

3.

272. Each ∠ at the Center is = 360 Degrees divided by the Number of Sides.

4.

273. The ∠ of the Polygon is = 180° — the ∠ at the Center. See a Table of ∠s in Art. 274.

5.

276. In a Hexagon the Side of the Polygon is = the Radius.

X. *Of the Properties of Circles.*

1.

134. If a Line drawn through the Center of a Circle biſect any Chord, it will cut it at Rt. ∠s : And if the Diameter is at Rt. ∠s to the Chord, it will biſect it.

2.

*134. If any Point which is not the Center be taken in the Diameter of a Circle, then of all the Lines that can be drawn from that Point to the Circumference, that in which the Center is, is the longeſt, and the other Part of the Diameter is the leaſt : And of the others, that which is nearer to that in which the Center is, is longer than one more diſtant.

3.

135. If Lines drawn from any Point without a Circle to the Circumference ; then, of thoſe which fall on the concave Part of the Circumference, the greateſt will be that which paſſes through the Center, and the nearer to it greater than the more remote : But of thoſe which fall on the convex Part of the Circumference, the leaſt is that which coincides with the Diameter produced, and the nearer to it is leſs than the more remote.

4.

136. If a Point be taken within a Circle from which there fall more than two equal Lines to the Circumference, that Point is the Center of the Circle.

5.

137. One Circle cannot cut another in more than two Points.

6.

139. Equal Lines in a Circle are equally diſtant from the Center : And thoſe equally diſtant from the Center are equal to each other.

7.

*139. The Diameter is the greateſt Line in a Circle ; and of any other, that which is nearer to the Center is always greater than that more remote.

8.

140. A Line greater than the Diameter drawn from any Point within the Circle muſt cut the Circumference.

9.

141. A Line meeting a Circle in a Point at Rt. ∠s to the Radius, is a Tangent to the Circle at that Point.

10.

142. If a Line touch a Circle in any Point, a Rt. Line drawn from the Center to that Point will be perpendicular to the touching Line.

11.

143. If a Line touches a Circle, and from the Point of Contact a Line be drawn perpendicular to the touching Line, the Center of the Circle ſhall be in that Line.

12.

145. The ∠ at the Center of a Circle is double of the ∠ at the Circumference upon the ſame Baſe; that is, upon the ſame Part of the Circumference.

13.

146. The ∠s in the ſame Segment of a Circle are = one another.

14.

147. The oppoſite ∠s of any quadrilateral Figure deſcribed in a Circle are together = 2 Rt. ∠s.

15.

148. The ∠ in a Semicircle is a Rt. ∠; but the ∠ in a Segment greater than a Semicircle is leſs than a Rt. ∠; and the ∠ in a Segment leſs than a Semicircle is greater than a Rt. ∠.

16.

149. In an oblique △ inſcribed in a Circle, the vertical ∠ exceeds, or is leſs than a Rt. ∠, by the ∠ made by the Baſe and the Diameter drawn from either Extremity of the Baſe.

17.

150. In equal Circles the Angles at the Circumference, and ſtanding upon equal Chords, are = each other: And if the Circumferences are equal, the Chords on which they ſtand will be equal alſo.

18.

151. If a Line touches a Circle, and from the Point of Contact a Line be drawn cutting the Circle, the ∠s made by this Line with the Line touching the Circle, ſhall be = the ∠s which are in the alternate Segment of the Circle.

19.

210. (See Pl. IV. Fig. 13.) 1ſt. As $AB : BC :: AD : AC$; 2d. As $BD : DA :: DA : DC$ (this is all we mean when we ſay AD is a geometric mean Proportional between BD and DC.

20.

211. Therefore 1ſt. $AB \times AC = BC \times AD$; 2d. $BD \times DC = DA^2$; 3d. $BC \times BD = AB^2$; 4th. $BC \times DC = AC^2$.

21.

212. If 2 Lines cut one another within a Circle, (as in Plate IV. Fig. 14.) the Rectangle of the Segments of one of them is = the Rectangle of the Segments of the other, *viz.* $AE \times EC = BE \times ED$. And if two Lines cutting a Circle, being produced, interſect each other without the Circle, (as in Pl. IV. Fig. 15.) then alſo will $AE \times EC = BE \times ED$.

22.

213. If from any Point C without a Circle, (Pl. IV. Fig. 16.) 2 Lines CA, CB, are drawn in ſuch a Manner that CB touches the Circle, and CA cuts it, then will $CA \times CD = BC^2$.

23.

214. In equal Circles, ∠s, whether at the Centers or Circumferences, have the ſame Ratio with the Circumferences on which they ſtand. Thus, let G and H (Pl. IV. F. 17.) be the Centers of the two Circles; then we affirm, that, as the Circumference BC : the Circumference EF :: the ∠ BGC : the ∠ EHF; and :: ∠ BAC : ∠ EDF.

24.

215. The ∠ BGC in the Center : 4 Rt. ∠s :: the Circumference or Arc BC on which it ſtands : the whole Circumference of the Circle.

25.

216. Hence the Circumferences DE, BC, (Pl. IV. Fig. 18.) of unequal Circles, which ſubtend equal Angles at the Centers, as ∠s DAE, BAC, are ſimilar; *viz.* As Arc DE : the whole Circumference of the Circle DEF :: BC : the Circumference of the Circle BCG. And if they are ſimilar, they will ſubtend equal Angles.

N. B. As to the Theorems relating to the Circumferences of Circles being as their Radii or Diameters. The Areas of ſimilar Plane Figures being as the Squares of their like Sides. &c. we intend to give them in a more proper Place, *viz.* when we treat on *Menſuration.*

XI. *Of Proportion.*

1.

187. Magnitudes have the ſame Ratio to one another which their Equimultiples have.

2.

188. If 4 Quantities are proportional, then any Equimultiples of the 2 firſt, will have the ſame Ratio as any Equimultiples of the two laſt.

3.

193. If 4 Lines are proportional, the Rectangle contained by the Extremes is = the Rectangle contained by the Means.

4.

198. If 4 Quantities are proportional, they will alſo be Proportionals taken inverſely, *viz.* if $a : b :: c : d$; then will $b : a :: d : c$.

5.

199, 200, 201, 202, 203. If 4 Quantities are proportional, they will alſo be proportional when taken alternately, compoundedly, by Diviſion, by Converſion, and mixtly, *viz.* if $a : b :: c : d$; then,

199. $a : c :: b : d$. Alternately.
200. $a + b : b :: c + d : d$. Compoundedly,
201. $a - b : b :: c - d : d$. By Diviſion,

202. $a : a - b :: c : c - d$. By Converſion.

203. $a + b : a - b :: c + d : c - d$. Mixtly.

6.

205. If 3 Lines are proportional, the Rectangle under the Extremes is = the Square on the Mean, *viz.* if $a : b :: b : c$, then $ac = b^2$.

7.

206. The Rectangles under proportional Lines are proportional. Thus, if $a : b :: c : d$, and $e : f :: g : h$; then $a \times e : b \times f :: c \times g : d \times h$.

8.

207. The Squares on 4 proportional Lines are Proportionals, *viz.* if $a : b :: c : d$; then will $a^2 : b^2 :: c^2 : d^2$.

9.

208. If 4 Squares are Proportionals, their Sides will be ſo too.

A LIST *of the* PROBLEMS.

PROBLEM 1.

111 *and* 112. To find the Areas of Parallelograms and Triangles.

2.

221. To draw a Line from a given Point = a given Line.

3.

222. From a greater Line to cut off a Part = a leſſer Line.

4.

223. About a given Point to deſcribe a Circle whoſe Radius ſhall be = a given Line.

5.

224. To make a △ whoſe Sides ſhall be reſpectively = three given Lines.

6.

225. To biſect a given ∠, that is, to divide it into two equal Parts.

7.

226. To biſect a given Line.

8.

227. The laſt Art. ſerves alſo to raiſe a Perpendicular on the Middle of a Line.

9.

228. To draw a Perpendicular to a given Line from any Point in it. See alſo Art. 315.

10.

229. To let fall a Perpendicular from a Point above to a Line below.

11.

230. To make an Angle = a given ∠.

12.

231. To conſtruct and ſhew the Uſe of a Protractor.

13.

234. To conſtruct and ſhew the Uſe of a Line of Chords.

14.

236. To draw a Line parallel to a given Line through a given Point.

15.

237. At any Diſtance to draw a Line || to another Line.

16.

239. To conſtruct a parallel Ruler.

17.

241. To triſect an Angle.

18.

242. To deſcribe a Parallelogram that ſhall be = a given △, and have one of its ∠s = a given ∠.

19.

243. To apply a Parallelogram to a given Line, which ſhall be = a given △, and have one of its ∠s = a given ∠.

20.

244. To deſcribe a Parallelogram = a given rectilineal Figure, and having an ∠ = a given rectilineal ∠.

21.

246. By this alſo may be deſcribed a Parallelogram = the Sum, or Difference, of any rwo given right-lined Figures.

22.

247. To deſcribe a Rectangle whoſe Sides ſhall be = two given Lines.

23.

248. Hence alſo to conſtruct a Square.

24.

249. To make a Square = any Number of given Squares.

25.

250. To make a Square = the Difference of two given Squares.

26.

251. To make a △ = any given quadrilateral Figure.

27.

252. To make a △ = any given 5-ſided Figure.

28.

253. To deſcribe a Circle through 3 given Points.

29.

254. To find the Center of a Circle.—— By having a Segment of a Circle to compleat the Circle. —— And to deſcribe a Circle about a given △.

30.

255. To draw a Tangent to a Circle from a given Point without it.

31.

256. To draw a Tangent from a given Point in the Circumference of the Circle.

32.

257. Upon a given Line to deſcribe a Segment of a Circle which ſhall contain a given Angle.

33.

259. To cut off a Segment from a given Circle which ſhall contain a given ∠.

34.

260. To inſcribe in a given Circle a Line = any given Line, leſs than the Diameter of the Circle.

35.

261. In a given Circle to inſcribe a △ equiangular to a given △.

36.

262. About a given Circle to deſcribe a △ equiangular to a given △.

37.

263. To inſcribe a Circle in a given △.

38.

264. To inſcribe a Square in a given Circle.

39.

265. To deſcribe a Square about a given Circle.

40.

266. To inſcribe a Circle in a given Square.

41.

267. To deſcribe a Circle about a given Square.

42.

275. In a given Circle to inſcribe a Polygon of a given Number of Sides.

43.

277. From a given Line to cut off any aliquot Part.

44.

278. To divide a given Line into any Number of equal Parts. Being of Uſe in making Scales of equal Parts.

45.

279. To find a third Proportional to two given Lines.

46.

280. To find a fourth Proportional to three given Lines.

47.

281. To find a mean Proportional to two given Lines.

48.

282. To divide a Line according to extreme and mean Proportion.

49.

283. To make a Square = any given right-line Figure.

50.

285. To find two Lines which ſhall have the ſame Ratio to each other as any two given rectilineal Figures.

51.

286. Upon a given Line to deſcribe a rectilineal Figure, ſimilar, and ſimilarly ſituated, to a given rectilineal Figure.

52.

288 To find the Height of an acceſſible Object by Means of two Poles of unequal Length.

53.

53.

290. To find the Height of an acceſſible Object by Means of any plane Reflecting Surface, as a Looking-Glaſs, or Bowl of Water.

54.

291. To deſcribe a Figure called an Oval.

55.

292. To deſcribe a Figure called by ſome an Helix, or Spiral Line.

56.

293. Having a round Table, to make it into a ſquare Table, and have a Hole in the Middle, without Waſte.

57.

294. Having a round Table to change it into two oval-like Tables, and have a Hole in the Middle of each, without Waſte of Timber.

58.

295. To fill a Hole exactly a Foot ſquare, with a Piece of Board in Length 16 Inches, and Breadth 9 Inches, by only cutting it into two Parts.

59.

296. To deſcribe Circles of different Radii with the ſame Extent of the Compaſſes.

60.

297. To deſcribe an Oval-like Figure at one Sweep of the Compaſſes.

61.

298. To cut a Hole in a Card large enough for a Man to paſs through.

62.

299. To deſcribe an Ellipſis of any given Length and Breadth.

63.

300. To deſcribe a Figure reſembling a Hen's Egg.

64.

305. Any two Sides of a right-angled △ being given, to find the other Side.

As many valuable Authors in their Works frequently refer to ſome Propoſitions of *Euclid*, we have added the following Tables, to ſhew by Inſpection what Articles in this Eſſay anſwer to the moſt uſeful Theorems in that Author, and the contrary.

A TABLE ſhewing by Inſpection what Article in this ESSAY anſwers to its correſponding Propoſition in *Euclid*.

Propoſitions in *Euclid.*	Articles in this Eſſay.	Propoſitions in *Euclid.*	Articles in this Eſſay.	Propoſitions in *Euclid.*	Articles in this Eſſay.	Propoſitions in *Euclid.*	Articles in this Eſſay.
Book I. Prop.		Book II Prop.		Bo. III. Prop.		Bo. VI. Prop.	
2	221	1	314	1	260	1	189
3	222	2	313	2	261	2	191
4	58	4	119	3	262	3	192
6	62	5	121	4	263	4	194
8	95	11	282	5	254	5	197
9	225	14	283	6	264	6	196
10	226			7	265	8	209
11	228	Bo. III. Prop.		8	266	9	277
12	229			9	267	10	278
13	64	2	254	10 11 12 13 14 15 16	relate to deſcribing Polygons ſee Article 275.	11	279
15	69	3	134			12	280
18	89	7	*134			13	281
19	90	8	135			16	193
20	70	9	136			17	205
21	91	10	137			18	286
22	224	14	*138			33	214
23	230	15	139				
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25	93	17	255 256	Book V Prop.			
26	94						
27	73	18	142	4	188		
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30	74	20	145	16	199		
31	236	21	146	17	201		
32	76	22	147	18	200		
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1C.32	87	30	225				
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34	97	32	151				
35	102	33	257				
36	105	34	259				
41	106	35	212				
42	242	36	213				
43	113						
44	243						
45	244						
46	248						
47	115						

A TABLE ſhewing what Propoſition in *Euclid* anſwers to its correſponding Article in this ESSAY.

Art. in this Eſſay.	Propoſ. in *Euclid*.		Art. in this Eſſay.	Propoſ. in *Euclid*.		Art. in this Eſſay.	Propoſ. in *Eulid*.		Art. in this Eſſay.	Propoſ. in *Euclid*.	
	Prop.	Book		Prop.	Book		Prop.	Book		Prop.	Book
58	4.	1	119	4.	2	199	16.	5	261	2.	4
59	5.	1	121	5.	2	200	18.	5	262	3.	4
62	6.	1	134	3.	3	201	17.	5	263	4.	4
64	13.	1	*134	7.	3	205	17.	6	264	6.	4
69	15.	1	135	8.	3	209	8.	6	265	7.	4
70	20.	1	136	9.	3	212	35.	3	266	8.	4
72	29.	1	137	10.	3	213	36.	3	267	9.	4
73	27.	1	*138	14.	3	214	33.	6	275 ſhews how to deſcribe Polygons, which *Euclid* does in 10, 11, 12, 13, 14, 15, 16, of 4th Book, but in a Method entirely different.		
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76	32.	1	141	16.	3	222	3.	1			
78	38.	1	142	18.	3	224	22.	1			
87	1C.32.1.		143	19.	3	225	9.	1			
88	2C.32.1.		145	20.	3	226	10.	1			
89	18.	1	146	21.	3	228	11.	1			
90	19.	1	147	22.	3	229	12.	1			
91	21.	1	148	31.	3	230	23.	1			
92	24.	1	151	32.	3	236	31.	1			
93	25.	1	187	15.	5	242	42.	1	277	9.	6
94	26.	1	188	4.	5	243	44.	1	278	10.	6
95	8.	1	189	1.	6	244	45.	1	279	11.	6
97	34.	1	191	2.	6	248	46.	1	280	12.	6
102	35.	1	192	3.	6	254	2.	3	281	13.	6
105	36.	1	193	16.	6		5.	4	282	11.	6
106	41.	1	194	4.	6	255 256	17.	3	283	14.	2
113	43.	1	196	6.	6				286	18.	6
115	47.	1	197	5.	6	257	33.	3	313	2.	2
						259	34.	3	314	1.	2
						260	1.	4			

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5. —————— Geometry.

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